Derivácia funkcie

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PRÍKLADY

$c' = 0$

$(x^n)' = n \cdot x^{n-1}$

1) $f(x) = 5$

$f'(x) = 0$

2) $f(x) = 5 \cdot x^2$

$f'(x) = (5x^2)' = 5 \cdot (x^2)' = 5 \cdot 2 \cdot x^{2-1} = 10x$

3) $f(x) = \sqrt{x} + 3x + 4 = x^{\frac{1}{2}} + 3x + 4$

$f'(x) = \left(x^{\frac{1}{2}} + 3x + 4\right)' = \left(x^{\frac{1}{2}}\right)' + (3x)' + 4' =$

$= \left(x^{\frac{1}{2}}\right)' + 3(x)' + 4' =$

$x^0 = 1$

$(x)' = 1$

$= \frac{1}{2} x^{\frac{1}{2}-1} + 3 \cdot 1 \cdot x^{1-1=0} + 0 =$

$= \frac{1}{2} x^{-\frac{1}{2}} + 3 = \frac{1}{2\sqrt{x}} + 3$

4) $f(x) = x^2 \cdot \sin x$

$f'(x) = (x^2)' \cdot \sin x + x^2 \cdot (\sin x)' =$

$= 2x \cdot \sin x + x^2 \cdot \cos x$

5) $f(x) = \frac{\sin x}{\cos x}$ $(= \operatorname{tg} x)$

$f'(x) = \frac{(\sin x)' \cos x - \sin x (\cos x)'}{\cos^2 x} = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = (\operatorname{tg} x)'$

1) $f(x) = 3x^2 + \sqrt{x\sqrt{x}} + 8 + \frac{4}{\sqrt[3]{x}} = 3x^2 + \left(x \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}} + 8 + \frac{4}{x^{\frac{1}{3}}} =$

$= 3x^2 + \left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}} + 8 + 4 \cdot x^{-\frac{1}{3}} =$

$= 3x^2 + x^{\frac{3}{2} \cdot \frac{1}{2}} + 8 + 4 \cdot x^{-\frac{1}{3}}$

$x^a \cdot x^b = x^{a+b}$

$(x^a)^b = x^{a \cdot b}$

$\frac{1}{x} = x^{-1}$

$$= 3x^2 + x^{\left(\frac{3}{2}\cdot\frac{1}{2}\right)} + 8 + 4\cdot x^{-\frac{1}{3}}$$

$$\frac{1}{x} = x^{-1} \qquad \frac{1}{x^a} = x^{-a} \qquad x^{5/9} = \sqrt[9]{x^5}$$

$$f'(x) = 3\cdot 2\cdot x^1 + \frac{3}{4}\cdot x^{\frac{3}{4}-1} + 0 + 4\cdot\left(-\frac{1}{3}\right)\cdot x^{-\frac{1}{3}-1}$$

$$= 6x + \frac{3}{4}x^{-\frac{1}{4}} - \frac{4}{3}x^{-\frac{4}{3}} =$$

$$6x + \frac{3}{4\sqrt[4]{x}} - \frac{4}{3\sqrt[3]{x^4}}$$

4) $f(x) = \sqrt{4-x^2} = (4-x^2)^{\boxed{\frac{1}{2}}}$

$$\boxed{[f(g(x))]'} = \underbrace{f'(g(x))}_{*}\cdot g'(x)$$

$$x \xrightarrow{g} 4-x^2 = t \xrightarrow{f} \sqrt{t} = \sqrt{4-x^2}$$

$$\neq f'(g'(x))\ !$$

$$f'(x) = \frac{1}{2}(4-x^2)^{\frac{1}{2}-1}\cdot(4-x^2)' = \frac{1}{2}(4-x^2)^{-\frac{1}{2}}\cdot(0-2x)$$

$$= \frac{1}{\cancel{2}\sqrt{4-x^2}}\cdot(-\cancel{2}x) = \frac{-x}{\sqrt{4-x^2}}\ \checkmark$$

PR. $f(x) = e^{4-x^2}$ $\qquad (e^x)' = e^x$

$$f'(x) = \underbrace{e^{4-x^2}}_{f'(g(x))}\cdot\underbrace{(4-x^2)'}_{g'(x)} = -2x\cdot e^{4-x^2}$$

$$(\ln x)' = \frac{1}{x}$$

5) $f(x) = \ln\left(\sqrt{4-x^2}\right)$

$$f'(x) = \underbrace{\frac{1}{\sqrt{4-x^2}}}_{f'(g(x))}\cdot\left(\sqrt{4-x^2}\right)' = \frac{1}{\sqrt{4-x^2}}\cdot\frac{-x}{\sqrt{4-x^2}} =$$

$$= \frac{-x}{4-x^2}$$

$$\log_a x^n = n\log_a x$$

INAK $f(x) = \ln(4-x^2)^{\frac{1}{2}} = \frac{1}{2}\ln(4-x^2)$

$$f'(x) = \frac{1}{2}\,\frac{1}{4-x^2}\cdot(4-x^2)' = \frac{1}{\cancel{2}}\,\frac{1}{4-x^2}(-\cancel{2}x)$$

$$= \frac{-x}{4-x^2}$$

6) $f(x) = \sqrt{\ln(4-x^2)} = \left(\ln(4-x^2)\right)^{\frac{1}{2}}$

$4-x^2$

$$f'(x) = \frac{1}{2}\left(\ln(4-x^2)\right)^{\frac{1}{2}-1} \cdot \left(\ln(4-x^2)\right)' =$$

$$= \frac{1}{\cancel{2}}\left(\ln(4-x^2)\right)^{-\frac{1}{2}} \cdot \frac{-\cancel{2}x}{4-x^2} = \frac{-x}{(4-x^2)\sqrt{\ln(4-x^2)}}$$

7) $f(x) = x^3 + 3^x + \boxed{x^x}$

$(x^3)' = 3x^2$

$(3^x)' = 3^x \cdot \ln 3$

$(a^x)' = a^x \cdot \ln a$

$(x^n)' = n \cdot x^{n-1}$

LOGARITMICKÁ DERIVÁCIA

$\left(f(x)^{g(x)}\right)'$

$$x^x = e^{\ln x^x} = e^{x \cdot \ln x}$$

$$(x^x)' = \left(e^{x \cdot \ln x}\right)' = e^{x \cdot \ln x} \cdot \left[x \cdot \ln x\right]' =$$

$$= e^{x \cdot \ln x}\left[1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}}\right] = \underbrace{e^{x \ln x}}\left[\ln x + 1\right]$$

$$= x^x(\ln x + 1)$$

DERIVÁCIA VYŠŠÍCH RÁDOV

PR: $f(x) = e^{x^2-2x}$

$$f'(x) = e^{x^2-2x} \cdot (x^2-2x)' = e^{x^2-2x}(2x-2) =$$

$$= (2x-2) \cdot e^{x^2-2x}$$

$$f''(x) = \left(f'(x)\right)' = \left[(2x-2) \cdot e^{x^2-2x}\right]' =$$

súčin

$$= (2x-2)' e^{x^2-2x} + (2x-2)\left(e^{x^2-2x}\right)'$$

súčin

$$= (2\cdot 1-0)\cdot e^{x^2-2x} + (2x-2)\cdot\left(e^{x^2-2x}\right)' =$$

$$= 2e^{x^2-2x} + \underbrace{(2x-2)(2x-2)}\cdot e^{x^2-2x} =$$

$$= \left[2 + (2x-2)^2\right]\cdot e^{x^2-2x}$$

PR:

$$f(x) = \sin x$$

$$f'(x) = \cos x = f^{V}(x) = f^{(4k+1)}(x)$$

$$f''(x) = (\cos x)' = -\sin x = f^{VI}(x) = f^{(4k+2)}(x)$$

$$f'''(x) = (-\sin x)' = -\cos x = f^{VII}(x) = f^{(4k+3)}(x)$$

$$f^{IV}(x) = (-\cos x)' = \sin x = f^{VIII}(x) = f^{(4k)}(x) \qquad k = 0,1,2,\ldots$$