Limita funkcie

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$\lim\limits_{x\to 1=a} \sqrt{(x-2)}$ $\quad (x-2) < 0$

$D(f): x-2 \geq 0 \iff x \geq 2$

$D(f) = \langle 2, \infty)$ $\quad a = 1 \notin D(f)$

$O_{\frac{1}{10}}(1) = (1-\frac{1}{10}, 1+\frac{1}{10}) =$
$= (0{,}9 ; 1{,}1)$

okolie bodu 1 o polomere $\varepsilon = \frac{1}{10}$

$1 \in O_{\frac{1}{10}}(1) = (0{,}9 ; 1{,}1)$

0,9 1 1,1

$\mathring{O}_{\frac{1}{10}}(1)$ 0,9 1 1,1

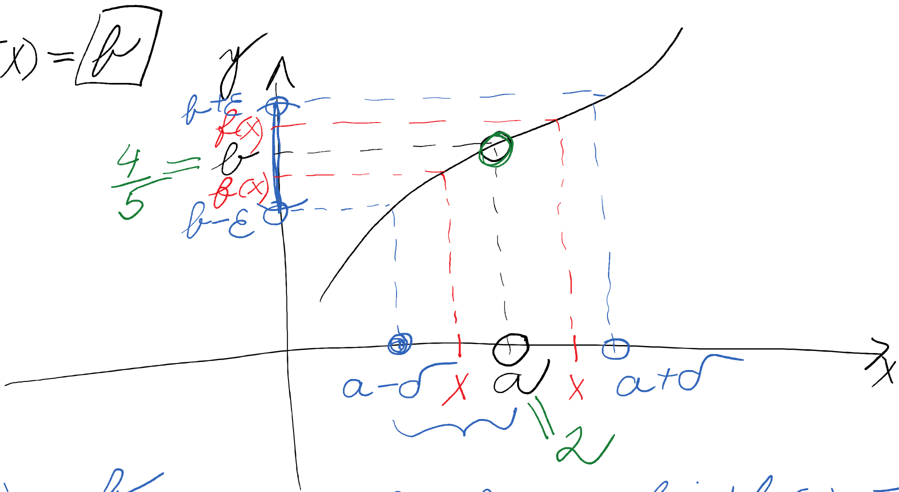
prstencové okolie bodu 1 o polomere $\varepsilon = \frac{1}{10}$

$1 \notin \mathring{O}_{\frac{1}{10}}(1) = (0{,}9 ; 1) \cup (1 ; 1{,}1)$

$A = D(f) = \langle 2, \infty)$

$a = 1$ nie je hromadný bod A (existuje okolie bodu a, že v ňom neleží žiaden bod z $D(f)$)

$\lim\limits_{x\to \boxed{a}} f(x) = \boxed{b}$



limita

zľava $\lim\limits_{x\to a^-} f(x) = b$

sprava $\lim\limits_{x\to a^+} f(x) = b$

$\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a} f(x)$

(existencia limity v bode a)

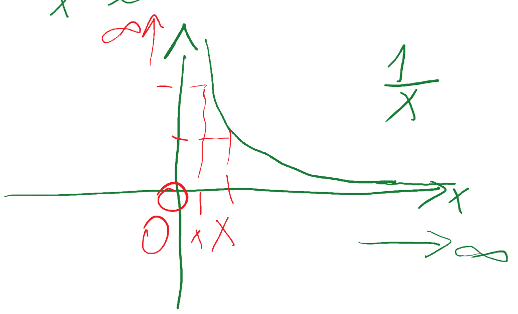
NEURČITÉ VÝRAZY:

PR1: $\lim\limits_{x\to 2} \frac{x^2-4}{x^2-5x+6} \overset{\boxed{\frac{0}{0}}}{=} \lim\limits_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+3)} = \frac{2+2}{2+3} = \frac{4}{5}$

$\lim\limits_{x\to\infty} x = \infty$

$\lim\limits_{x\to\infty} \frac{1}{x} = 0$

$$\lim_{x\to\infty}\frac{1}{x} = \underline{\underline{0}}$$



1.) $\lim_{x\to 0^+} x = 0$ $\quad x>0$ $\qquad$ 2.) $\lim_{x\to 0^-} x = 0$ $\quad x<0$

$\lim_{x\to 0^+}\frac{1}{x} = \infty$ $\qquad$ $\lim_{x\to 0^-}\frac{1}{x} = -\infty$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1-\cos^2 x$

PR2 $$\lim_{x\to 0}\frac{\sin x\,\mathrm{tg}\,x}{1-\cos^2 x}\ \boxed{\frac{0}{0}} = \lim_{x\to 0}\frac{\cancel{\sin x}\cdot\frac{\cancel{\sin x}}{\cos x}}{\cancel{\sin^2 x}} = \lim_{x\to 0}\frac{1}{\cos x} = \underline{\underline{1}}$$

PR3 $$\lim_{x\to 0}\frac{\sin 3x\cdot\frac{1}{x}}{\sin 2x\cdot\frac{1}{x}}\ \boxed{\frac{0}{0}}\ \lim_{x\to 0}\frac{\frac{3}{3}\,\frac{\sin 3x}{x}}{\frac{2}{2}\,\frac{\sin 2x}{x}} =$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$$

$$= \lim_{\boxed{x\to 0}}\frac{3\,\frac{\sin 3x}{3x}\to 1}{2\,\frac{\sin 2x}{2x}\to 1} = \underline{\underline{\frac{3}{2}}}$$

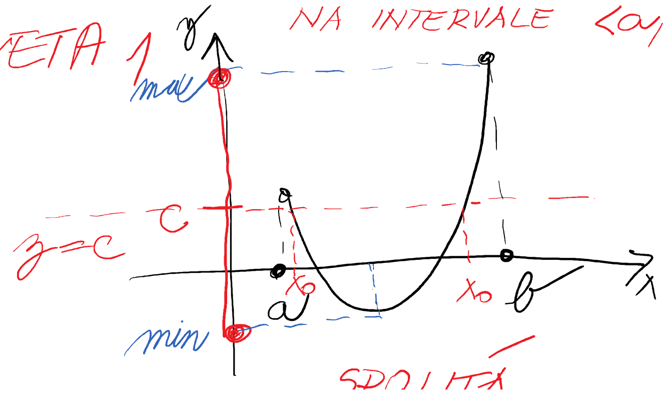
PR4 $$\lim_{x\to\infty}\frac{\boxed{5}x^2+3x-4}{\boxed{2}\boxed{x^2}-x}\ \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty}\frac{\cancel{x^2}\left(5+\frac{3}{x}\to 0-\frac{4}{x^2}\to 0\right)}{\cancel{x^2}\left(2-\frac{1}{x}\to 0\right)} = \frac{5}{2}$$

PR5 $$\lim_{x\to\infty}\frac{5x^{\boxed{3}}+3x-4}{2x^{\boxed{2}}-x}\ \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty}\frac{\cancel{x^2}\left(5x\to\infty+\frac{3}{x}\to 0-\frac{4}{x^2}\to 0\right)}{\cancel{x^2}\left(2-\frac{1}{x}\to 0\right)} = \frac{\infty}{2}\ \boxed{\infty}$$

PR6 $$\lim_{x\to\infty}\frac{5x^{\boxed{2}}+3x-4}{2x^{\boxed{3}}-x}\ \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty}\frac{\cancel{x^3}\left(\frac{5}{x}\to 0+\frac{3}{x^2}\to 0-\frac{4}{x^3}\to 0\right)}{\cancel{x^3}\left(2-\frac{1}{x^2}\to 0\right)} = \frac{0}{2} = \boxed{0}$$

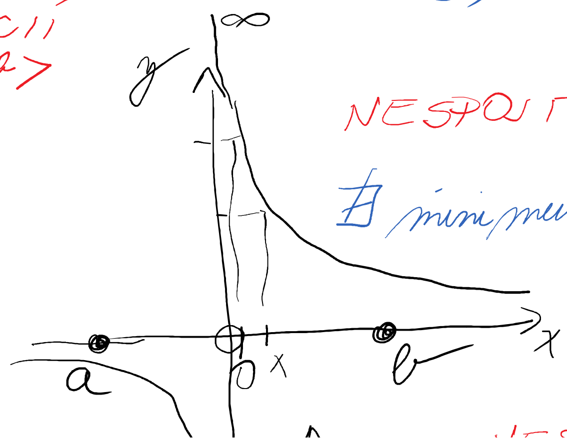
VLASTNOSTI SPOJITÝCH FUNKCIÍ NA INTERVALE $\langle a,b\rangle$

VETA 1



NESPOJITÁ

$\nexists$ minimum, maximum

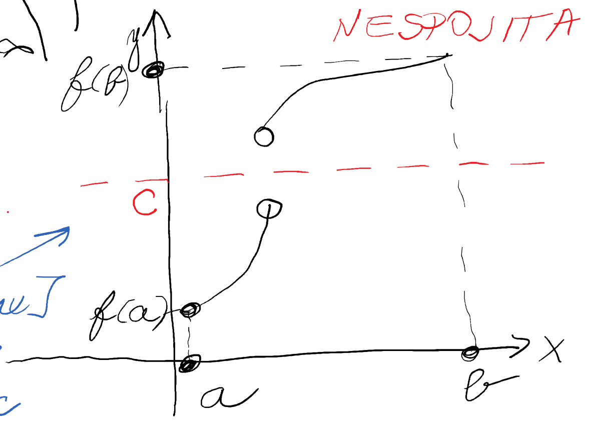


SPOJITÁ

$\exists$ minimum, maximum

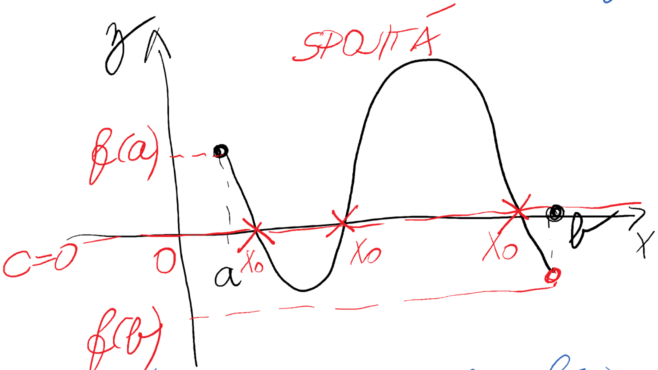
VETA 2

$$\forall c \in \langle min, max\rangle \; \exists x_0 \in \langle a, b\rangle$$
$$f(x_0) = c$$

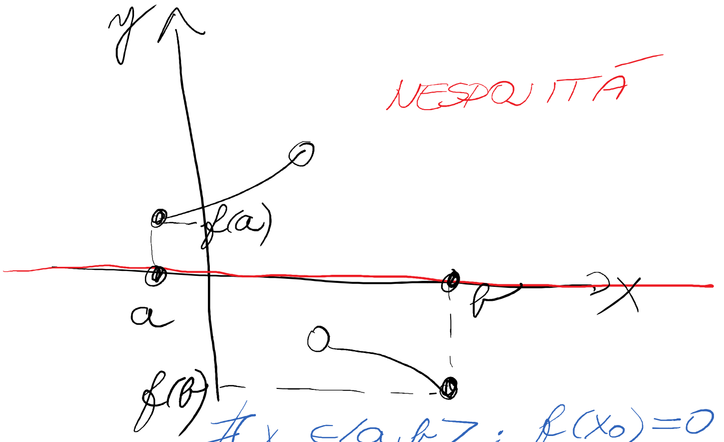


$\exists c \in \langle min, max]$
$\nexists x_0 \in \langle a, b\rangle$
aby : $f(x_0) = c$

VETA 3



$f(a) \cdot f(b) < 0 \quad \exists x_0 \in \langle a, b\rangle : f(x_0) = 0$



$\nexists x_0 \in \langle a, b\rangle : f(x_0) = 0$

ASYMPTOTA BEZ SMERNICE $x = a$ (a bod nespojitosti)

$$f(x) = \frac{x-2}{x-3} \qquad D(f) = R - \{3\}$$

$a = 3$ je bod nespojitosti

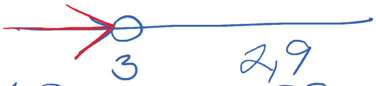
$$\lim_{x \to 3} f(x) = ?$$

$$\lim_{x \to \boxed{3^-}} \frac{x-2 \to 1}{x-3 \to 0^+} = \boxed{+\infty}$$

aspoň jedno $\pm\infty$

$$\frac{1}{-\frac{1}{10}} = -10 \qquad \frac{1}{-\frac{1}{100}} = -100$$

zľava

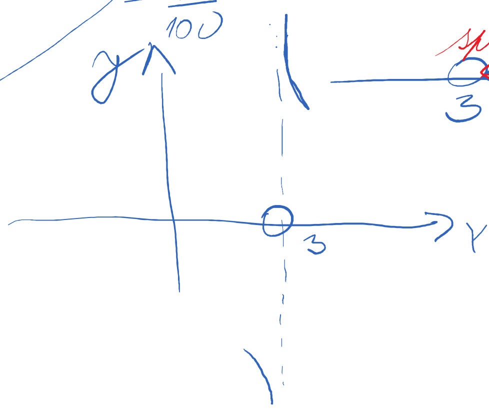


2,9
2,99
2,999

$$\lim_{x \to \boxed{3^+}} \frac{x-2 \to 1}{x-3 \to 0^-} = \boxed{-\infty}$$

sprava

3,1
3,01
3,001



$\Rightarrow \quad x = 3$ je ABS

ASYMPTOTY SO SMERNICOU $y = kx + q$

$$f(x) = \frac{4x^2}{2x-1}$$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{4x^2}{2x-1}}{x} = \lim_{x\to\infty} \frac{4x^{\boxed{2}}}{2x^{\boxed{2}} - x} = \frac{4}{2} = \boxed{2}$$ konečné ✓

$$q = \lim_{x\to\infty} \left(f(x) - k\cdot x\right) = \lim_{x\to\infty} \left(\frac{4x^{\boxed{2}}}{2x^{\boxed{1}} - 1} - 2\cdot x\right) \overset{\infty-\infty}{=}$$

$$= \lim_{x\to\infty} \frac{4x^2 - 2x(2x-1)}{2x-1} = \lim_{x\to\infty} \frac{\cancel{4x^2} - \cancel{4x^2} + \boxed{2}x^1}{\boxed{2}x^1 - 1} = \frac{2}{2} = \boxed{1}$$ konečné ✓

ASS $x \to \infty$ $y = 2x + 1$ pre smer $x \to \infty$

$x \to -\infty$ $y = 2x + 1$ pre smer $x \to -\infty$

(rovnaké výsledky limít)