$(-5)+7=2$, $(-5)\cdot 7=-35$

$$\lim_{x\to 5}\frac{x^2+2x-35}{x^2-8x+15}\left[\frac{0}{0}\right]=\lim_{x\to 5}\frac{(x-5)(x+7)}{(x-5)(x-3)}=\frac{12}{2}=6$$

$(-5)\cdot(-3)=15$, $(-5)+(-3)=-8$

$$\lim_{x\to\infty}\frac{3x^2+2x-35}{6x^2-8x+15}\left[\frac{\infty}{\infty}\right]=\frac{3}{6}=\frac{1}{2}$$

$$\lim_{x\to\infty}\frac{3x+2}{6x^2-8x+15}\left[\frac{\infty}{\infty}\right]=0$$

$$\lim_{x\to\infty}\frac{3x^4+7x-1}{2x^2+5x+10}\left[\frac{\infty}{\infty}\right]=\infty \qquad (a-b)(a+b)=a^2-b^2$$

$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\cdot\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}=$$

$$=\lim_{x\to 0}\frac{(2+x)-2}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\lim_{x\to 0}\frac{x}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\frac{1}{2\sqrt{2}}$$

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty-\infty$

$$\lim_{x\to\infty}\left(\sqrt{x-2}-\sqrt{x}\right)[\infty-\infty]=\lim_{x\to\infty}\left(\sqrt{x-2}-\sqrt{x}\right)\cdot\frac{\sqrt{x-2}+\sqrt{x}}{\sqrt{x-2}+\sqrt{x}}=$$

$$=\lim_{x\to\infty}\frac{x-2-x}{\sqrt{x-2}+\sqrt{x}}=\lim_{x\to\infty}\frac{-2}{\sqrt{x-2}+\sqrt{x}}\left[\frac{-2}{\infty}\right]=0$$

$\frac{1}{\infty}=0$

$$\lim_{x\to\infty}\left(\sqrt{x-2}+\sqrt{x}\right)[\infty+\infty]=\infty$$

$$\lim_{x\to -2}\frac{\sqrt{6+x}-2}{x+2}\left[\frac{0}{0}\right]\cdot\frac{\sqrt{6+x}+2}{\sqrt{6+x}+2}=\lim_{x\to -2}\frac{6+x-4}{(x+2)\left(\sqrt{6+x}+2\right)}=$$

$$=\lim_{x\to -2}\frac{1}{\sqrt{6+x}+2}=\frac{1}{4}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{\sin(kx)}{k\cdot x}=1 \qquad k\in\mathbb{R}$$

$$\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}2\cdot\frac{\sin 2x}{2\cdot x}=2\cdot 1=2$$

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}\cdot\frac{\frac{1}{3x\cdot 4}}{\frac{1}{4x\cdot 3}}=\lim_{x\to 0}\frac{\frac{\sin 3x}{3x}\cdot\frac{1}{4}}{\frac{\sin 4x}{4x}\cdot\frac{1}{3}}=\frac{\frac{1}{4}}{\frac{1}{3}}=\frac{3}{4}$$

$$\lim_{x\to 0}\frac{tg\,2x}{tg\,5x}\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\frac{\sin 2x}{\cos 2x}}{\frac{\sin 5x}{\cos 5x}}\cdot\frac{\frac{1}{2x\cdot 5}}{\frac{1}{5x\cdot 2}}=\lim_{x\to 0}\frac{\frac{\sin 2x}{2x}\cdot\frac{1}{5\cos 2x}}{\frac{\sin 5x}{5x}\cdot\frac{1}{2\cos 5x}}=\frac{2}{5}$$

je ohraničená fcia $\in\langle 0,1\rangle$

$$\lim_{x\to\infty}\frac{\cos^2 x}{x}=\lim_{x\to\infty}\frac{1}{x}\cdot\cos^2 x=0$$

$\frac{1}{\infty}=0$

$0\cdot\infty$ - neurčitý

$$\lim_{x\to 0}\frac{\sin x}{x^3}\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{x^2}=\lim_{x\to 0}\frac{1}{x^2}=\infty$$

$\lim_{x\to 0}\frac{1}{x}$ - neexistuje

$$\lim_{x\to 0^+}\frac{1}{x}=\infty \qquad \lim_{x\to 0^-}\frac{1}{x}=-\infty$$

$$\lim_{x\to 0^+}\frac{1}{x^2}=\infty \qquad \lim_{x\to 0^-}\frac{1}{x^2}=\infty$$

$$\lim_{x\to 2^+}\frac{1}{(x-2)_{\to 0^+}}=\infty \qquad \lim_{x\to -3^+}\frac{1}{(x+3)_{\to 0^+}}=\infty$$

$$\lim_{x\to 2^-}\frac{1}{(x-2)_{\to 0^-}}=-\infty \qquad \lim_{x\to -3^-}\frac{1}{(x+3)_{\to 0^-}}=-\infty$$

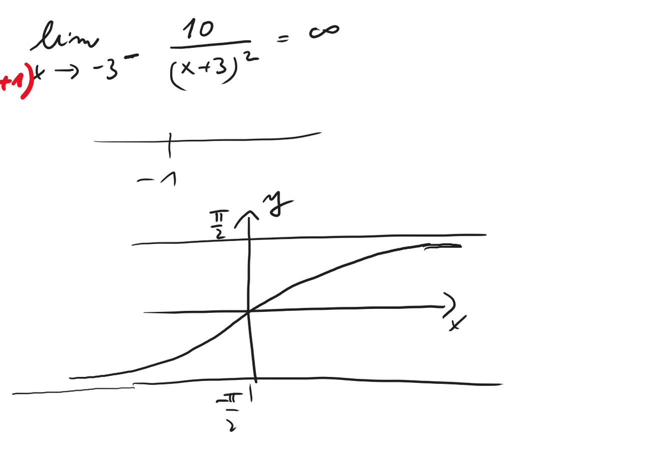
$$\lim_{x\to 2^+}\frac{-7}{x-2}=-7\cdot\infty=-\infty \qquad \lim_{x\to -3^+}\frac{+10}{(x+3)^2}=\infty$$

$$\lim_{x\to 2^-}\frac{-2}{x-2}=-7\cdot(-\infty)=\infty \qquad \lim_{x\to -3^-}\frac{10}{(x+3)^2}=\infty$$

$$\lim_{x\to 99}\log(x+1)=\log\lim_{x\to 99}(x+1)=\log 100=2$$

$$\lim_{x\to -1^+}\operatorname{arctg}\frac{1}{(1+x)_{\to 0^+}}=\frac{\pi}{2}$$

$$\lim_{x\to -1^-}\operatorname{arctg}\frac{1}{(1+x)_{\to 0^-}}=-\frac{\pi}{2}$$



ASYMPTOTY KU GRAFU FUNKCIE $y=f(x)$

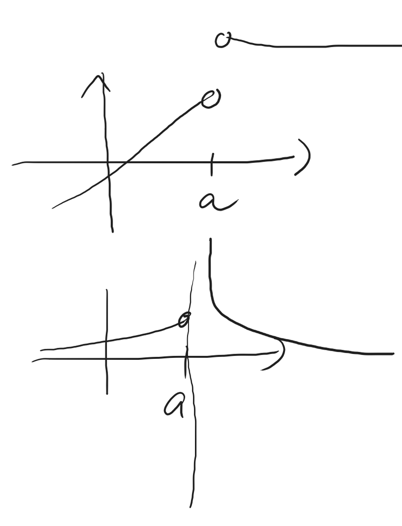
asymptota bez smernice (ABS): $x=a$

1. $a\notin D(f)$
2. $\lim_{x\to a^+}f(x)=\pm\infty$ alebo $\lim_{x\to a^-}f(x)=\pm\infty$

asymptota so smernicou (ASS): $y=kx+q$

$$k=\lim_{x\to\infty}\frac{f(x)}{x} \qquad (x\to-\infty)$$

$$q=\lim_{x\to\infty}\left(f(x)-kx\right)$$



$f(x)=\frac{x}{x^2+4}$ $\quad D(f)=\mathbb{R}\Rightarrow$ ABS neexistuje

ASS: $k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{\frac{x}{x^2+4}}{x}=\lim_{x\to\infty}\frac{1}{x^2+4}\left[\frac{1}{\infty}\right]=0$

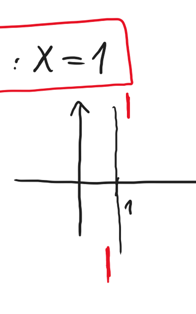
$q=\lim_{x\to\infty}\left(f(x)-kx\right)=\lim_{x\to\infty}\frac{x}{x^2+4}=0$

$y=0$ $\quad (x\to-\infty$ TO ISTÉ$)$

$f(x)=\frac{x^2-2x+2}{x-1}$ $\quad D(f)=\mathbb{R}-\{1\}$

$\lim_{x\to 1^+}f(x)=\lim_{x\to 1^+}\frac{x^2-2x+2}{x-1}\left[\frac{1}{0^+}\right]=\infty\Rightarrow$ **ABS: $x=1$**

$\lim_{x\to 1^-}f(x)=\lim_{x\to 1^-}\frac{x^2-2x+2}{x-1}\left[\frac{1}{0^-}\right]=-\infty$



ASS: $k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2-2x+2}{x(x-1)}=1$ $\quad (x(x-1)=x^2-x)$

$q=\lim_{x\to\infty}\left(f(x)-kx\right)=\lim_{x\to\infty}\left(\frac{x^2-2x+2}{x-1}-x\right)=\lim_{x\to\infty}\frac{x^2-2x+2-x(x-1)}{x-1}=$

$=\lim_{x\to\infty}\frac{x^2-2x+2-x^2+x}{x-1}=\lim_{x\to\infty}\frac{-x+2}{x-1}=-1$

ASS: $y=x-1$ $\quad (x\to-\infty$ TO ISTÉ$)$

$y=\frac{(x+2)^2}{x}$ $\quad D(f)=\mathbb{R}-\{0\}$

ABS: $\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}\frac{(x+2)^2}{x}\left[\frac{4}{0^+}\right]=\infty$ $\quad$ **ABS: $x=0$**

$\lim_{x\to 0^-}f(x)=\lim_{x\to 0^-}\frac{(x+2)^2}{x}\left[\frac{4}{0^-}\right]=-\infty$

ASS: $k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{(x+2)^2}{x^2}=1$ $\quad (x^2+\dots)$

$q=\lim_{x\to\infty}\left(f(x)-kx\right)=\lim_{x\to\infty}\left(\frac{(x+2)^2}{x}-x\right)=\lim_{x\to\infty}\frac{x^2+4x+4-x^2}{x}=$

$=\lim_{x\to\infty}\frac{4x+4}{x}=4$

ASS: $y=x+4$ $\quad (x\to-\infty$ TO ISTÉ$)$

$y=x+e^{-x}$ $\quad D(f)=\mathbb{R}\Rightarrow$ ABS neexistuje

ASS: $k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x+e^{-x}}{x}=\lim_{x\to\infty}\left(1+\frac{1}{e^x\cdot x}\right)=1$

$q=\lim_{x\to\infty}\left(f(x)-kx\right)=\lim_{x\to\infty}\left(x+e^{-x}-x\right)=\lim_{x\to\infty}\frac{1}{e^x}=0$

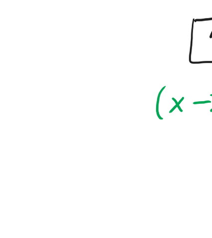
ASS: $y=x$ $\quad (x\to-\infty$ INÉ ... L'Hospitalovo pravidlo$)$ NESKÔR

$f(x)=x\cdot\ln x$ $\quad D(f)=(0,\infty)$

ABS: $\lim_{x\to 0^+}f(x)=$ NESKÔR

~~$x\to-\infty$~~, ~~$x\to 0^-$~~

ASS: $\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\ln x=\infty\Rightarrow$ ASS neex.



$y=\frac{x^2+1}{x}$ $\quad D(f)=\mathbb{R}-\{0\}$

ABS: $\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}\frac{x^2+1}{x}\left[\frac{1}{0^+}\right]=\infty\Rightarrow$ **ABS: $x=0$**

$\lim_{x\to 0^-}f(x)=\lim_{x\to 0^-}\frac{x^2+1}{x}\left[\frac{1}{0^-}\right]=-\infty$

ASS: $k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2+1}{x^2}=1$

$q=\lim_{x\to\infty}\left(f(x)-kx\right)=\lim_{x\to\infty}\left(\frac{x^2+1}{x}-x\right)=\lim_{x\to\infty}\frac{x^2+1-x^2}{x}\left[\frac{1}{\infty}\right]=0$

ASS: $y=x$

$(x\to-\infty$ TO ISTÉ$)$