MATEMATIKA I.

Lineárne a kvadratické rovnice s absolútnou hodnotou

$|a| = \begin{cases} a, & a \geq 0 \\ -a & a < 0 \end{cases}$

Pr. $|3x-1| = |3x-2|$

$3x=1$, $x=\frac{1}{3}$; $x=\frac{2}{3}$

NB: $\frac{1}{3}, \frac{2}{3}$

	$(-\infty, \frac{1}{3})$	$(\frac{1}{3}, \frac{2}{3})$	$(\frac{2}{3}, \infty)$
$3x-1$	$-$	$+$	$+$
$3x-2$	$-$	$-$	$+$

$x \in (-\infty, \frac{1}{3})$:
$-3x+1 = -3x+2$
$1 \neq 2$
$K_1 = \emptyset$

$x \in (\frac{1}{3}, \frac{2}{3})$:
$3x-1 = -3x+2$
$6x = 3$
$x = \frac{1}{2} \in (\frac{1}{3}, \frac{2}{3})$
$K_2 = \{\frac{1}{2}\}$

$x \in (\frac{2}{3}, \infty)$:
$3x-1 = 3x-2$
$-1 \neq -2$
$K_3 = \emptyset$

pre $x = \frac{1}{3}$ dostávame
$|1-1| = |1-2|$
$0 \neq 1$

pre $x = \frac{2}{3}$ dostávame
$|2-1| = |2-2|$
$1 \neq 0$

$K = K_1 \cup K_2 \cup K_3 = \{\frac{1}{2}\}$

Pr. $|x^2+1| = |2x+1|$, $x^2+1 > 0$

$x^2+1 = |2x+1|$

NB: $-\frac{1}{2}$

	$I_1 = (-\infty, -\frac{1}{2})$	$(-\frac{1}{2}, \infty) = I_2$
$2x+1$	$-$	$+$
	$x^2+1 = -2x-1$	$x^2+1 = 2x+1$
	$x^2+2x+2 = 0$	$x^2-2x = 0$
	$(x+1)^2+1 = 0$, $(x+1)^2+1 > 0$	$x(x-2) = 0$
	$\emptyset$	$x_1 = 0 \in I_2$
		$x_2 = 2 \in I_2$

$x^2+2x+1 = (x+1)^2$

$x \in \{0, 2\}$

$x = -\frac{1}{2}$: $|(-\frac{1}{2})^2+1| \stackrel{?}{=} |2(-\frac{1}{2})+1|$
$\frac{5}{4} \neq 0$

LINEÁRNE A KVADRATICKÉ NEROVNICE

Pozn. $ax+b=0$, $a \neq 0$
$ax^2+bx+c=0$, $a \neq 0$

$ax+b \; \leq, <, >, \geq \; 0$

Pr. $2x+5 > 0 \quad /-5$
$2x > -5 \quad /:2$
$x > -\frac{5}{2}$
$x \in (-\frac{5}{2}, \infty)$

Pr. $-3x+6 \leq 0 \quad /-6$
$-3x \leq -6 \quad /:(-3)$
$x \geq 2$
$x \in \langle 2, \infty)$

① $x^2+6x+9 \geq 0$
$(x+3)^2 \geq 0$
$x \in \mathbb{R}$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

② $x^2-4x+4 \leq 0$
$(x-2)^2 \leq 0 \Leftrightarrow x-2=0$
$x=2$

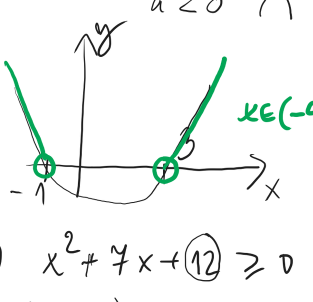
③ $x^2-2x-3 > 0$ (1 · 3)
$(x-3)(x+1) > 0$

$ax^2+bx+c=0$
$a(x-x_1)(x-x_2)=0$
$D = b^2-4ac$

$D>0$: $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$, 2 rieš.
$D=0$: $x_1 = x_2 = \frac{-b}{2a}$, 1 rieš.
$D<0$: v $\mathbb{R}$ riеš. nex.

$x^2+px+q=0$
$(x-x_1)(x-x_2)=0$
$x_1 \cdot x_2 = q$
$x_1 + x_2 = -p \rightarrow -x_1-x_2=p$

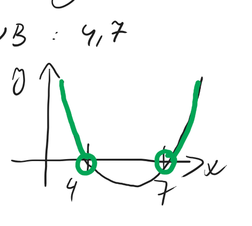
$y = ax^2+bx+c$ parabola: $a>0$ ∪, $a<0$ ∩



$x \in (-\infty, -1) \cup (3, \infty)$

④ $x^2+7x+12 \geq 0$ (1.12, 2.6, 3.4)
$(x+3)(x+4) \geq 0$
NB: $-3, -4$
(+) -4 (−) -3 (+)
$x \in (-\infty, -4\rangle \cup \langle -3, \infty)$

⑤ $x^2-11x+28 > 0$ (1 . 28, 2 . 14, 4 . 7)
$(x-4)(x-7) > 0$
NB: 4, 7
$x \in (-\infty, 4) \cup (7, \infty)$



⑥ $x^2+2x+2 < 0$
$(x+1)^2+1 < 0$, $(x+1)^2+1 > 0$ → x − neex.

$x^2+2x+2=0$
$(x+1)^2+1=0$

$D = b^2-4ac = 4-4\cdot 2 = -4 < 0$

$(a-b)^2 = a^2-2ab+b^2$

⑦ $x^2-x+1 > 0$
$(x-\frac{1}{2})^2+\frac{1}{2} > 0$, > 0 → $x \in \mathbb{R}$

$x^2-x+1=0$
$x^2-x+\frac{1}{2}$

$D = b^2-4ac = 1-4 = -3 < 0$
v $\mathbb{R}$ $\sqrt{-3}$ nie je def.

FUNKCIE

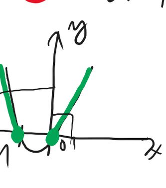
$D(f) = ?$ Určme definičný obor danej fcie $y = f(x)$

① $y = \frac{1}{x-2}$
$x-2 \neq 0$
$x \neq 2$
$D(f) = \mathbb{R} - \{2\}$

② $y = \frac{5x+10}{x+3}$
$x+3 \neq 0$
$x = -3$
$D(f) = \mathbb{R} - \{-3\}$

③ $y = \frac{x}{(x-1)(x+1)}$
$(x-1)(x+1) \neq 0$
$x \neq 1$ a $x \neq -1$
$D(f) = \mathbb{R} - \{\pm 1\}$

④ $y = \sqrt{x^2+x}$
$x^2+x \geq 0$
$x(x+1) \geq 0$
NB: $-1, 0$
$x \in (-\infty, -1\rangle \cup \langle 0, \infty)$



$y = \sqrt[n]{\ast}$ n-párne, $\ast \geq 0$

⑤ $y = \sqrt[4]{\frac{5x+10}{x+3}}$
$\frac{5x+10}{x+3} \geq 0 \quad /:5$

$A \cdot B > 0 \Leftrightarrow (A>0 \wedge B>0) \vee (A<0 \wedge B<0)$
$+ \cdot +$, $- \cdot -$
$<$: $+ \cdot -$, $- \cdot +$

$\frac{A}{B} > 0$: $\frac{+}{+}$, $\frac{-}{-}$; $\frac{A}{B} < 0$: $\frac{+}{-}$, $\frac{-}{+}$

$\frac{x+2}{x+3} \geq 0$ NB: $-3, -2$
$\frac{x+2}{x+3}$: (+) -3 (−) -2 (+)
$x \in (-\infty, -3) \cup \langle -2, \infty)$

⑥ $\frac{x(x-1)(x+7)}{(x+4)(x-5)} < 0$ (ĽS)
NB: $0, 1, -7, -4, 5$
ĽS: (−) -7 (+) -4 (−) 0 (+) 1 (−) 5 (+)
$x \in (-\infty, -7) \cup (-4, 0) \cup (1, 5)$

⑦ $y = \sqrt{\frac{10}{x+1}}$; $\frac{10}{x+1} \geq 0 \Leftrightarrow x+1 > 0$
$x > -1$
$x \in (-1, \infty) = D(f)$

⑧ $y = \sqrt[6]{(x-5)(x+6)}$
$(x-5)(x+6) \geq 0$
(+) -6 (−) 5 (+)
$D(f) = (-\infty, -6\rangle \cup \langle 5, \infty)$

⑨ $y = \sqrt[3]{x^3-1}$
$y = \sqrt[n]{\ast}$ n-nepárne, $\ast$ − ľub.
$x \in \mathbb{R}$, $D(f) = \mathbb{R}$

⑩ $y = \sqrt[5]{\frac{1}{(x+1)(x-1)}}$
$(x+1)(x-1) \neq 0$
$x \neq \pm 1$
$D(f) = \mathbb{R} - \{\pm 1\}$

⑪ $y = \sqrt[3]{x^2+2x-4}$, $x \in \mathbb{R} = D(f)$

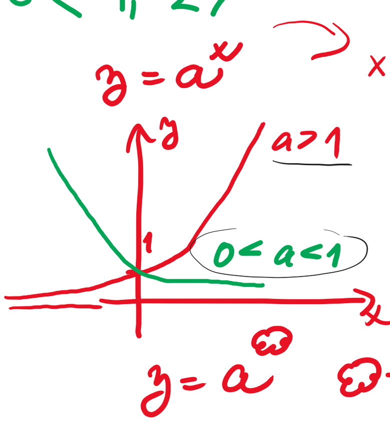
⑫ $y = \sqrt[15]{\frac{1}{x^2+2x-3}}$
$x^2+2x-3 \neq 0$ (3 · 1)
$(x-1)(x+3) \neq 0$
$D(f) = \mathbb{R} - \{-3, 1\}$

⑬ $y = \sqrt{x^2+6x+8}$
$x^2+6x+8 \geq 0$ (2 · 4, 1 · 8)
$(x+2)(x+4) \geq 0$
(+) -4 (−) -2 (+)
$D(f) = (-\infty, -4\rangle \cup \langle -2, \infty)$

⑭ $y = \sqrt{-x^2-6x-8}$
$-x^2-6x-8 \geq 0 \quad /\cdot(-1)$
$x^2+6x+8 \leq 0$
$x \in \langle -4, -2 \rangle$

⑮ $y = 2^{x^2-2x-3}$, $D(f) = \mathbb{R}$

$y = a^x \rightarrow x \in \mathbb{R}$



$y = a^{\ast}$, $\ast$ − ľub.

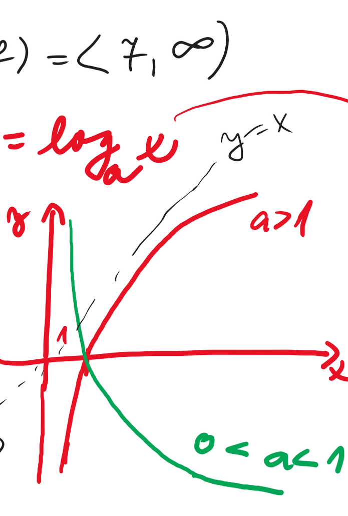
⑯ $y = \left(\frac{1}{2}\right)^{\frac{1}{x+10}}$
$x+10 \neq 0$
$x = -10$
$D(f) = \mathbb{R} - \{-10\}$

⑰ $y = e^{\frac{x-1}{x+1}}$
$x+1 \neq 0$
$D(f) = \mathbb{R} - \{-1\}$

⑱ $y = \left(\frac{1}{5}\right)^{\sqrt{x-7}}$
$x-7 \geq 0$
$x \geq 7$, $D(f) = \langle 7, \infty)$

⑲ $y = \log_2(x^2-4)$
$x^2-4 > 0$
$x^2 > 4$
$|x| > 2$
$|x-0| > 2$
-2, 0, 2
$D(f) = (-\infty, -2) \cup (2, \infty)$

$y = \log_a x \rightarrow x > 0$



$|x-9|$ vzdialenosť x od čísla 9

$y = \log_a \ast$, $\ast > 0$

⑳ $y = \log_{\frac{1}{3}} \frac{5}{x^2+10x}$
$\frac{5}{x^2+10x} > 0 \Leftrightarrow x^2+10x > 0$
$x(x+10) > 0$
(+) -10 (−) 0 (+)
$D(f) = (-\infty, -10) \cup (0, \infty)$

$y = \log \frac{-5}{x^2+10x}$, $x^2+10x < 0$

㉑ $y = 4^{\ln(5-x)}$
$5-x > 0$
$5 > x$
$D(f) = (-\infty, 5)$

㉒ $y = \ln \frac{x-4}{x+4}$
$\frac{x-4}{x+4} > 0$
(+) -4 (−) 4 (+)
$D(f) = (-\infty, -4) \cup (4, \infty)$

㉓ $y = \ln(x^2-11x-12)$
$x^2-11x-12 > 0$
$(x-12)(x+1) > 0$
(+) -1 (−) 12 (+)
$D(f) = (-\infty, -1) \cup (12, \infty)$

S akou pravdepodobnosťou by ste odporučili Whiteboard priateľovi alebo kolegovi?

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