

Derivácia funkcie

$$\begin{aligned}
 (x^n)' &= n \cdot x^{n-1} & (\sin x)' &= \cos x & (\arcsin x)' &= \frac{1}{\sqrt{1-x^2}} \\
 (a^x)' &= a^x \cdot \ln a & (\cos x)' &= -\sin x & (\arccos x)' &= -\frac{1}{\sqrt{1-x^2}} \\
 (e^x)' &= e^x & (tg x)' &= \frac{1}{\cos^2 x} & (\operatorname{arctg} x)' &= \frac{1}{1+x^2} \\
 (\log_a x)' &= \frac{1}{x \ln a} & (\cot x)' &= -\frac{1}{\sin^2 x} & (\operatorname{arccot} x)' &= -\frac{1}{1+x^2} \\
 (\ln x)' &= \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 [a \cdot f(x) \pm b \cdot g(x)]' &= a \cdot f'(x) \pm b \cdot g'(x) \\
 [f(x) \cdot g(x)]' &= f'(x) \cdot g(x) + f(x) \cdot g'(x) \\
 \left[\frac{f(x)}{g(x)} \right]' &= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}
 \end{aligned}$$

Zderivujme:

- 1) $f(x) = 3x^5 - 7x^3 + 11x^2 - 7x + 35$
 $f'(x) = 3 \cdot 5 \cdot x^4 - 7 \cdot 3 \cdot x^2 + 11 \cdot 2 \cdot x - 7 + 0$
- 2) $f(x) = \frac{1}{x^4} - 2\sqrt{x} + \sqrt[5]{x^3} - \frac{1}{x} = x^{-4} - 2x^{\frac{1}{2}} + x^{\frac{3}{5}} - x^{-1}$
 $f'(x) = -4x^{-5} - 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} + \frac{3}{5} \cdot x^{-\frac{2}{5}} + x^{-2}$
- 3) $f(x) = \frac{1}{\sqrt[4]{x^9}} - \frac{1}{\sqrt{x}} + \frac{7}{x^3} - \frac{1}{x^{10}} = x^{-\frac{9}{4}} - x^{-\frac{1}{2}} + 7x^{-3} - x^{-10}$
 $f'(x) = -\frac{9}{4} x^{-\frac{13}{4}} + \frac{1}{2} x^{-\frac{3}{2}} - 21x^{-4} + 10x^{-11}$
- 4) $f(x) = \sqrt[4]{x^9} + \sqrt{x} + 7x^3 - x^{10} = x^{\frac{9}{4}} + x^{\frac{1}{2}} + 7x^3 - x^{10}$
 $f'(x) = \frac{9}{4} x^{\frac{5}{4}} + \frac{1}{2} x^{-\frac{1}{2}} + 21x^2 - 10x^9$
- 5) $f(x) = 2^x + \left(\frac{1}{5}\right)^x + e^x - 3 \cdot \ln x + 7 \cdot \log_3 x - 2 \log_{\frac{1}{7}} x$
 $f'(x) = 2^x \cdot \ln 2 + \left(\frac{1}{5}\right)^x \cdot \ln \frac{1}{5} + e^x - 3 \cdot \frac{1}{x} + 7 \cdot \frac{1}{x \ln 3} - 2 \cdot \frac{1}{x \ln \frac{1}{7}}$
- 6) $f(x) = 10^x \cdot x^{10}$
 $f'(x) = 10^x \cdot \ln 10 \cdot x^{10} + 10^x \cdot 10 \cdot x^9$
- 7) $f(x) = \sin x \cdot \cos x$
 $f'(x) = \cos x \cdot \cos x + \sin x \cdot (-\sin x)$
- 8) $f(x) = (3x^4 - 10x^2 + 11) \cdot 20^x$
 $f'(x) = (12x^3 - 20x) \cdot 20^x + (3x^4 - 10x^2 + 11) \cdot 20^x \cdot \ln 20$
- 9) $f(x) = \frac{4x^2}{x^2 + 3x}$
 $f'(x) = \frac{4 \cdot (x^2 + 3x) - 4x^2 \cdot (2x + 3)}{(x^2 + 3x)^2}$
- 10) $f(x) = \frac{e^x}{\sin x}$
 $f'(x) = \frac{e^x \cdot \sin x - e^x \cdot \cos x}{\sin^2 x}$
- 11) $f(x) = \frac{\ln x}{x}$
 $f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2}$
- 12) $f(x) = x^5 \cdot \cos x$
 $f'(x) = 5x^4 \cdot \cos x + x^5 \cdot (-\sin x)$
- 13) $f(x) = e^x \cdot \arcsin x$
 $f'(x) = e^x \cdot \arcsin x + e^x \cdot \frac{1}{\sqrt{1-x^2}}$
- 14) $f(x) = x \cdot \arccos x + \operatorname{arctg} x - \frac{x^2 + x}{\operatorname{arccot} x}$
 $f'(x) = \arccos x + x \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right) + \frac{1}{1+x^2} - \frac{(2x+1) \cdot \operatorname{arccot} x - (x^2+x) \cdot \left(-\frac{1}{1+x^2}\right)}{(\operatorname{arccot} x)^2}$

Derivácia zložených funkcií

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

- 15) $f(x) = (3x^2 + 11x - 7)^{15}$
 $f'(x) = 15(3x^2 + 11x - 7)^{14} \cdot (6x + 11)$
- 16) $f(x) = 3^{\sin x}$
 $f'(x) = 3^{\sin x} \cdot \ln 3 \cdot \cos x$
- 17) $f(x) = \cos(\ln x)$
 $f'(x) = -\sin(\ln x) \cdot \frac{1}{x}$
- 18) $f(x) = (\ln x)^3$
 $f'(x) = 3(\ln x)^2 \cdot \frac{1}{x}$
- 19) $f(x) = \ln^3 x = (\ln x)^3$
 $f'(x) = \frac{\ln x^3}{x^3} = \frac{\ln(x^3)}{x^3}$
 $f'(x) = \frac{1}{x^3} \cdot 3x^2$
- 21) $f(x) = \frac{4x \cdot e^x}{\cos^3(x \cdot e^x)}$
 $f'(x) = \frac{1}{\cos^3(x \cdot e^x)} \cdot [e^x + x \cdot e^x]$
- 22) $f(x) = \log^5 x^3 \cdot [\log(x^3)]^5$
 $f'(x) = 5 \cdot (\log x^3)^4 \cdot \frac{1}{x^3 \cdot \ln 10} \cdot 3x^2$
- 23) $f(x) = \ln(\ln(\ln(2x)))$
 $f'(x) = \frac{1}{\ln(\ln(2x))} \cdot \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot 2$
- 24) $f(x) = \sqrt{\cos x^2} = (\cos x^2)^{\frac{1}{2}}$
 $f'(x) = \frac{1}{2} (\cos x^2)^{-\frac{1}{2}} \cdot (-\sin x^2) \cdot 2x$
- 25) $f(x) = \operatorname{arctg} \left(\frac{2x+5}{3x-4} \right)$
 $f'(x) = \frac{1}{1 + \left(\frac{2x+5}{3x-4}\right)^2} \cdot \frac{2 \cdot (3x-4) - (2x+5) \cdot 3}{(3x-4)^2}$
- 26) $f(x) = \log_4(x^3 \cdot \cos x)$
 $f'(x) = \frac{1}{x^3 \cdot \cos x \cdot \ln 4} \cdot [3x^2 \cdot \cos x + x^3 \cdot (-\sin x)]$
- 27) $f(x) = \ln \frac{4x+1}{4x-1}$
 $f'(x) = \frac{1}{\frac{4x+1}{4x-1}} \cdot \frac{4(4x-1) - (4x+1) \cdot 4}{(4x-1)^2}$
- 28) $f(x) = \frac{e^x \cdot \ln x}{(3x+2)^4}$
 $f'(x) = \frac{(e^x \cdot \ln x)' \cdot (3x+2)^4 - e^x \cdot \ln x \cdot 4(3x+2)^3}{(3x+2)^8}$
- 29) $f(x) = \sin^4(\operatorname{arctg} \sqrt{x})^3 = (\sin(\operatorname{arctg} \sqrt{x}))^3$
 $f'(x) = 4 \cdot (\sin(\operatorname{arctg} \sqrt{x}))^2 \cdot \cos(\operatorname{arctg} \sqrt{x}) \cdot 3 \cdot (\operatorname{arctg} \sqrt{x})^2 \cdot \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2} x^{-\frac{1}{2}}$
- 30) $f(x) = \frac{\ln(2^x - x^7)}{(\cos x)^3}$
 $f'(x) = \frac{\frac{1}{2^x - x^7} \cdot (2^x \ln 2 - 7x^6) \cdot (\cos x)^3 - \ln(2^x - x^7) \cdot 3(\cos x)^2 \cdot (-\sin x)}{(cos x)^6}$

Derivácia vyšších rádov

$$\begin{aligned}
 f(x) &= 3x^4 - 2x^3 + 7x^2 - 11x + 10 & f'(x) &= 12x^3 - 6x^2 + 14x - 11 \\
 f'(x) &= 12x^3 - 6x^2 + 14x - 11 & f''(x) &= 36x^2 - 12x + 14 \\
 f''(x) &= (f'(x))' = 36x^2 - 12x + 14 & f'''(x) &= 72x - 12 \\
 f'''(x) &= 72x - 12 & f^{(4)}(x) &= 72
 \end{aligned}$$

$[f(x)]$ konst.

$$\begin{aligned}
 (\text{konst.}) f(x) & \quad \ln a^b = b \ln a \\
 [f(x)]^{g(x)} &= e^{\ln [f(x)]^{g(x)}} = e^{g(x) \cdot \ln f(x)} \\
 &= \frac{e^{g(x) \cdot \ln f(x)}}{(konst.)^{g(x)}}
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= x^{\sin x} = e^{\sin x \cdot \ln x} \\
 f'(x) &= e^{\sin x \cdot \ln x} \left[\cos x \cdot \ln x + \sin x \cdot \frac{1}{x} \right]
 \end{aligned}$$