Určme definičný obor danej funkcie:

(1.) $y = \sqrt{\ln \frac{5}{x-3}}$

$x-3 \neq 0 \land \ln \frac{5}{x-3} \geq 0$

$x \neq 3$

$\frac{5}{x-3} \geq 1$

$\frac{5}{x-3} - 1 \geq 0$

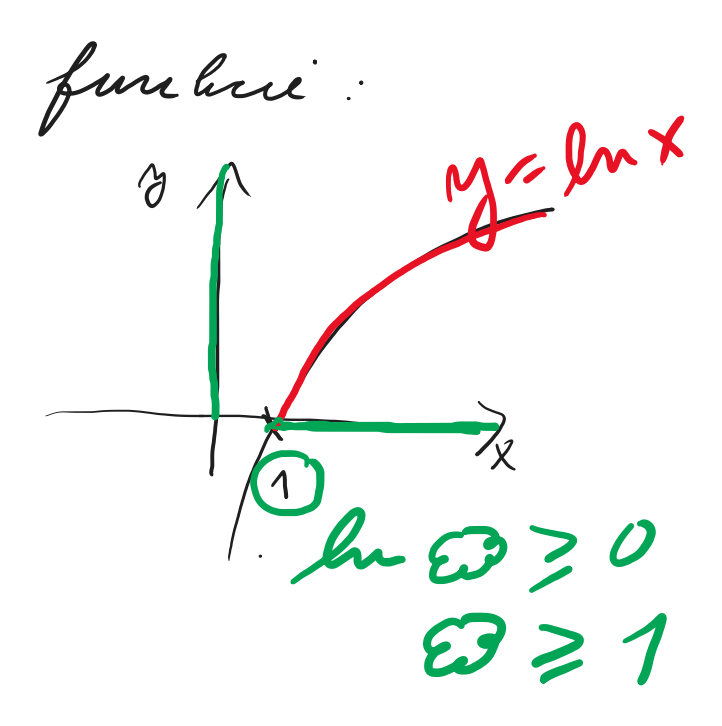
$\frac{5-x+3}{x-3} \geq 0$

$\frac{-x+8}{x-3} \geq 0 \quad /\cdot(-1)$

$\frac{x-8}{x-3} \leq 0$ NB: 3, 8

$x \in (3, 8\rangle$

$D(f) = (3, 8\rangle$



(2.) $y = \sqrt[3]{\log(x+2)}$ $\quad x+2>0$, $x>-2$ $\quad D(f) = (-2, \infty)$ $\quad \log ☺ = 0$, $☺ = 1$

(3.) $y = \sqrt[5]{\frac{1}{\log(x-7)}}$ $\quad x-7>0$, $x>7$ $\land \log(x-7) \neq 0$, $x-7 \neq 1$, $x \neq 8$

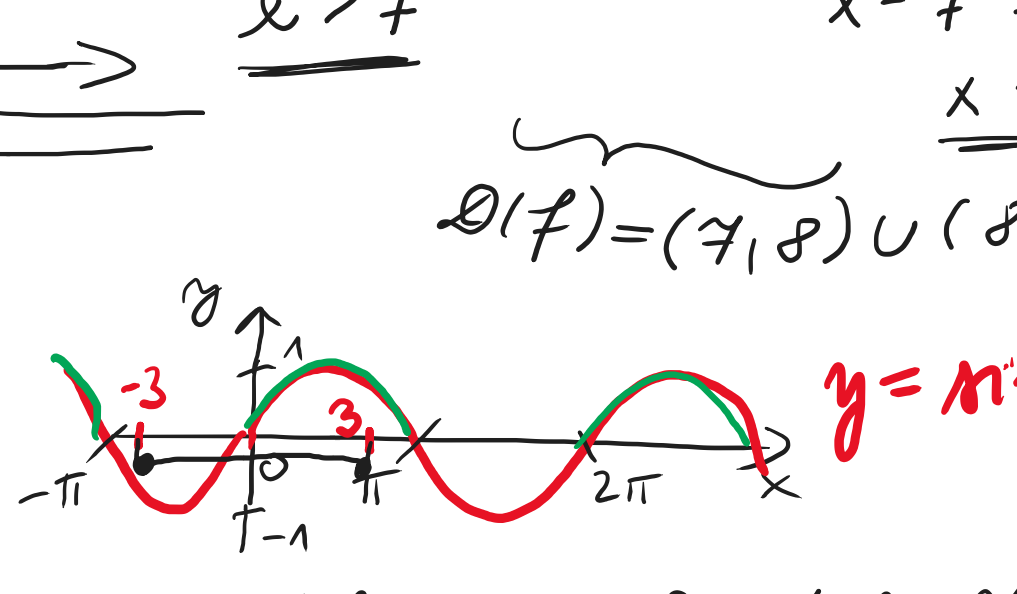
$D(f) = (7, 8) \cup (8, \infty)$

$y = \frac{1}{\sin x}$

$\sin x \neq 0$

$x \neq k\pi, k \in \mathbb{Z}$

$D(f) = \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}$



(4.) $y = \sqrt{\sin x} + \sqrt{9-x^2}$

$\sin x \geq 0 \land 9 - x^2 \geq 0$

$9 \geq x^2 \quad /\sqrt{\ }$

$3 \geq |x|$

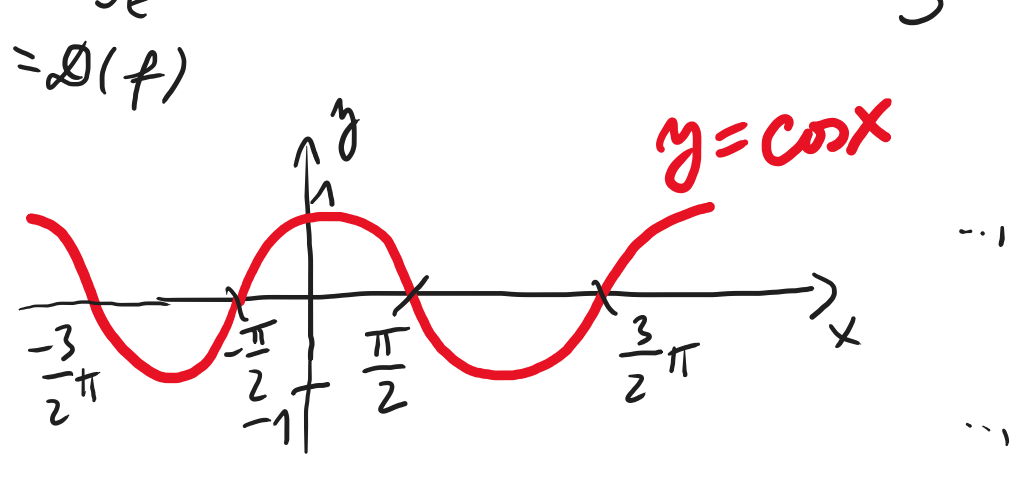
$|x| \leq 3 \rightarrow |x-0| \leq 3$

$x \in \langle -3, 3\rangle$

PRIENIK JE $\langle 0, 3\rangle = D(f)$

$|x - x_0| \leq k$

(5.) $y = \frac{1}{\cos 2x}$



$k \in \mathbb{Z}$: $2k$ PÁRNE: $\ldots, -4, -2, 0, 2, 4, \ldots$; $2k+1$ NEPÁRNE: $\ldots, -3, -1, 1, 3, 5, \ldots$

$\cos 2x \neq 0$

$2x \neq \frac{\pi}{2}(2k+1) \quad k \in \mathbb{Z}$

$x \neq \frac{\pi}{4}(2k+1)$

$D(f) = \mathbb{R} - \{\frac{\pi}{4}(2k+1); k \in \mathbb{Z}\}$

(6.) $y = tg\sqrt{2x}$

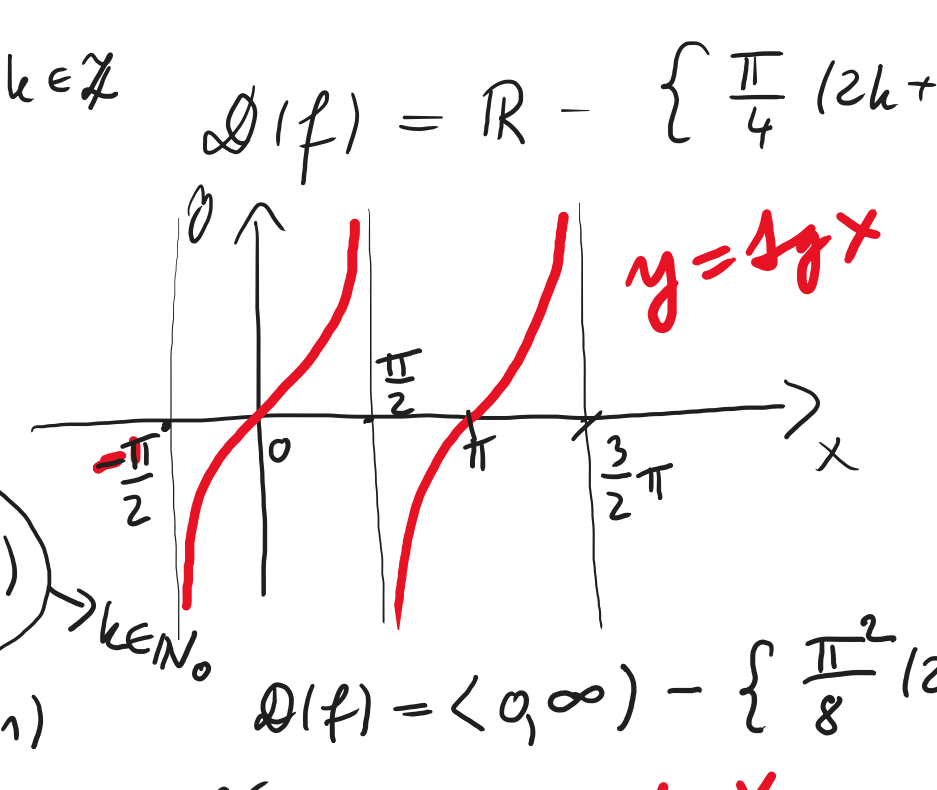
$x \geq 0$

$\land \sqrt{2x} \neq \frac{\pi}{2}(2k+1) \quad k \in N_0$

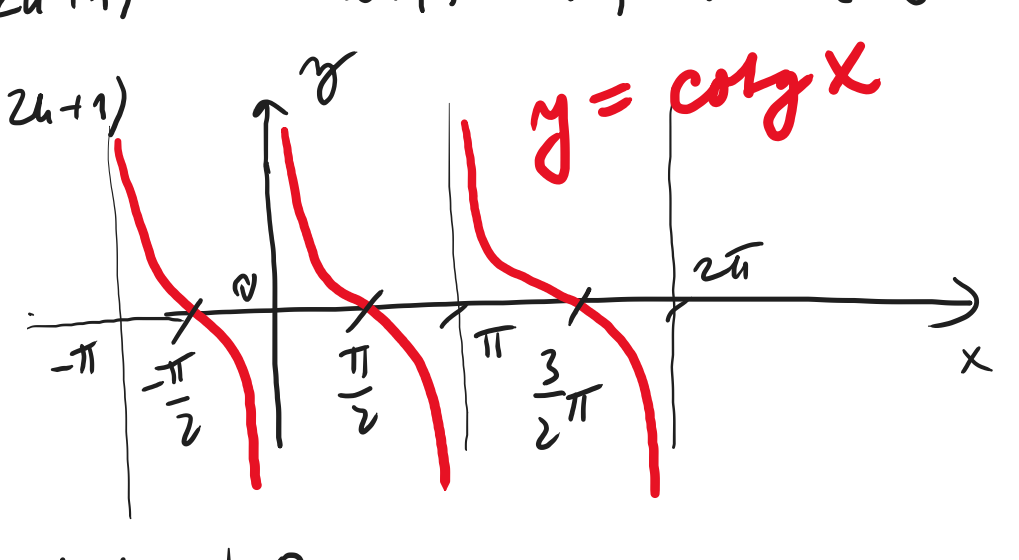
$2x \neq \frac{\pi^2}{4}(2k+1)$

$x \neq \frac{\pi^2}{8}(2k+1)$

$D(f) = \langle 0, \infty) - \{\frac{\pi^2}{8}(2k+1); k \in N_0\}$



(7.) $y = \frac{x}{cotg 4x}$



$4x \neq k\pi \land cotg 4x \neq 0$

$x \neq \frac{\pi}{4}k$ $\quad 4x \neq \frac{\pi}{2}(2k+1)$, $x \neq \frac{\pi}{8}(2k+1)$

$D(f) = \mathbb{R} - \{\frac{\pi}{4}k; \frac{\pi}{8}(2k+1); k \in \mathbb{Z}\}$

(8.) $y = \log_3(2\cos x - \sqrt{3})$, $x \in \langle 0, 2\pi)$

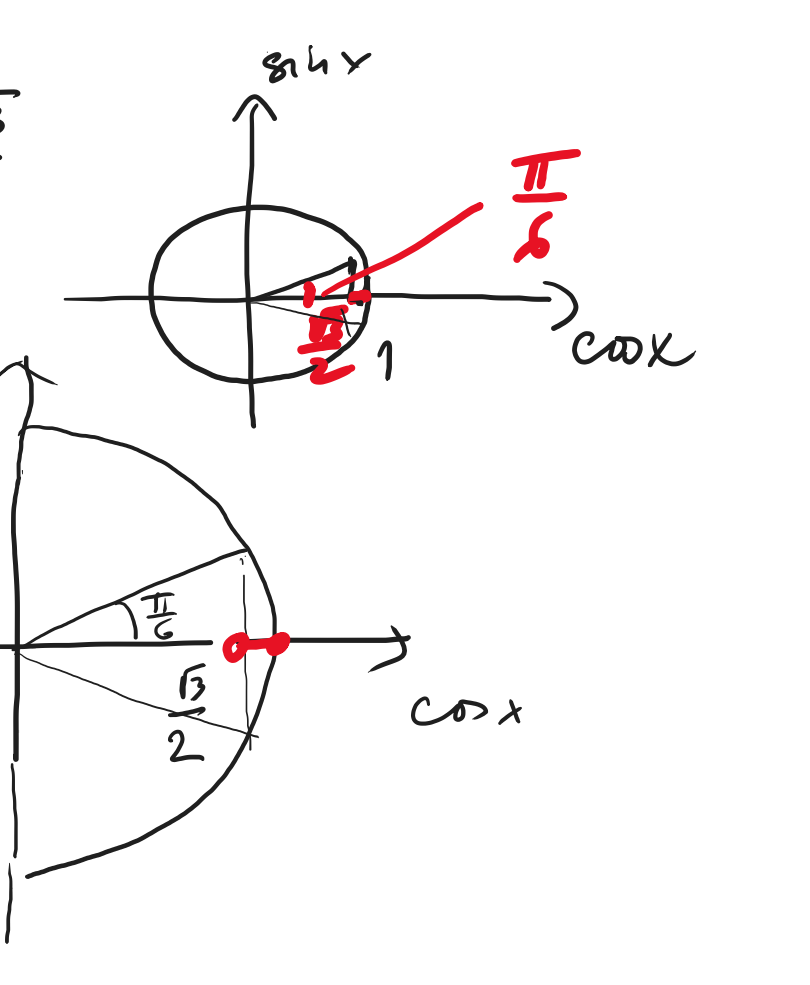
$2\cos x - \sqrt{3} > 0$

$2\cos x > \sqrt{3}$

$\cos x > \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$x \in \langle 0, \frac{\pi}{6}) \cup (\frac{11}{6}\pi, 2\pi)$

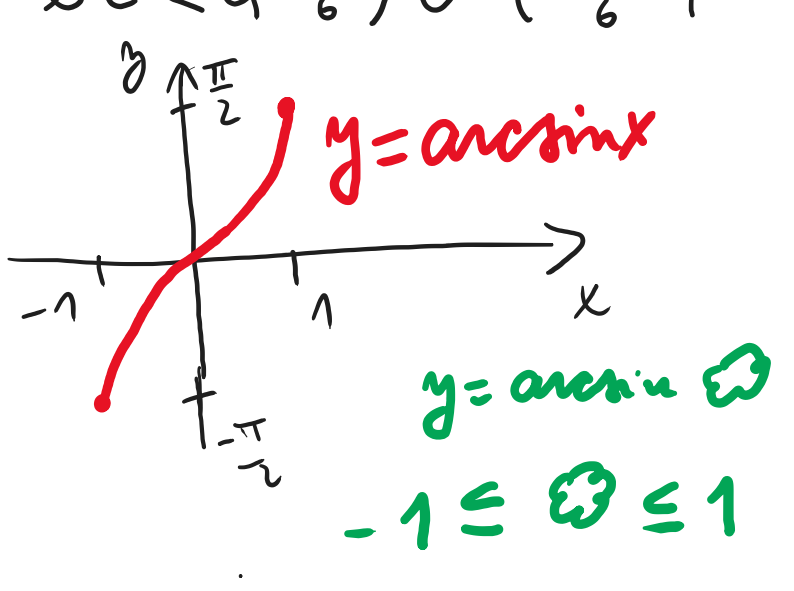


(9.) $y = \arcsin(x+2)$

$-1 \leq x+2 \leq 1 \quad /-2$

$-3 \leq x \leq -1$

$D(f) = \langle -3, -1\rangle$



(10.) $y = \arcsin \frac{3x-2}{5}$

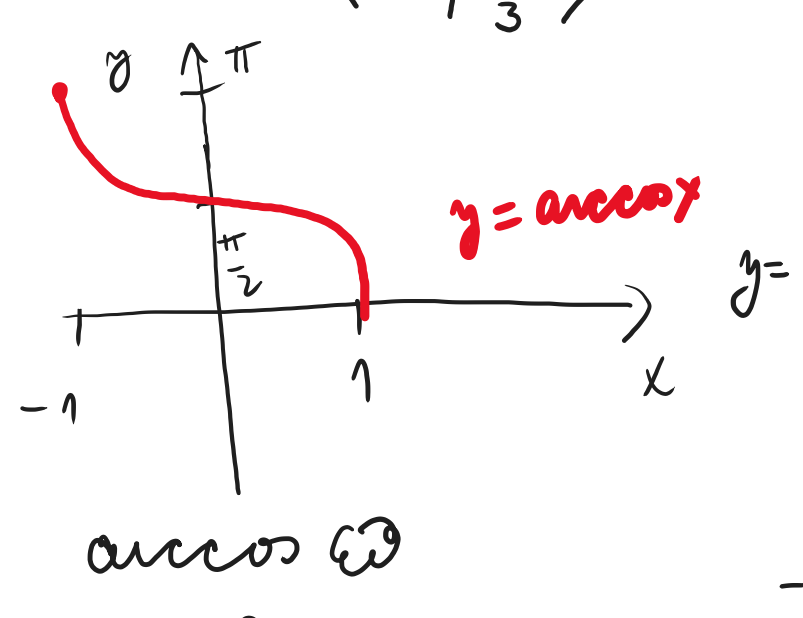
$-1 \leq \frac{3x-2}{5} \leq 1 \quad /\cdot 5$

$-5 \leq 3x-2 \leq 5 \quad /+2$

$-3 \leq 3x \leq 7 \quad /:3$

$-1 \leq x \leq \frac{7}{3}$

$x \in \langle -1, \frac{7}{3}\rangle$



(11.) $y = \arcsin \frac{3x-2}{5} + \sqrt{5-3x}$

$-1 \leq \frac{3x-2}{5} \leq 1 \land 5 - 3x \geq 0$

$5 \geq 3x \quad /:3$

$x \in \langle -1, \frac{7}{3}\rangle$ $\quad x \leq \frac{5}{3}$

PRIENIK

$D(f) = \langle -1, \frac{5}{3}\rangle$

(12.) $y = \arccos \frac{1-x}{2}$

$-1 \leq \frac{1-x}{2} \leq 1 \quad /\cdot 2$

$-2 \leq 1-x \leq 2 \quad /-1$

$-3 \leq -x \leq 1 \quad /\cdot(-1)$

$3 \geq x \geq -1$

$-1 \leq x \leq 3$ $\quad D(f) = \langle -1, 3\rangle$

(13.) $y = \arccos(x^2-16)$

$-1 \leq x^2 - 16 \leq 1$

$15 \leq x^2 \leq 17$

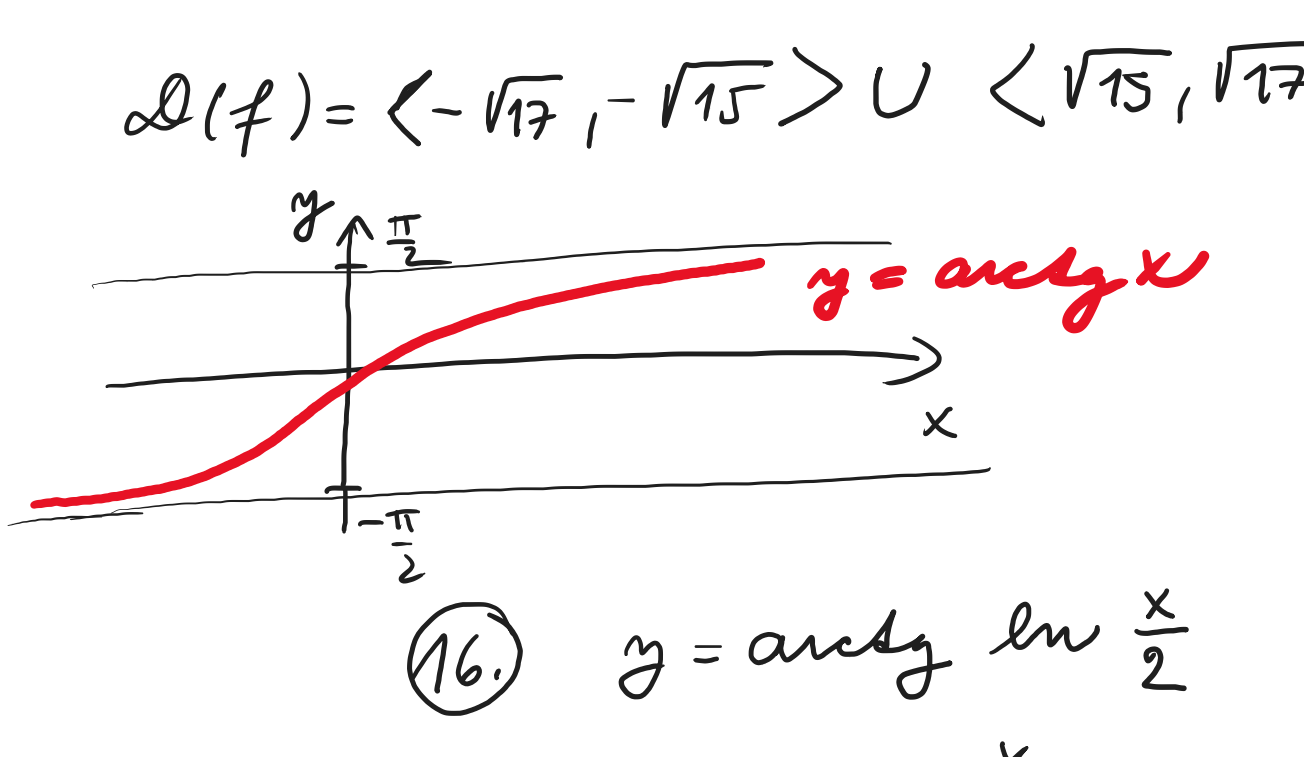
$\sqrt{15} \leq |x| \leq \sqrt{17}$

PRIENIK

$D(f) = \langle -\sqrt{17}, -\sqrt{15}\rangle \cup \langle \sqrt{15}, \sqrt{17}\rangle$

(14.) $y = arctg(x+13)$

$x \in D(f)$



(15.) $y = arctg\sqrt{x+1}$

$x+1 \geq 0$

$x \geq -1$

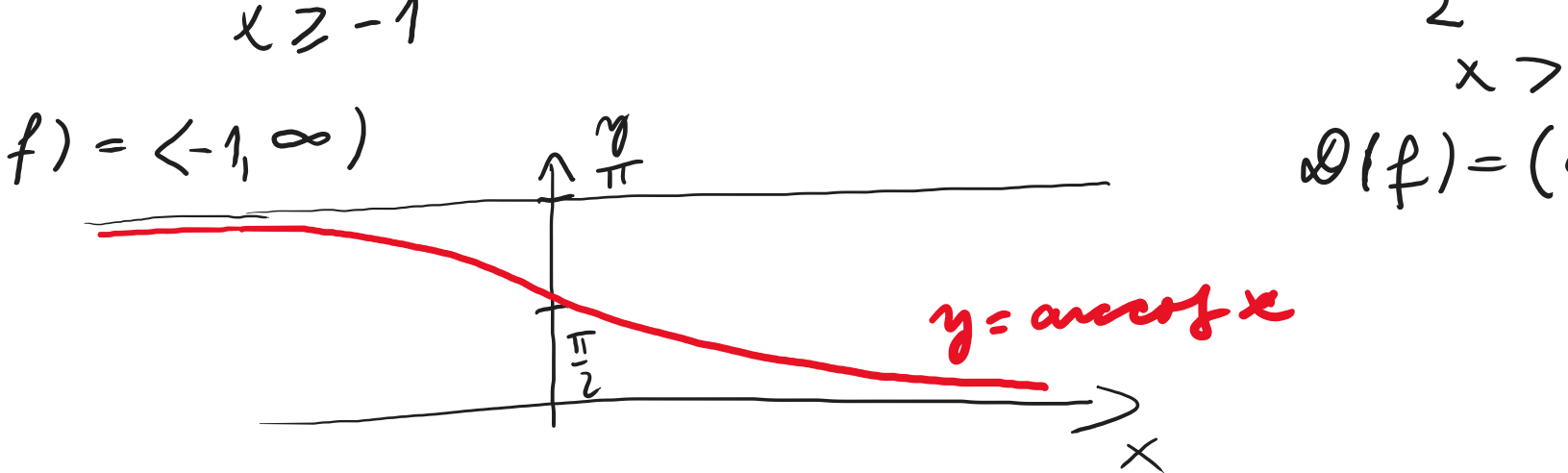
$D(f) = \langle -1, \infty)$

(16.) $y = arctg \ln \frac{x}{2}$

$\frac{x}{2} > 0$

$x > 0$

$D(f) = (0, \infty)$



(17.) $y = arccotg \frac{1}{x-2}$

$x \neq 2$

$D(f) = \mathbb{R} - \{2\}$

(18.) $y = arccotg \sqrt[3]{\frac{x+1}{x+2}}$

$x+2 \neq 0$, $x \neq -2$ $\quad D(f) = \mathbb{R} - \{-2\}$

(19.) $y = arccotg \sqrt[4]{\frac{x+1}{x-2}}$

$\frac{x+1}{x-2} \geq 0$ NB: $-1, 2$

$D(f) = (-\infty, -1\rangle \cup (2, \infty)$

(20.) $y = \frac{\sqrt{x^2-x-2}}{\ln x} + \arcsin \frac{1-2x}{4}$

$x^2 - x - 2 \geq 0 \land x > 0 \land -1 \leq \frac{1-2x}{4} \leq 1 \quad /\cdot 4 \land \ln x \neq 0$

$(x-2)(x+1) \geq 0$ $\quad -4 \leq 1-2x \leq 4 \quad /-1$ $\quad x \neq 1$

NB: $2, -1$ $\quad -5 \leq -2x \leq 3 \quad /:(-2)$

$\frac{5}{2} \geq x \geq -\frac{3}{2}$

$-\frac{3}{2} \leq x \leq \frac{5}{2}$

∩ PRIENIK

prienik všetkých 4 intervalov

$D(f) = \langle 2, \frac{5}{2}\rangle$

(21.) $y = \frac{\sqrt{x^2+4x-5}}{\ln(3-2x)} + arctg(e^x+2)$

$x^2+4x-5 \geq 0 \land 3-2x > 0 \land \ln(3-2x) \neq 0$

$(x+5)(x-1) \geq 0$ $\quad 3 > 2x$ $\quad 3-2x \neq 1$

$x < \frac{3}{2}$ $\quad -2x \neq -2$, $x \neq 1$

∩ prienik

$D(f) = (-\infty, -5\rangle \cup (1, \frac{3}{2})$

(22.) $y = \frac{x}{\sqrt{2x+8}} + \arccos(3x+10)$

$\ln x = \log_e x$

$\log x = \log_{10} x$

$2x+8>0 \land -1 \leq 3x+10 \leq 1$

$2x > -8$ $\quad -11 \leq 3x \leq -9$

$x > -4$ $\quad -\frac{11}{3} \leq x \leq -3$

(23.) $y = \frac{\log_2(3x+21)}{x^3-3x^2+2x}$ $\quad D(f) = \langle -\frac{11}{3}, -3\rangle$

$3x+21>0 \land x^3-3x^2+2x \neq 0$

$3x > -21$ $\quad x(x^2-3x+2) \neq 0$

$x > -7$ $\quad x(x-2)(x-1) \neq 0 \Leftrightarrow$

$\Leftrightarrow x \neq 0 \quad x \neq 2 \quad x \neq 1$

$D(f) = (-7, 0) \cup (0, 1) \cup (1, 2) \cup (2, \infty)$

$ax^2+bx+c=0 \longrightarrow x^2+px+q=0$

$(x-x_1)(x-x_2)=0$

$x^2 - x_1x - x_2x + x_1x_2 = 0$

$q = x_1 \cdot x_2$

$p = -x_1 - x_2$

$x^2+2x-3=0$ (3·1)

$(x-1)(x+3)=0$

$x^2-6x+5=0$ (5·1)

$(x-5)(x-1)=0$

$x^2+x-6=0$ (6·1, 2·3)

$(x-2)(x+3)=0$

$x^2+7x+12=(x+3)(x+4)$ (1·12, 2·6, 3·4)

$x^2-11x-12=(x+1)(x-12)$

$x^2+8x+12=(x+6)(x+2)$

$x^2-x-12=(x+4)(x-3)$

$x^2+x+2 \neq (x+\ )(x+\ )$ $\quad D=\ldots, x_{1,2}=\ldots$

$x^2+x-2=(x-1)(x+2)$ (2·1)

(2) $x^2+3x+4=0$

$x^2+\frac{3}{2}x+2=0$

$D=\ldots \quad x_{1,2}=\ldots$