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PR1: $R(x) = \frac{2x^2+10x-18}{x^4-2x^3-5x^2+6x}$

$$\frac{2x^2+10x-18}{x(x-1)(x-3)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-3} + \frac{D}{x+2} \quad / \cdot x(x-1)(x-3)(x+2)$$

$$2x^2+10x-18 = A(x-1)(x-3)(x+2) + Bx(x-3)(x+2) + Cx(x-1)(x+2) + Dx(x-1)(x-3)$$

DOSADZOVACIA METÓDA

$x=0$	$-18 = A(-1)(-3)\cdot 2$	$6A=-18$	$A=-3$
$x=1$	$-6 = B\cdot 1\cdot(-2)\cdot 3$	$-6B=-6$	$B=1$
$x=3$	$30 = C\cdot 3\cdot 2\cdot 5$	$30C=30$	$C=1$
$x=-2$	$-30 = B(-2)(-3)(-5)$	$-30D=-30$	$D=1$

$$\frac{2x^2+10x-18}{x^4-2x^3-5x^2+6x} = -\frac{3}{x} + \frac{1}{x-1} + \frac{1}{x-3} + \frac{1}{x+2}$$

PR2 $R(x) = \frac{5x^2+12x+1}{x^3+3x^2-4}$

$$\frac{5x^2+12x+1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \quad / \cdot (x-1)(x+2)^2$$

$$5x^2+12x+1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$x=1$	$18 = A\cdot 3^2$	$9A=18$	$A=2$
$x=-2$	$-3 = -3C$		$C=1$

DOSADÍME AKÉKOĽVEK ČÍSLO, KTORÉ SME EŠTE NEPOUŽILI

$x=0$ $\quad 1 = A\cdot 2^2 + B(-1)\cdot 2 + C\cdot(-1)$

A DOSADÍME $A=2$ a $C=1$

$1 = 2\cdot 4 + B(-2) + 1\cdot(-1)$ $\quad -2B=-6$ $\quad B=3$

$$\frac{5x^2+12x+1}{(x-1)(x+2)^2} = \frac{2}{x-1} + \frac{3}{x+2} + \frac{1}{(x+2)^2}$$

PR3 $R(x) = \frac{x^4+5x^3+11x^2+8x-4}{x^3+3x^2-4}$

4>3 NIE JE RYDZORACIONÁLNA

DELÍME

$(x^4+5x^3+11x^2+8x-4) : (x^3+3x^2-4) = x+2 + \frac{5x^2+12x+1}{x^3+3x^2-4}$

DELÍME

$$(x^4+5x^3+11x^2+8x-7) : (x^3+3x^2-4) = x+2 + \frac{5x^2+12x+1}{x^3+3x^2-4}$$

$$-(x^4+3x^3 \quad -4x)$$

$$2x^3+11x^2+12x-7$$

$$-(2x^3+6x^2 \quad -8)$$

$$5x^2+12x+1$$

PR2

PR4 $R(x) = \dfrac{5x^2-5x+9}{x^3-3x^2+4x-12}$

$$\frac{5x^2-5x+9}{(x-3)(x^2+4)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+4} \quad / \cdot (x-3)(x^2+4)$$

$$5x^2-5x+9 = A(x^2+4) + (Bx+C)(x-3)$$

$x=3$ $\quad 39 = 13A \quad A=3$

$x=0$ $\quad 9 = 3\cdot 4 + C\cdot(-3) \quad -3C=-3 \quad C=1$

$x=1$ $\quad 9 = 3\cdot 5 + (B\cdot 1+1)(-2) \quad -2B=-4 \quad B=2$

$$\frac{5x^2-5x+9}{(x-3)(x^2+4)} = \frac{3}{x-3} + \frac{2x+1}{x^2+4}$$

PR5 $R(x) = \dfrac{5x^2-x+8}{x^3+4x}$

$$\frac{5x^2-x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} \quad / \cdot x(x^2+4)$$

$$5x^2-x+8 = A(x^2+4) + (Bx+C)x$$

$x=0$ $\quad 8 = 4A \quad A=2$

$x=1$ $\quad 12 = 2\cdot 5 + (B+C)\cdot 1 \quad B+C=2$

$x=-1$ $\quad 14 = 2\cdot 5 + B - C \quad B-C=4$

RIEŠIME SÚSTAVU $\quad \begin{matrix} B+C=2 \\ B-C=4 \end{matrix}$

SČÍTAME $\quad 2B=6 \quad B=3$

$C = 2-B = 2-3 = -1$

$$\frac{5x^2-x+8}{x^3+4x} = \frac{2}{x} + \frac{3x-1}{x^2+4}$$

PRG $R(x) = \dfrac{2x^4+x^3-x+1}{x^3+1}$

$(2x^4+x^3-x+1) : (x^3+1) = 2x+1 - \dfrac{3x}{x^3+1}$

$-(2x^4+2x)$

x^3-3x+1

$-(x^3+1)$

$-3x$

$$\frac{-3x}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad / \cdot (x+1)(x^2-x+1)$$

$$-3x = A(x^2-x+1) + (Bx+C)(x+1)$$

$x=-1$ $\quad 3 = A \cdot 3 \quad A = 1$

$x=0$ $\quad 0 = 1+C \quad C = -1$

$x=1$ $\quad -3 = 1 \cdot 1 + (B-1) \cdot 2$

$2B = -2 \quad B = -1$

$$\frac{2x^4+x^3-x+1}{x^3+1} = 2x+1 + \frac{1}{x+1} - \frac{x+1}{x^2-x+1}$$

PR 7

$$R(x) = \frac{3x^3-14x^2+27x-16}{(x^2-4x+4)(x^2-2x)} = \frac{3x^3-14x^2+27x-16}{(x-2)^2 \cdot x(x-2)}$$

$$\frac{3x^3-14x^2+27x-16}{x(x-2)^3} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3} \quad / \cdot x(x-2)^3$$

$$3x^3-14x^2+27x-16 = A(x-2)^3 + Bx(x-2)^2 + Cx(x-2) + Dx$$

$x=0$ $\quad -16 = -8A \quad A = 2$

$x=2$ $\quad 6 = 2D \quad D = 3$

$x=1$ $\quad 0 = 2(-1) + B \cdot 1 + C(-1) + 3 \rightarrow B - C = -1$

$x=-1$ $\quad -60 = -27 \cdot 2 - B \cdot 9 - C(-3) - 3 \rightarrow 3B - C = 1$

RIEŠIME SÚSTAVU

$$\begin{aligned} B - C &= -1 \\ 3B - C &= 1 \end{aligned}$$

$\ominus \quad 2B = 2 \qquad B = 1$

$C = B + 1 = 1 + 1 = 2$

$$\frac{3x^3 - 14x^2 + 27x - 16}{x(x-2)^3} = \frac{2}{x} + \frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{3}{(x-2)^3}$$

PR8

$$R(x) = \frac{5x^2 - x + 12}{x^3 + x^2 + 3x - 5}$$

$$\frac{5x^2 - x + 12}{(x-1)(x^2+2x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5} \quad / \cdot (x-1)(x^2+2x+5)$$

$$5x^2 - x + 12 = A(x^2+2x+5) + (Bx+C)(x-1)$$

POROVNÁVACIA METÓDA $\longrightarrow$ ROZNÁSOBÍME

$$5x^2 - x + 12 = Ax^2 + 2Ax + 5A + Bx^2 - Bx + Cx - C$$

$$5x^2 - x + 12 = (A+B)\cdot x^2 + (2A - B + C)x + (5A - C)\cdot x^0$$

POROVNÁME KOEFICIENTY PRI ROVNAKÝCH MOCNINÁCH

$x^2: \quad 5 = A + B$

$x^1: \quad -1 = 2A - B + C$

$x^0: \quad 12 = 5A - C$ VYRIEŠIME SÚSTAVU

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 2 & -1 & 1 & -1 \\ 5 & 0 & -1 & 12 \end{array}\right) \begin{array}{l} \\ -2R_1 \\ -5R_1 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & -3 & 1 & -11 \\ 0 & -5 & -1 & -13 \end{array}\right) \begin{array}{l} \\ \cdot(-1) \\ \cdot(-1) \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 3 & -1 & 11 \\ 0 & 5 & 1 & 13 \end{array}\right) \cdot 3 - 5R_2 \approx \left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 3 & -1 & 11 \\ 0 & 0 & 8 & -16 \end{array}\right)$$

$8C = -16 \qquad C = -2$

$3B - C = 11 \qquad 3B = C + 11 = -2 + 11 \qquad 3B = 9 \qquad B = 3$

$A + B = 5 \qquad A = 5 - B = 5 - 3 = 2$

$$\frac{5x^2 - x + 12}{x^3 + x^2 + 3x - 5} = \frac{2}{x-1} + \frac{3x-2}{x^2+2x+5}$$