

PR
 a) x ... počet kusov modelu A
 y ... počet ks modelu B
 z ... počet ks modelu C

40x ... minút pre x kusov modelu A na linke 1

30y ... minút pre y ks modelu B

25z ... minút pre z ks modelu C

$$40x + 30y + 25z = 4500$$

$$25x + 20y + 20z = 3050$$

$$10x + 10y + 5z = 1200$$

$$A^* = \left(\begin{array}{ccc|c} 40 & 30 & 25 & 4500 \\ 25 & 20 & 20 & 3050 \\ 10 & 10 & 5 & 1200 \end{array} \right) \cdot \frac{1}{5} \approx \left(\begin{array}{ccc|c} 8 & 6 & 5 & 900 \\ 5 & 4 & 4 & 610 \\ 2 & 2 & 1 & 240 \end{array} \right) \cdot \frac{1}{5} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 8 & 6 & 5 & 900 \\ 2 & 2 & 1 & 240 \end{array} \right) \begin{array}{l} -8R_1 \\ -2R_1 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 6 & -11 & -140 \\ 0 & 2 & -3 & -20 \end{array} \right) \begin{array}{l} \\ \\ -2R_2 \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 2 & -3 & -20 \\ 0 & 6 & -11 & -140 \end{array} \right) \begin{array}{l} \\ \\ -3R_2 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 2 & -3 & -20 \\ 0 & 0 & -2 & -80 \end{array} \right) \cdot \left(-\frac{1}{2}\right) \approx$$

$R(A^*) = 3 = R(A) \Rightarrow \exists$ riešenie $\Rightarrow \exists$ práve 1 riešenie

$R(A^*) = n$ JORDANOVA ELIMINÁCIA

$$\approx \left(\begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 2 & -3 & -20 \\ 0 & 0 & 1 & 40 \end{array} \right) \begin{array}{l} -2R_3 \\ +3R_3 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 2 & 0 & 100 \\ 0 & 0 & 1 & 40 \end{array} \right) \cdot \frac{1}{2} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 40 \end{array} \right) \begin{array}{l} x=50 \\ y=50 \\ z=40 \end{array}$$

$$\bar{x} = (50, 50, 40)^T$$

je treba vyrobiť 50 ks modelu A
 50 ks modelu B
 40 ks modelu C

iný spôsob: $x + 2z = 130$
 $2y - 3z = -20$
 $2z = -20 + 120 = 100$

iný spôsob:

$$\begin{array}{r} x + 2z = 130 \\ 2y - 3z = -20 \end{array} \rightarrow \begin{array}{l} z = 40 \end{array}$$

$$\begin{array}{l} 2y - 3 \cdot 40 = -20 \\ 2y = -20 + 120 = 100 \\ y = 50 \\ x + 2 \cdot 40 = 130 \\ x = 130 - 80 = 50 \\ x = 50 \end{array}$$

$$\begin{array}{l} 40x + 30y + 25z = 4500 \\ 25x + 20y + 20z = 3050 \end{array}$$

$$A^* = \left(\begin{array}{ccc|c} 40 & 30 & 25 & 4500 \\ 25 & 20 & 20 & 3050 \end{array} \right) \cdot \frac{1}{5} \approx \left(\begin{array}{ccc|c} 8 & 6 & 5 & 900 \\ 5 & 4 & 4 & 610 \end{array} \right) \cdot 8 - 5R_1 \approx$$

$$\approx \left(\begin{array}{ccc|c} 8 & 6 & 5 & 900 \\ 0 & 2 & 7 & 380 \end{array} \right)$$

$$r(A^*) = r(A) = 2$$

$$n = 3 > r(A) \Rightarrow \text{J } \infty \text{ voln } \checkmark$$

$$n - r(A) = 3 - 2 = 1 \text{ voln } \checkmark$$

$$\begin{array}{l} 8x + 6y + 5z = 900 \\ 2y + 7z = 380 \end{array}$$

$$\begin{array}{l} z = t \\ 2y + 7t = 380 \\ 2y = 380 - 7t \\ y = \frac{380 - 7t}{2} \\ y = 190 - \frac{7}{2}t \end{array}$$

$$\begin{array}{l} 8x + 6 \cdot (190 - \frac{7}{2}t) + 5t = 900 \\ 8x = 900 - 6(190 - \frac{7}{2}t) - 5t \\ x = \frac{1}{8}[-240 + 16t] \\ x = -30 + 2t \end{array}$$

$$\bar{x} = (-30 + 2t, 190 - \frac{7}{2}t, t)^T \quad t \in \mathbb{R}$$

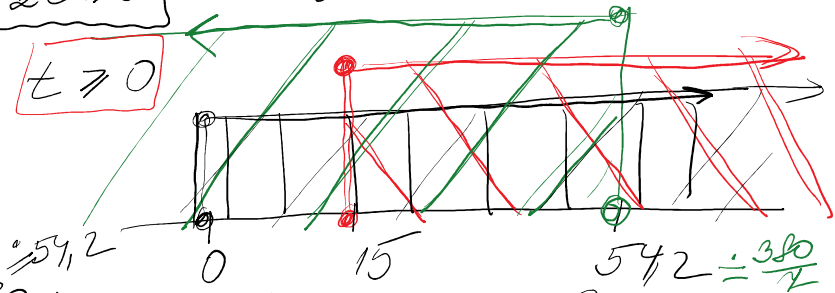
$$t=1: \quad x = -30 + 2 \cdot 1 = -28; \quad y = 190 - \frac{7}{2} \cdot 1 = \frac{373}{2} \quad \text{nevhodné}$$

$$\begin{array}{l} x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{array}$$

$$-30 + 2t \geq 0 \Leftrightarrow 2t \geq 30 \Leftrightarrow t \geq 15$$

$$190 - \frac{7}{2}t \geq 0 \Leftrightarrow \frac{7}{2}t \leq 190 \Leftrightarrow t \leq \frac{380}{7} \approx 54,2$$

$$t \geq 0$$



$$t \in \langle 15, \frac{380}{7} \rangle \cap \{2k; k \in \mathbb{Z}\}$$

$$= \{16, 18, 20, \dots, 54\} \quad \text{parne celé čísla } \{ \dots, -4, -2, 0, 2, 4, 6, \dots \}$$

$$\bar{x} = (-30 + 2t, 190 - \frac{7}{2}t, t)^T \quad t \in \{16, 18, 20, \dots, 54\}$$

20 riešení

PR 21

- a) x ... počet ks hodiniek modelu A
 y ... počet ks hodiniek modelu B
 z ... počet ks hodiniek modelu C

$$\begin{aligned} 10x + 8y + 12z &= 620 \\ 15x + 6y + 16z &= 750 \\ 5x + 2y + 8z &= 330 \end{aligned}$$

$$A^* = \left(\begin{array}{ccc|c} 10 & 8 & 12 & 620 \\ 15 & 6 & 16 & 750 \\ 5 & 2 & 8 & 330 \end{array} \right) \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \approx \left(\begin{array}{ccc|c} 5 & 2 & 8 & 330 \\ 15 & 6 & 16 & 750 \\ 10 & 8 & 12 & 620 \end{array} \right) \begin{array}{l} \\ -3R_1 \\ -2R_1 \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 5 & 2 & 8 & 330 \\ 0 & 0 & -8 & -240 \\ 0 & 4 & -4 & -40 \end{array} \right) \begin{array}{l} \\ \cdot (-\frac{1}{8}) \\ \cdot \frac{1}{4} \end{array} \approx \left(\begin{array}{ccc|c} 5 & 2 & 8 & 330 \\ 0 & 1 & -1 & -10 \\ 0 & 0 & +1 & +30 \end{array} \right) \begin{array}{l} -8R_3 \quad (-2R_2) \\ +2R_3 \\ \end{array}$$

$r(A) = r(A) = 3 = n \Rightarrow \exists$ práve 1 riešenie

$$\begin{aligned} 5x + 2y + 8z &= 330 \\ y - z &= -10 \\ \boxed{z} &= \boxed{30} \end{aligned}$$

$$\begin{aligned} y - 30 &= -10 \\ \boxed{y} &= \boxed{20} \end{aligned}$$

$$\begin{aligned} 5x + 2 \cdot 20 + 8 \cdot 30 &= 330 \\ 5x + 40 + 240 &= 330 \\ 5x &= 50 \\ \boxed{x} &= \boxed{10} \end{aligned}$$

$$\bar{x} = (10, 20, 30)^T$$

je treba vyrobiť

10 ks modelu A
 20 ks modelu B
 30 ks modelu C

b) $10x + 8y + 12z = 620$
 $15x + 6y + 16z = 750$

$$A^* = \left(\begin{array}{ccc|c} 10 & 8 & 12 & 620 \\ 15 & 6 & 16 & 750 \end{array} \right) \cdot \frac{1}{2} \approx \left(\begin{array}{ccc|c} 5 & 4 & 6 & 310 \\ 15 & 6 & 16 & 750 \end{array} \right) \begin{array}{l} \\ -3R_1 \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 5 & 4 & 6 & 310 \\ 0 & -6 & -2 & -180 \end{array} \right) \cdot (-\frac{1}{2}) \approx \left(\begin{array}{ccc|c} 5 & 4 & 6 & 310 \\ 0 & 3 & 1 & 90 \end{array} \right)$$

$r(A^*) = r(A) = 2 \Rightarrow \exists$ riešenie
 $n = 3 > r(A) \Rightarrow \exists \infty$ veľa riešení
 $n - r(A) = 3 - 2 = 1$ voľná premenná

$$5x + 4y + 6z = 310$$

$$\boxed{y = t}$$

$$\begin{aligned} 5x + 4y + 6z &= 310 \\ 3y + z &= 90 \end{aligned}$$

$$y = z$$

$$z = 90 - 3y = 90 - 3z$$

$$5x + 4z + 6(90 - 3z) = 310$$

$$\begin{aligned} 5x &= 310 - 4z - 6(90 - 3z) = 310 - 4z - 540 + 18z = \\ &= -230 + 14z \end{aligned}$$

$$x = \frac{-230 + 14z}{5} = -46 + \frac{14}{5}z$$

$$\bar{x} = \left(-46 + \frac{14}{5}z, z, 90 - 3z \right)^T \quad z \in \mathbb{R}$$

$$x \geq 0$$

$$y \geq 0$$

$$z \geq 0$$

$$-46 + \frac{14}{5}z \geq 0$$

$$z \geq 0$$

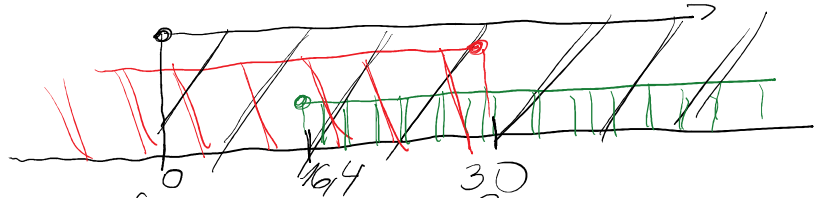
$$90 - 3z \geq 0$$

$$\frac{14}{5}z \geq 46$$

$$z \geq 16,4$$

$$3z \leq 90$$

$$z \leq 30$$



$$z \in \langle 16,4 ; 30 \rangle \cap \{5k, k \in \mathbb{Z}\}$$

$$z \in \{20, 25, 30\}$$

celok sečne nabitky 5
3 riešená {..., -10, -5, 0, 5, 10, 15, ...}

$$\bar{x} = \left(-46 + \frac{14}{5}z, z, 90 - 3z \right)^T$$

$$z_1 = 20 \quad \bar{x} = \left(-46 + \frac{14}{5} \cdot 20, 20, 90 - 3 \cdot 20 \right)^T = (10, 20, 30)^T$$

$$z_2 = 25 \quad \bar{x} = \left(-46 + \frac{14}{5} \cdot 25, 25, 90 - 3 \cdot 25 \right)^T = (24, 25, 15)^T$$

$$z_3 = 30 \quad \bar{x} = \left(-46 + \frac{14}{5} \cdot 30, 30, 90 - 3 \cdot 30 \right)^T = (38, 30, 0)^T$$