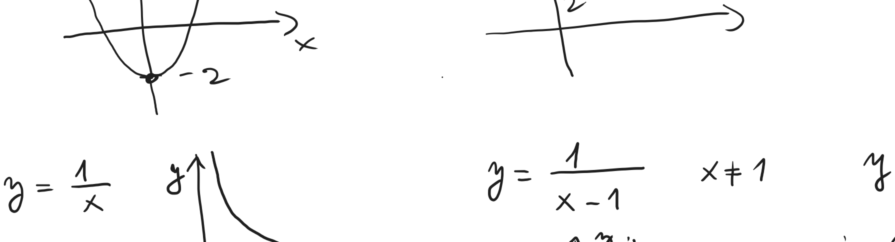
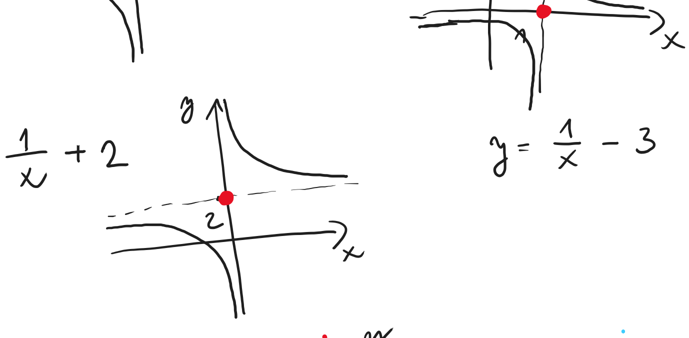
GRAFY

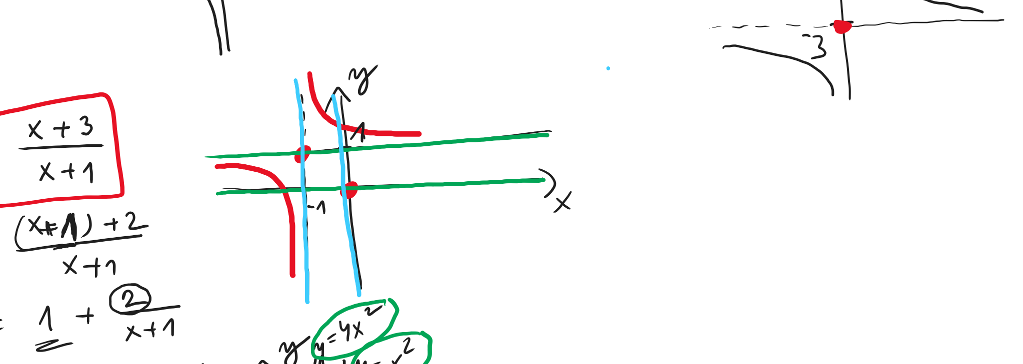
$y = x^2$ $\quad y = (x+3)^2$ $\quad y = (x-3)^2$



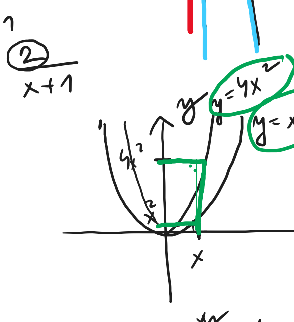
$y+2 = x^2$, $y = x^2 - 2$ $\quad y-2 = x^2$, $y = x^2 + 2$



$y = \frac{1}{x}$ $\quad y = \frac{1}{x-1}$ $\quad x \neq 1$ $\quad y = \frac{1}{x+2}$ $\quad x \neq -2$



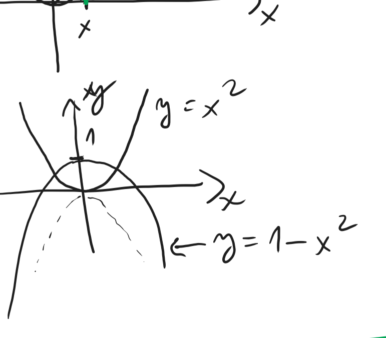
$y = \frac{1}{x} + 2$ $\quad y = \frac{1}{x} - 3$



$y = \frac{x+3}{x+1}$

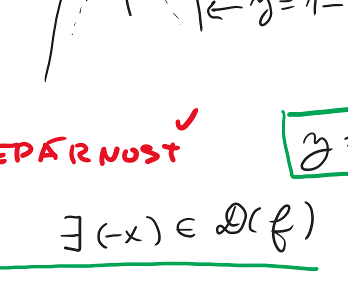
$y = \frac{(x+1)+2}{x+1}$

$y = 1 + \frac{2}{x+1}$



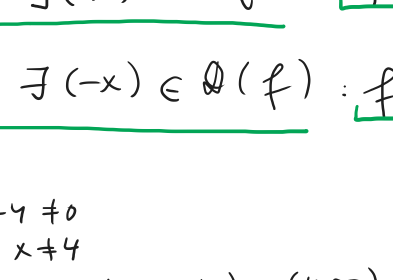
$y = x^2$

$y = 2x^2$



$y = x^2$

$y = 1 - x^2$



PÁRNOSŤ, NEPÁRNOSŤ

$y = f(x)$

$\forall x \in D(f) \quad \exists (-x) \in D(f) : f(-x) = f(x)$ funkcia $f(x)$ je párna (o_y)

$\forall x \in D(f) \quad \exists (-x) \in D(f) : f(-x) = -f(x)$ funkcia $f(x)$ je nepárna ($[0,0]$)

$y = \frac{1}{x-4}$ $\quad x - 4 \neq 0$, $x \neq 4$

$D(f) = (-\infty, 4) \cup (4, \infty) \Rightarrow f(x)$ nie je párna ani nepárna

(číselná os, bod 4 vylúčený)

pre $x = -4 \in D(f)$ $\quad (-x) = 4 \notin D(f)$

$f(x) = 2x^3 + 3x \qquad D(f) = \mathbb{R} \qquad f(☺) = 2 \cdot ☺^3 + 3☺$

$f(-x) = 2 \cdot (-x)^3 + 3(-x) = 2 \cdot (-x^3) - 3x = -2x^3 - 3x = -(2x^3 + 3x) = -f(x)$

fcia je nepárna

$f(x) = -3x^4 + 5x^2 - 11 \qquad D(f) = \mathbb{R}$

$f(-x) = -3(-x)^4 + 5(-x)^2 - 11 = -3x^4 + 5x^2 - 11 = f(x)$ $\qquad x^2 = (-x)^2 \neq -x^2$

fcia je párna

$f(x) = \frac{x-10}{x} \qquad D(f) = \mathbb{R} - \{0\} \qquad \frac{☺ - 10}{☺} = f(☺)$

$f(-x) = \frac{-x-10}{-x} = \frac{-(x+10)}{-x} = \frac{x+10}{x}$ $\neq f(x)$, $\neq -f(x)$ fcia nie je párna ani nepárna

$f(x) = 2 \cdot \sin x + 3\cos x \qquad D(f) = \mathbb{R}$ $\qquad \sin(-x) = -\sin x$, $\cos(-x) = \cos x$

$f(-x) = 2\sin(-x) + 3\cos(-x) = -2\sin x + 3\cos x$ $\neq f(x)$ ani párna, $\neq -f(x)$ ani nepárna

$f(x) = \sin x \cdot \cos x \qquad D(f) = \mathbb{R}$

$f(-x) = \sin(-x) \cdot \cos(-x) = (-\sin x) \cdot \cos x = -\sin x \cos x = -f(x)$

fcia je nepárna

$f(x) = -x^2 + \sin x^5 \qquad D(f) = \mathbb{R}$

$f(-x) = -(-x)^2 + \sin(-x)^5 = -x^2 + \sin(-x^5) = -x^2 - \sin x^5$ $\neq f(x)$, $\neq -f(x)$

ani párna ani nepárna

$f(x) = -x^2 + \sin x^4 \qquad D(f) = \mathbb{R}$

$f(-x) = -(-x)^2 + \sin(-x)^4 = -x^2 + \sin x^4 = f(x)$ je párna

$f(x) = x \cdot \cos x \qquad D(f) = \mathbb{R}$

$f(-x) = -x \cdot \cos(-x) = -x \cdot \cos x = -f(x)$ fcia je nepárna

$f(x) = \ln \frac{2+x}{2-x} \qquad \frac{2+x}{2-x} > 0 \quad$ NB: $-2, 2$

$- \quad (-2) \quad + \quad (2) \quad -$

$D(f) = (-2, 2)$

$f(-x) = \ln \frac{2-x}{2+x} = \ln \left(\frac{2+x}{2-x}\right)^{-1} = -\ln \frac{2+x}{2-x} = -f(x)$

fcia je nepárna

(ak $D(f) = \langle -2, 2)$, tak nie je ani párna ani nepárna)

$\ln a^b = b \ln a$

$f(x) = e^{2-x} \qquad D(f) = \mathbb{R}$

$f(-x) = e^{2+x}$ $\neq f(x)$, $\neq -f(x)$ fcia nie je párna ani nepárna

$f(x) = \ln(1+x) \qquad 1 + x > 0$, $x > -1 \qquad D(f) = (-1, \infty)$ $\Rightarrow$ ani párna ani nepárna

LIMITA FUNKCIE

$\lim_{x \to -2} (4x^3 - 2x^2 + 11x) = -62$

$f(x) = 4x^3 - 2x^2 + 11x$

$f(-2) = 4 \cdot (-8) - 2 \cdot 4 + 11 \cdot (-2) = -32 - 8 - 22 = -62$

$\lim_{x \to \infty} (14x^2 + x + 18)$

$\lim_{x \to -2} \frac{x^2-4}{x+2} \left[\frac{0}{0}\right] = \lim_{x \to -2} \frac{(x-2)(x+2)}{x+2} = -4$

$f(x) = \frac{x^2-4}{x+2}$

$f(-2) = \frac{0}{0}$

NEURČITÝ VÝRAZ: $\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty$

$f(\infty) = \ldots$

$\infty + \infty = \infty$

$\lim_{x \to -1} \frac{x^2 + x - 2}{2x^3 + x^2 - x - 2} = \frac{1-1-2}{-2+1-1-2} = 1$

$\lim_{x \to 2} \frac{x^2-4}{x^2-3x+2} \left[\frac{0}{0}\right] = \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x-1)} = \frac{4}{1} = 4$

$\lim_{x \to 4} \frac{x^2 + 7x - 44}{x^2 - 6x + 8} \left[\frac{0}{0}\right] = \lim_{x \to 4} \frac{(x-4)(x+11)}{(x-4)(x-2)} = \frac{15}{2}$

$\lim_{x \to 1} \frac{x^3 - 4x^2 + 5x - 2}{x^5 - 3x + 2} \left[\frac{0}{0}\right] = \lim_{x \to 1} \frac{(x-1)(x^2-3x+2)}{(x-1)(x^4+x^3+x^2+x-2)} = \frac{0}{2} = 0$

$(x^3 - 4x^2 + 5x - 2) : (x-1) = x^2 - 3x + 2$

$-x^3 + x^2$

$-3x^2 + 5x - 2$

$+3x^2 - 3x$

$2x - 2$

$-2x + 2$

0

$(x^5 - 3x + 2) : (x-1) = x^4 + x^3 + x^2 + x - 2$

$-x^5 + x^4$

$x^4 - 3x + 2$

$-x^4 + x^3$

$x^3 - 3x + 2$

$-x^3 + x^2$

$x^2 - 3x + 2$

$-x^2 + x$

$-2x + 2$

$+2x - 2$

$\lim_{x \to \infty} \frac{7x^2 + 2x + 15}{13x^2 + 6} \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{x^2(7 + \frac{2}{x} + \frac{15}{x^2})}{x^2(13 + \frac{6}{x^2})} = \frac{7}{13}$

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{5x - 4}{2x^5 - 7x^3 + 11} \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{x^5(\frac{5}{x^4} - \frac{4}{x^5})}{x^5(2 - \frac{7}{x^2} + \frac{11}{x^5})} = \frac{0}{2} = 0$

$\lim_{x \to \infty} \frac{10x^7 + 11x^5 - 3x}{4x^3 + 2x - 1} \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{x^3(10x^4 + 11x^2 - \frac{3}{x^2})}{x^3(4 + \frac{2}{x^2} - \frac{1}{x^3})} = \frac{\infty}{4} = \infty$

$$\lim_{x \to \infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_1 x + b_0} = \begin{cases} \frac{a_n}{b_m} & n = m \\ 0 & n < m \\ \infty & n > m \end{cases}$$