NEURČITÝ INTEGRÁL

Katedra matematiky a teoretickej informatiky,
Technická univerzita v Košiciach

Definícia

Funkcia F sa nazýva primitívna funkcia k funkcii f na intervale I, ak pre všetky $x \in I$ je $F'(x) = f(x)$.

Veta

Nech funkcia F je primitívna funkcia k funkcii f na intervale I a $c \in \mathbb{R}$, potom aj funkcia $G(x) = F(x) + c$ je primitívnou funkciou k funkcii f na intervale I.

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Nech funkcia f je spojitá na intervale I, potom k nej na intervale I existuje primitívna funkcia F.

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Množinu $\{F + c,\ c \in \mathbb{R}\}$ *všetkých primitívnych funkcií k funkcii f nazývame neurčitý integrál. Píšeme*

$$\int f(x)\,dx = F(x) + c$$

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Veta (o lineárnosti neurčitého inegrálu)

Nech k funkciám f a g existujú primitívne funkcie na intervale I, nech $a, b \in R$. *Potom existuje primitívna funkcia k funkcii* $af + bg$ *na intervale I a platí*

$$\int (af(x) + bg(x))\,\mathrm{d}x = a\int f(x)\,\mathrm{d}x + b\int g(x)\,\mathrm{d}x.$$

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$$\int (af(x) + bg(x))\, \mathrm{d}x = a \int f(x)\, \mathrm{d}x + b \int g(x)\, \mathrm{d}x.$$

- $\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + c$
- $\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + c$
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Veta

Nech $\varphi : (a, b) \to (\alpha, \beta)$ je spojite diferencovateľná funkcia. Nech $F(t)$ je primitívna funkcia k funkcii $f(t)$ na (α, β) Potom funkcia $F(\varphi(x))$ je primitívna k funkcii $f[\varphi(x)]\varphi'(x)$ na (a, b).

$$\int f[\varphi(x)]\varphi'(x)\,\mathrm{d}x = \left| \begin{array}{c} \varphi(x) = t \\ \varphi'(x)\,\mathrm{d}x = \mathrm{d}t \end{array} \right| = \int f(t)\,\mathrm{d}t.$$

- $\int \frac{f'(x)}{f(x)}\,\mathrm{d}x = \ln|f(x)| + c$