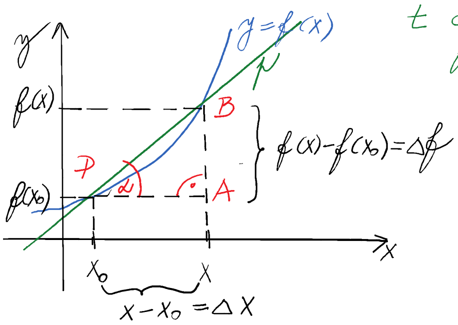
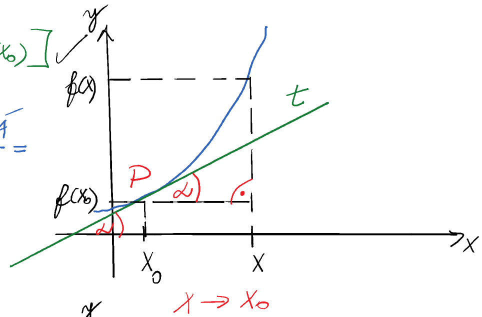
DOTYČNICA KU GRAFU FUNKCIE



t dotyčnica ku grafu funkcie v bode $P[x_0, f(x_0)]$ ✓

ΔABP:

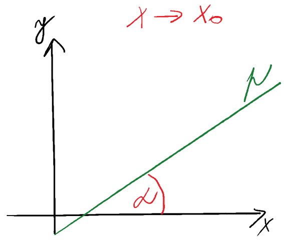
$$\mathrm{tg}\,\alpha = \frac{\text{PROTIĽAHLÁ}}{\text{PRIĽAHLÁ}} = \frac{\Delta f}{\Delta x}$$



$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = k$$

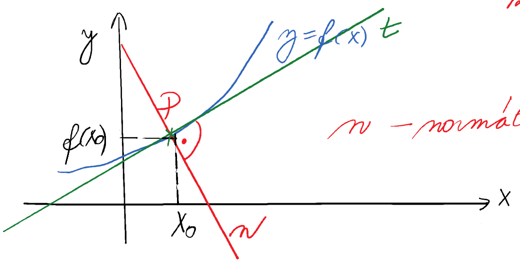
$h \to 0$

$$k = f'(x_0)$$



$y = kx + q$

$k = \mathrm{tg}\,\alpha$



n − normála

$t: \; y = k_t x + q_t$

$n: \; y = k_n x + q_n$

$$t \perp n \Rightarrow k_t \cdot k_n = -1$$

$$k_n = -\frac{1}{k_t}$$

$$k_n = -\frac{1}{f'(x_0)}$$

DOTYČNICA t: $y - f(x_0) = f'(x_0)(x - x_0)$

NORMÁLA n: $y - f(x_0) = -\frac{1}{f'(x_0)} \cdot (x - x_0)$

R $f(x) = x \cdot \ln x$ $\quad \ln x^n = n \ln x$

$P = [e^2, ?]$ $\quad \boxed{x_0 = e^2}$ ✓

$f(x_0) = f(e^2) = e^2 \cdot \ln e^2 = 2e^2 \ln e = 2e^2$ $\quad (\ln e = 1)$

$P = [e^2, 2e^2]$ $\quad \boxed{f(x_0) = 2e^2}$ ✓

$f'(x_0) = ?$

$f'(x) = (x \cdot \ln x)' = 1 \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$f'(x_0) = f'(e^2) = \ln e^2 + 1 = 2 \ln e + 1 = 2 + 1 = 3$

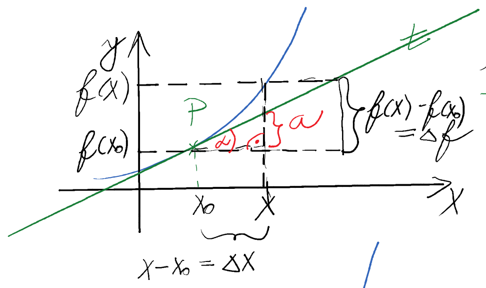
$f'(x_0) = 3$ ✓

$t: \; y - 2e^2 = 3(x - e^2)$

$y = \boxed{3}x \boxed{-e^2}$ $\quad y = kx + q$

$n: \; y - 2e^2 = -\frac{1}{3}(x - e^2)$ $\quad k_n = -\frac{1}{k_t}$

DIFERENCIÁL FUNKCIE

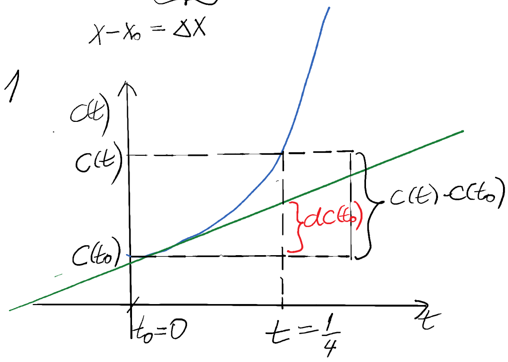


$tg\alpha = \frac{a}{\Delta X} = f'(x_0) \quad / \cdot \Delta X$

$a = f'(x_0)\Delta X = df(x_0)$

$a = df(x_0)$

PR 1



$t_0 = 0$

$t = \frac{1}{4}$

$\Delta t = t - t_0 = \frac{1}{4} - 0 = \frac{1}{4}$

$\Delta C = C(t) - C(t_0)$ PRESNE

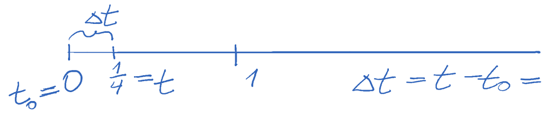
$\Delta C \doteq dC(t_0) = C'(t_0)\Delta t$

ODHAD

1) $t_0 = 0$ ✓ $\quad C(t) = 50t^2 + 100t + 10\,000$

$t = \frac{3}{12} = \frac{1}{4}$

$dC(t_0) = ?$



$\Delta t = t - t_0 = \frac{1}{4} - 0 = \frac{1}{4}$

$C'(t) = 100t + 100$ ✓ $\quad dC(t_0) = \underbrace{C'(t_0)}_{100} \cdot \underbrace{\Delta t}_{\frac{1}{4}} \quad (\doteq \Delta C)$

$C'(t_0) = C'(0) = \boxed{100}$

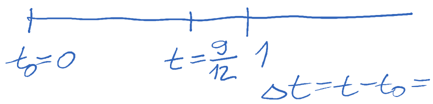
$dC(t_0) = 100 \cdot \frac{1}{4} = 25$ ks

POČAS NASLED. 3 MESIACOV VZRASTIE NÁKLAD O 25 KS.

2) $t_0 = 0$

$t = \frac{9}{12} = \frac{3}{4}$

$dC(t_0) = ?$



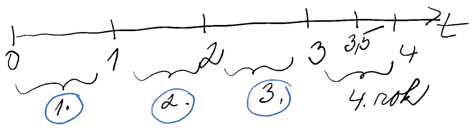
$\Delta t = t - t_0 = \frac{9}{12} - 0 = \frac{3}{4}$

$C'(t_0) = 100$

$dC(t_0) = \underbrace{100}_{C'(t_0)} \cdot \underbrace{\frac{3}{4}}_{\Delta t} = 75$ ks

POČAS NASLEDUJÚCICH 9 MESIACOV VZRASTIE NÁKLAD O 75 KS.

3)



$\Delta t = t - t_0 = 3{,}5 - 3 = 0{,}5$

$t_0 = 3$

$t = 3{,}5$

$dC(t_0) = ?$

$dC(t_0) = C'(t_0)\Delta t$

$\Delta t = t - t_0 = 3{,}5 - 3 = \frac{1}{2}$

$C'(t) = 100t + 100$

$C'(t_0) = C'(3) = 100 \cdot 3 + 100 = 400$

$dC(t_0) = C'(t_0)\Delta t = 400 \cdot \frac{1}{2} = 200$ ks.

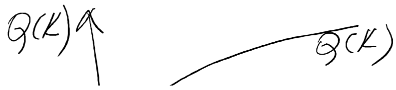
ODHAD

PRESNE: $\Delta C = C(t) - C(t_0) = C(3{,}5) - C(3) =$

$= 212{,}5 \doteq 212$

PR2: $Q(K) = 1200\sqrt{K}$

K = 400

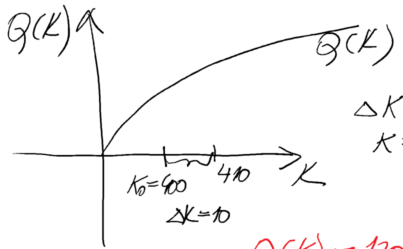


PR 2: $Q(K) = 1200\sqrt{K}$

1) $K_0 = 400$

$\Delta K = 10$

$dQ(K) = ?$



$\Delta K = K - K_0$

$K = K_0 + \Delta K$

$dQ(K_0) = Q'(K_0)\,\Delta K$

$Q(K) = 1200\,K^{\frac{1}{2}}$

$Q'(K) = 1200 \cdot \frac{1}{2} \cdot K^{\frac{1}{2}-1}$

$Q'(K) = 1200 \cdot \frac{1}{2} \cdot K^{-\frac{1}{2}} = \frac{600}{\sqrt{K}}$

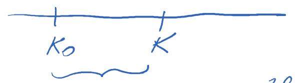
$Q'(K_0) = Q'(400) = \frac{600}{\sqrt{400}} = \frac{600}{20} = 30$

$dQ(K_0) = 30 \cdot 10 = 300$

DENNÁ PRODUKCIA VZRASTIE O 300 KS.

2) $\Delta K = -20$ $\quad K = 380$

$K_0 = 400$



$\Delta K = K - K_0 = 380 - 400 = -20$

$dQ(K_0) = ?$

$Q'(K_0) = Q'(400) = 30$

$dQ(K_0) = 30 \cdot (-20) = -600$

DENNÁ PRODUKCIA SA ZNÍŽI O 600 KS.

3) $K_0 = 400$

$dQ(K_0) = 180$

$K = ?$ $\quad (\Delta K ?)$

$\Delta K = K - K_0$

$K = K_0 + \Delta K$

$dQ(K_0) = \underbrace{Q'(K_0)}_{30} \cdot \Delta K = 180$

$30\,\Delta K = 180$

$\Delta K = 6$

$K = 400 + 6 = 406$

KAP. INV. SA MUSÍ ZVÝŠIŤ NA HODNOTU 406 000

L'HOSPITALOVO PRAVIDLO

$x^3 - 4x^2 = x^2(x-4)$

$\lim_{x \to a} \frac{f(x)}{g(x)}$ $\qquad \lim_{x \to a} \frac{f'(x)}{g'(x)}$ ∃

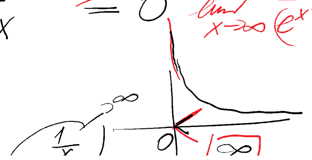
PR 1 $\lim_{x \to 0} \frac{\sin x}{x} \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{\cos x}{1} = 1$ $\qquad \lim_{x \to 0} \frac{(\sin x)'}{x'}$

PR 2 a) $\lim_{x \to \infty} \frac{x^3 - 4x^2}{x^2 - 1} \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{3x^2 - 8x}{2x} \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{6x - 8}{2} = \infty$

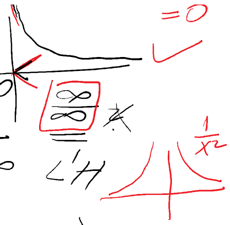
b) $\lim_{x \to \infty} \frac{5x^2 - 2x + 1}{3x^2 + 4x} \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{10x - 2}{6x + 4} \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim \frac{10}{6} = \frac{5}{3}$

PR 3 $\lim_{x \to \infty} \frac{x}{e^{x^2}} \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{1}{e^{x^2} \cdot 2x} \left[\frac{1}{\infty}\right] = 0$

$\lim_{x \to \infty} \frac{0}{(e^{x^2} \cdot 2x)'} = 0$



PR4 $\lim_{x\to 0^+} x^2 e^{\frac{1}{x}} \overset{0\cdot\infty}{=} \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x^2}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$



$= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'}{\left(x^{-2}\right)'} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}\left(-1\cdot x^{-2}\right)}{-2x^{-3}} =$

$= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{-2x^{-3}} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{2\cdot\frac{1}{x}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$

$= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'}{2\left(\frac{1}{x}\right)'} = \lim_{x\to 0^+} \frac{1}{2}\cdot e^{\frac{1}{x}} = \infty$

PR5 $\lim_{x\to\infty}\left(\frac{x^3-4x^2}{x^2-1} - x\right) \overset{\infty-\infty}{=}$ SPOLOČNÝ MENOVATEĽ $\lim_{x\to\infty} \frac{x^3-4x^2-x(x^2-1)}{x^2-1} =$

$= \lim_{x\to\infty} \frac{x^3-4x^2-x^3+x}{x^2-1} = \lim_{x\to\infty} \frac{-4x^2+x}{1\cdot x^2-1} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}}$

$= \lim_{x\to\infty} \frac{-8x+1}{2x} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{-8}{2} = -4$

ASYMPTOTY SO SMERNICOU

ASS: $y = kx + q$

$k = \lim_{x\to\infty} \frac{f(x)}{x}$ $\qquad f(x) = \frac{x^3-4x^2}{x^2-1}$

$k = \lim_{x\to\infty} \frac{\frac{x^3-4x^2}{x^2-1}}{x} = \lim_{x\to\infty} \frac{x^3-4x^2}{x^3-x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$

$= \lim_{x\to\infty} \frac{3x^2-8x}{3x^2-1} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{6x-8}{6x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$

$= \lim_{x\to\infty} \frac{6}{6} = 1$

$q = \lim_{x\to\infty} \left(f(x) - k\cdot x\right)$

$q = \lim_{x\to\infty} \left(\frac{x^3-4x^2}{x^2-1} - 1\cdot x\right) \overset{\infty-\infty}{=}$

$= \lim_{x\to\infty} \frac{x^3-4x^2-x(x^2-1)}{x^2-1}$ VIĎ PR 5 $= \dots = -4$

$y = 1\cdot x - 4 \qquad x\to\infty$