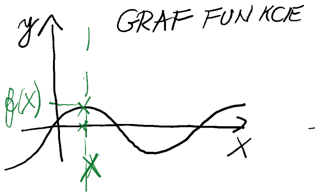
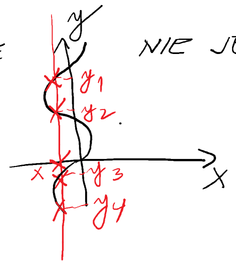
Pojem funkcie a funkcie ekonomickej analýzy

streda 11. októbra 2023 9:47

GRAF FUNKCIE





NIE JE GRAFOM FUNKCIE

$y = f(x)$!

↑ NEZÁVISLÁ PREMENNÁ

↑ ZÁVISLÁ PREMENNÁ

$D(f)$ ... DEFINIČNÝ OBOR

$H(f)$ ... OBOR HODNÔT

PRÍKLADY

$y = x^2 - 2x \quad D(f) = R$

$f(x) = \dfrac{x+1}{x^2-9} \quad D(f): x^2 - 9 \neq 0 \quad x \neq \pm 3 \quad D(f) = R - \{\pm 3\}$

$f(x) = \log_3 x \quad D(f): x > 0 \quad D(f) = R^+ = (0, \infty)$

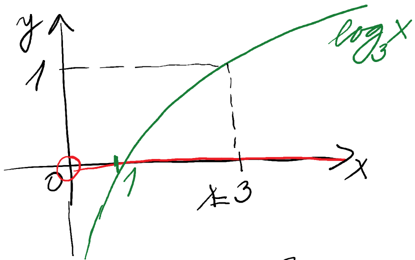
$y = \sqrt{x-2} \quad D(f): x - 2 \geq 0 \quad x \geq 2 \quad D(f) = \langle 2, \infty)$

PRAVIDLÁ NA URČOVANIE $D(f)$

1. VÝRAZ V MENOVATELI RÔZNY OD NULY $(\neq 0)$
2. VÝRAZ POD LOGARITMOM (ARGUMENT) KLADNÝ (> 0)
3. VÝRAZ POD PÁRNOU ODMOCNINOU NEZÁPORNÝ (≥ 0)

GRAF FUNKCIE

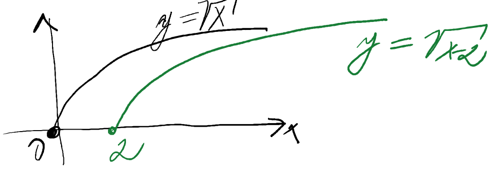
$y = \log_3 x$



$D(f) = (0, \infty)$

$H(f) = R$

$y = \sqrt{x-2}$



$D(f) = \langle 2, \infty)$

$H(f) = \langle 0, \infty)$

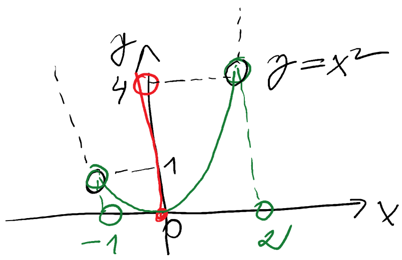
OHRANIČENÁ FUNKCIA

$f: \quad S \rightarrow f(S)$

1) $y = x^2 \quad D(f) = R$

$S = (-1, 2) \subset D(f)$

$f(S) = \langle 0, 4)$



ZDOLA OHRANIČENÁ $\quad k = -10 \quad f(x) \geq -10 \quad x \in S$

ZHORA OHRANIČENÁ $\quad K = 5 \quad f(x) \leq 5 \quad x \in S$

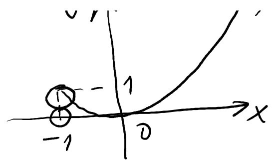
$\Rightarrow f(x)$ je ohraničená na S

2) $y = x^2$

2) $y=x^2$

$S=(-1,\infty)$

$f(S)=\langle 0,\infty)$

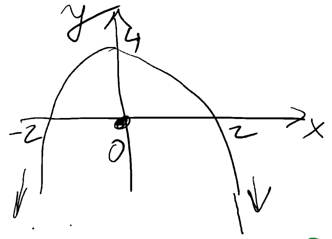


ZDOLA OHRANIČENÁ
ZHORA NEOHRANIČENÁ } NIE JE OHRANIČENÁ

3) $y=4-x^2$

$S=\langle 0,\infty)$

$f(S)=(-\infty,4\rangle$



ZHORA OHRANIČENÁ
ZDOLA NEOHRANIČENÁ } NIE JE OHRANIČENÁ

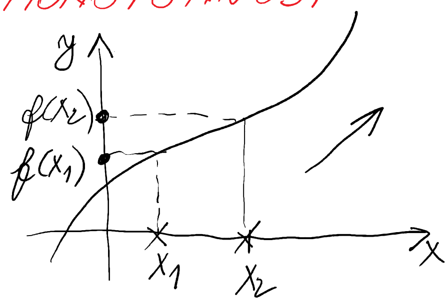
EXTRÉMY : MAXIMUM / MINIMUM

1) $\min\limits_{x\in(-1,2)} x^2 = 0$ $\qquad \nexists \max$

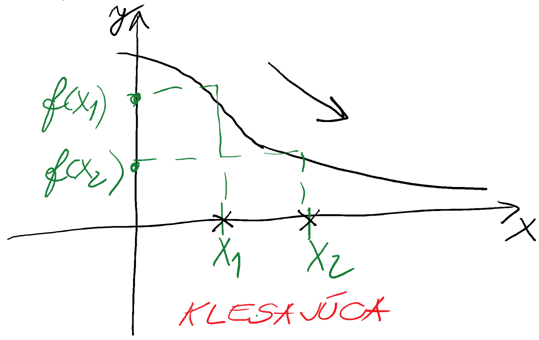
2) $\min\limits_{x\in(-1,\infty)} x^2 = 0$ $\qquad \nexists \max$

3) $\nexists \min\limits_{x\in\langle 0,\infty)} (4-x^2)$ $\qquad \max\limits_{x\in\langle 0,\infty)} (4-x^2) = 4$

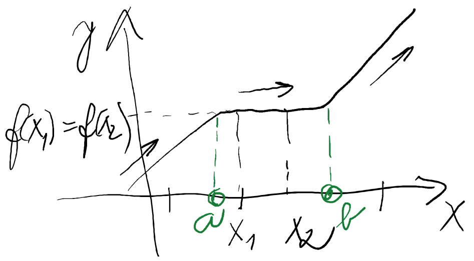
MONOTÓNNOSŤ : RASTÚCOSŤ / KLESAJÚCOSŤ



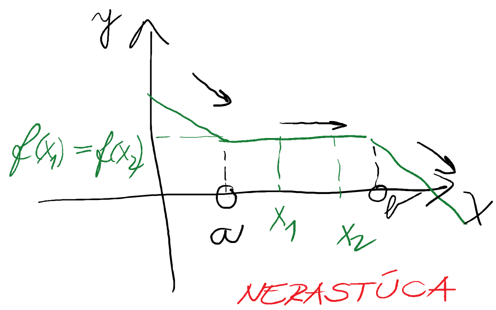
RASTÚCA



KLESAJÚCA



NEKLESAJÚCA

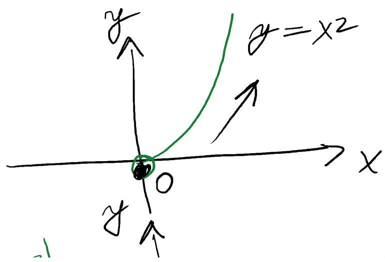


NERASTÚCA

PR. $y=x^2$ $\qquad D(f)=R$

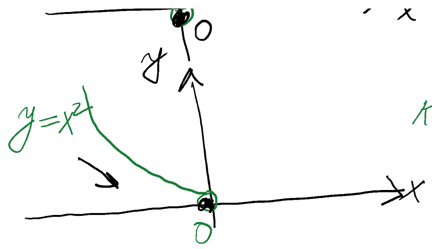
1) $S=\langle 0,\infty)$

2) $S=(-\infty,0\rangle$



RASTÚCA

1) $S = \langle 0, \infty)$
2) $S = (-\infty, 0\rangle$

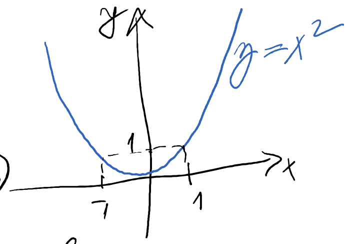


KLESAJÚCA

PÁRNOSŤ / NEPÁRNOSŤ

$D(f) = \mathbb{R}$

$\forall x \in D(f) \quad -x \in D(f)$



$f(-x) = f(x)$

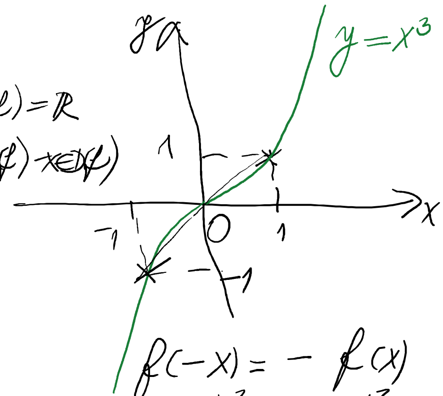
$(-x)^2 = x^2$

$x^2 = x^2$

$\cos(-x) = \cos x$

$D(f) = \mathbb{R}$

$\forall x \in D(f) \quad -x \in D(f)$



$f(-x) = -f(x)$

$(-x)^3 = -x^3$

$-x^3 = -x^3$

$\sin(-x) = -\sin x$

PERIODICKOSŤ

$\sin(x + 2\pi) = \sin x \qquad p = 2\pi$

$\cos(x + 2\pi) = \cos x \qquad p = 2\pi$

$\mathrm{tg}(x + \pi) = \mathrm{tg}\, x \qquad p = \pi$

$\mathrm{cotg}(x + \pi) = \mathrm{cotg}\, x \qquad p = \pi$

FUNKCIE EKONOMICKEJ ANALÝZY

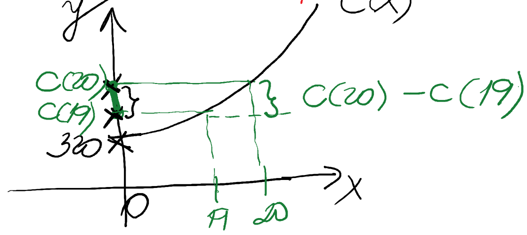
$$C(x) = \underbrace{x^3 - 2x^2 - 10x}_{V(x)} + \underbrace{320}_{K} = \underset{\substack{\uparrow \\ \text{počet ks} \\ q}}{x}\,\underbrace{(x^2 - 2x - 10)}_{p} + 320$$

náklad na 1 ks

$\boxed{p \cdot q} = V(x)$

1) $C(20) = 20^3 - 2 \cdot 20^2 - 10 \cdot 20 + 320$
$= 7\,320$

2) $C(20) - C(19) = 1053$



$S = \langle 0, \infty)$

3) $AC(x) = \dfrac{C(x)}{x}$

$AC(20) = \dfrac{C(20)}{20} = 366$

$AC(30) = \dfrac{C(30)}{30} = 840{,}67$

BOD ZLOMU

R. $C(x) = 0{,}1x^2 + 35x + 15\,000$

$p = 385 - 0{,}9x$ (eur)

$q = x$ (ks)

$R(x) = Z$

$$\boxed{\begin{aligned} R(x) &= C(x) \\ R(x) &> C(x) \end{aligned}}$$

$R(x) = p \cdot q = (385 - 0{,}9x) \cdot x$

$R(x) = (385 - 0{,}9x)x \geq 0{,}1x^2 + 35x + 15\,000 = C(x)$

$385x - 0{,}9x^2 \geq 0{,}1x^2 + 35x + 15\,000$

$x^2 - 350x + 15\,000 \leq 0$

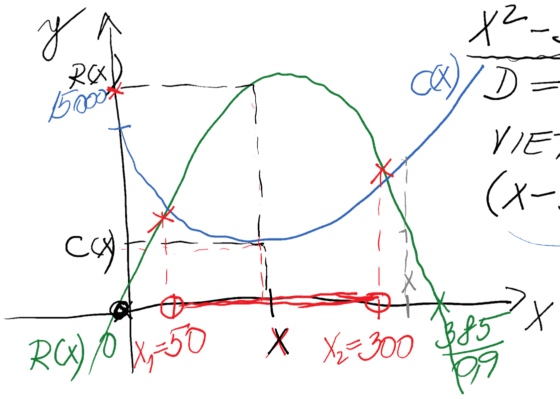
$D = b^2 - 4ac \qquad x_{1,2} = \dfrac{-b \pm \sqrt{D}}{2a}$

VIETOVE VZŤAHY

$(x-50)(x-300) \leq 0$

$x_1 = 50$

$x_2 = 300$



$\boxed{R(x) > C(x)}$

$(x-50)(x-300) < 0$

$+ \quad 50 \quad - \quad 300 \quad +$

$x \in (50, 300) \quad R(x) > C(x)$ je zóna zisková

EQUILIBRIUM – ROVNOVÁŽNY STAV

$D: \; q = 249 - 2p - p^2$

$S: \; q = 33 + 4p + p^2$

$D(p) = 249 - 2p - p^2$

$S(p) = 33 + 4p + p^2$

$D(p) = S(p)$

$249 - 2p - p^2 = 33 + 4p + p^2$

$2p^2 + 6p - 216 = 0 \quad /:2$

$p^2 + 3p - 108 = 0$

$(p+12)(p-9) = 0$

$p_1 = -12 < 0 \quad \emptyset$

$p_2 = 9 = p_E$

ROVNOVÁŽNA CENA je 9eur

$(q_E = 9^2 + 4 \cdot 9 + 33 = 150)$

$q_E = D(9) = S(9)$

$S: \; p^2 + 4p + 33 =$

$= (p+2)^2 - 2^2 + 33 =$

$= (p+2)^2 + 29 \quad V[-2, 29]$

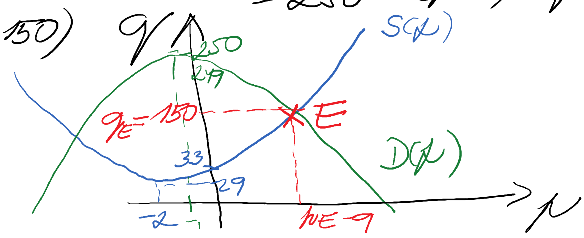
$D: \; 249 - 2p - p^2 =$

$= -[p^2 + 2p - 249] =$

$= -[(p+1)^2 - 1^2 - 249] =$

$= -[(p+1)^2 - 250] =$

$= 250 - (p+1)^2 \quad V = [-1, 250]$

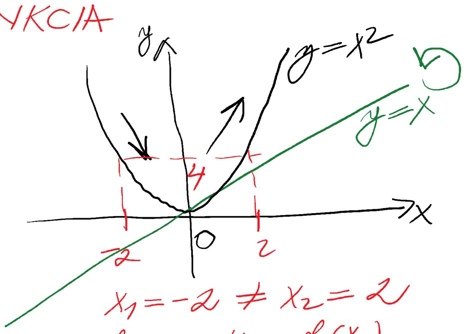


INVERZNÁ FUNKCIA

INVERZNÁ FUNKCIA

1) $y=x^2$

$D(f)=R$

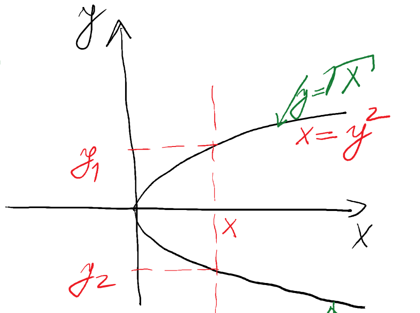


$x_1=-2 \neq x_2=2$

$f(x_1)=4=f(x_2)$

NIE JE PROSTÁ

$\nexists$ INVERZNÁ FUNKCIA NA $D(f)$



NIE JE FUNKCIA

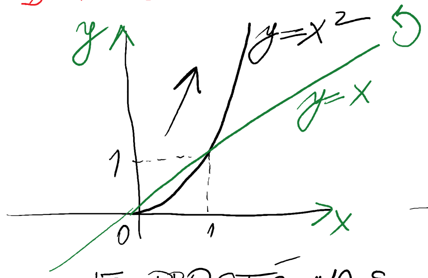
$y=-\sqrt{x}$

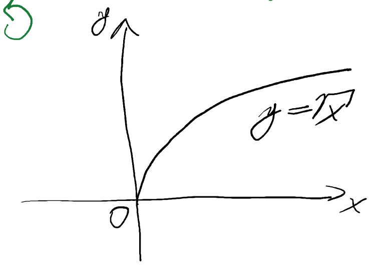
f: $y=x^2$

$S=\langle 0,\infty)$

$x=y^2$

f^{-1}: $y=\sqrt{x}$



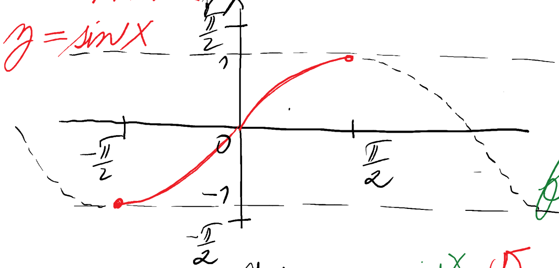


JE PROSTÁ NA S

$\exists$ INVERZNÁ FUNKCIA

INVERZNÉ FUNKCIE KU GONIOMETRICKÝM FUNKCIÁM

$y=\sin x$



$\boxed{D(f)=\langle -\frac{\pi}{2}, \frac{\pi}{2}\rangle}$

$H(f)=\langle -1, 1\rangle$

$\nearrow$ RASTÚCA

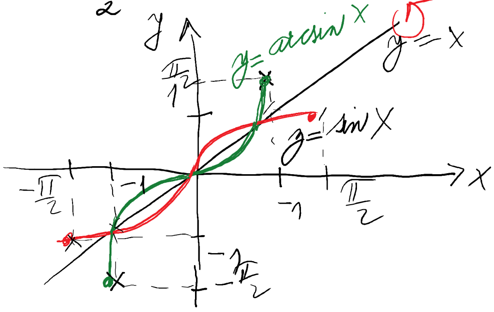
$\exists f^{-1}$

f^{-1}: $y = \arcsin x$

$D(f^{-1}) = H(f) = \langle -1, 1\rangle$

$H(f^{-1}) = D(f) = \langle -\frac{\pi}{2}, \frac{\pi}{2}\rangle$

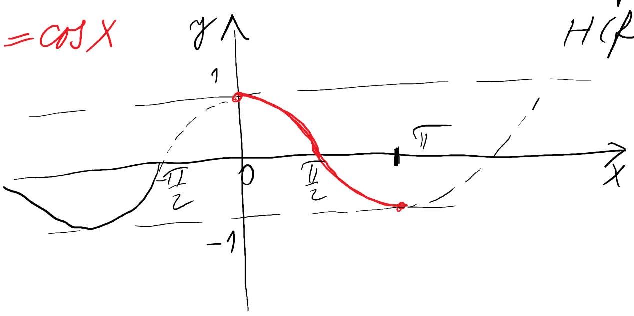
$\sin\frac{\pi}{2}=1 \quad \arcsin 1 = \frac{\pi}{2}$



$D(f) = \langle 0,\pi\rangle$ $\searrow$ KLESAJÚCA

$H(f) = \langle -1,1\rangle$ $\exists f^{-1}$

$y=\cos x$



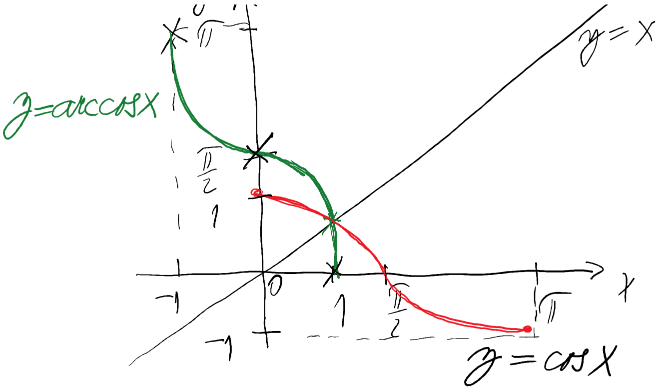
f^{-1}: $y = \arccos x$

$D(f^{-1}) = H(f) = \langle -1, 1\rangle$ ✓

$H(f) = D(f^{-1}) = \langle 0,\pi\rangle$

$\arccos 0 = \frac{\pi}{2}$

$\arccos -1 = \pi$



$\arccos(-1) = \pi$

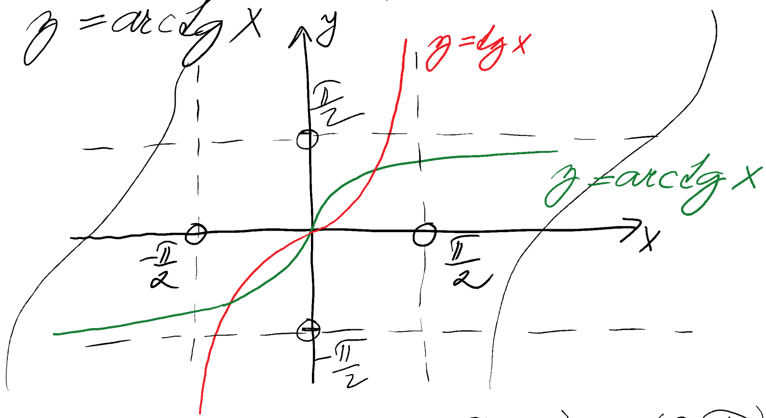
$\arccos 1 = 0$

$y = \operatorname{tg} x$

$D(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = H(f^{-1})$

$H(f) = \mathbb{R} = D(f^{-1})$

$f^{-1}: \; y = \operatorname{arctg} x$



$D(f) = (0, \pi) = H(f^{-1})$

$H(f) = \boxed{\mathbb{R} = D(f^{-1})}$

$y = \operatorname{cotg} x$