Marginálna analýza, elasticita

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MARGINÁLNA ANALÝZA

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) \doteq \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$\Delta x = 1$

$$f'(x) \doteq f(x+1) - f(x)$$

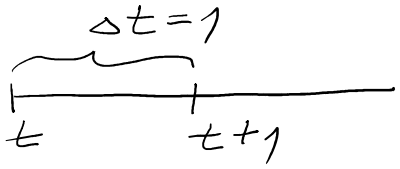
$$Mf(x) = f'(x)$$

R. $C(t) = 50t^2 + 100t + 10\,000$

1) tempo rastu po t rokoch
2) prírastok počas 1. roka
3) prírastok počas 6. roka

1) $MC(t) = C'(t)$

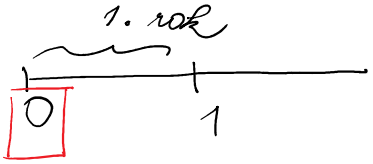
$C'(t) = 100t + 100$



2) $t_0 = 0$

$MC(t_0) = C'(t_0)$

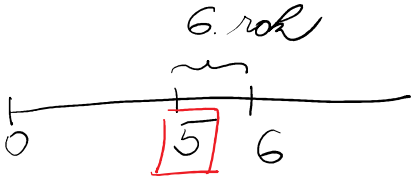
$C'(0) = 100 \cdot 0 + 100 = 100$



3) $t_0 = 5$

$MC(t_0) = C'(t_0)$

$C'(5) = 100 \cdot 5 + 100 = 600$



R. $C(x) = x^2 + 15x + 5000$ ✓

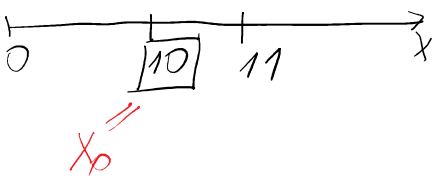
1) 11. výrobok odlad
2) presne

$x_0 = 10$

1) $MC(x) = C'(x)$

$MC(10) = ?$

$C'(x) = 2x + 15$



$MC(10) = C'(10) = 2 \cdot 10 + 15 = 35$ približne

2) $C(11) - C(10) = 5286 - 5250 = 36$ presne

R. $Q(K) = 1200 \cdot \sqrt{K} = 1200 \cdot K^{\frac{1}{2}}$

$K_0 = 400$ tisíc

$\Delta K = 1$

$MQ(K_0) = ?$

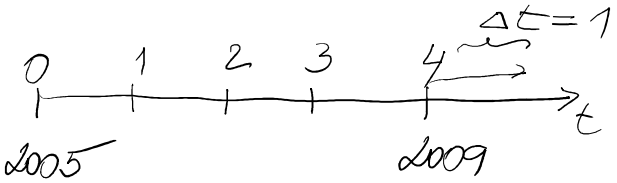
$MQ(K_0) = Q'(K_0)$

$Q'(K) = 1200 \cdot \frac{1}{2} K^{-\frac{1}{2}} = \frac{600}{\sqrt{K}}$

$MQ(400) = Q'(400) = \frac{600}{\sqrt{400}} = \frac{600}{20} = 30$

DENNÁ PRODUKCIA SA ZVÝŠI PRIBLIŽNE O 30 KS.

2. $R(t) = 20t^2 + 1000t + 2000$



1) ročné tempo rastu na začiatku 2009

2) priemerné ...

1) $MR(4) = ?$

$MR(t) = R'(t)$

$R'(t) = 40t + 1000$

$MR(4) = R'(4) = 40 \cdot 4 + 1000 = 1160$ tisíc eur

2) $\frac{R'(4)}{R(4)} \cdot 100\% = \frac{1160}{16\,336} \cdot 100\% = 18{,}3\%$

ELASTICITA FUNKCIE

$\mathcal{E}(f(x)) = \frac{f'(x)}{f(x)} \cdot x$

R. $Q(K) = 1200\sqrt{K}$

$\Delta K = \frac{K}{100}$

$\mathcal{E}(Q(K)) = ?$

$Q'(K) = \frac{600}{\sqrt{K}}$

$\mathcal{E}(Q(K)) = \frac{Q'(K)}{Q(K)} \cdot K = \frac{\frac{600}{\sqrt{K}}}{1200\sqrt{K}} \cdot K =$

$$E(Q(K)) = \frac{Q'(K)}{Q(K)} \cdot K = \frac{\frac{1}{2\sqrt{K}}}{1200\sqrt{K}} \cdot K =$$

$$= \frac{600}{2\,1200\,\sqrt{K}\sqrt{K}} \cdot K = \frac{K}{2K} = \frac{1}{2}\,\%$$

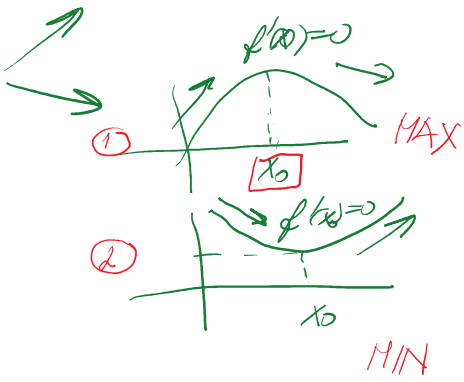
ELASTICITA DOPYTU

$q = D(p)$ $\quad p > 0,\ D(p) > 0$

$f'(x) > 0$ FUNKCIA $f(x)$ RASTIE

$f'(x) < 0$ ✓ KLESÁ

$f'(x_0) = 0$ NEMENÍ SA



$$E_D(p_0) = -\frac{D'(p_0)}{D(p_0)} \cdot p_0$$

PR $q = 240 - 2p \quad (= D(p))$

$0 < p < 120$

$q > 0$

$240 - 2p > 0$

$240 > 2p$

$120 > p$

1) $E_D = ?$

2) $E_D(100) = ?$

3) $E_D(50) = ?$

4) $E_D = 1$

1) $E_D = -\frac{D'(p)}{D(p)} \cdot p = -\frac{-2}{240-2p} \cdot p = \frac{2p}{240-2p}$ ✓ !

$= \frac{2p}{2(120-p)} = \frac{p}{120-p}$ ✓

$p = p_0 + \frac{p_0}{100} = 100+1 = 101$

2) $E_D(100) = \frac{100}{120-100} = \frac{100}{20} = 5\,\% > 1$ ELASTICKÁ

ZVÝŠENIE CENY O 1%, ZNÍŽI POČET PREDANÝCH KS O 5%

3) $E_D(50) = \frac{50}{120-50} = \frac{50}{70} \doteq 0{,}71\,\% < 1$ NEELAST.

4) $E_D = 1$

$$\frac{p}{120-p} = 1$$

$$p = 120 - p$$

$$2p = 120$$

$$p = 60$$

JEDNOTKOVÁ ELASTICITA

VPLYV ELASTICITY DOPYTU NA CELKOVÉ PRÍJMY

PR. $q = 240 - 2p$

$0 < p < 120$

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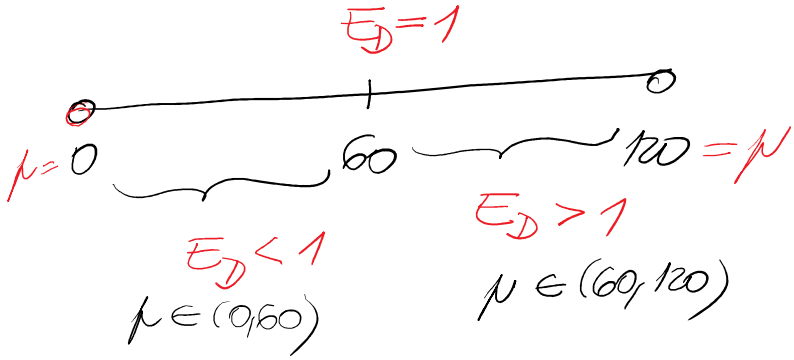
1) $E_D > 1$

$E_D < 1$

$E_D = 1$

$E_D = \frac{p}{120-p} = 1$

$p = 60$



$E_D > 1$ $\quad \frac{p}{120-p} > 1 \quad / \cdot (120-p) > 0$

$p > 120 - p$

$2p > 120$

$p > 60$

$240 - 2p > 0$

$120 - p > 0$

$E_D < 1$ $\quad \frac{p}{120-p} < 1 \quad / \cdot (120-p)$

$p < 120 - p$

$2p < 120$

$p < 60$

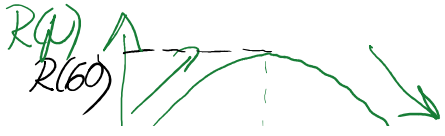
2) intervaly rastu a klesania $R(p)$ pomocou E_D

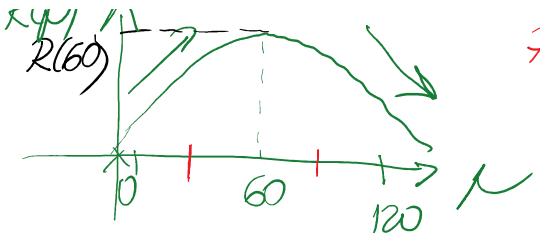
$R'(p) = D(p)\,[1 - E_D]$ $\quad$ (1°)

PRÍJMY RASTÚ

KLESAJÚ

NEMENIA





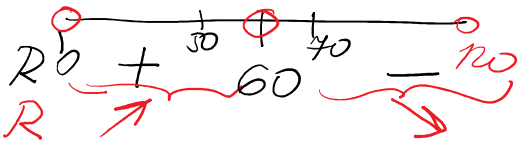
PRÍJMY RASTÚ KLESAJÚ NEMENIA

$\mu \in (0,60)$ PRÍJMY RASTÚ
$\mu \in (60,120)$ KLESAJÚ
$\mu = 60$ PRÍJEM JE MAXIMÁLNY

3) $R(\mu) = \mu \cdot q = \mu \cdot D(\mu) =$
$= \mu(240 - 2\mu) = 240\mu - 2\mu^2$

$R'(\mu) = 240 - 4\mu$

$R'(\mu) = 0 \quad 240 - 4\mu = 0$
$4\mu = 240$
$\mu = 60$



METÓDA NULOVÝCH BODOV

$R'(50) = 240 - 4 \cdot 50 = 40 > 0 \quad \mu \in (0,60)$
$R'(70) = 240 - 4 \cdot 70 = -40 < 0 \quad \mu \in (60,120)$

KLASICKY

$R'(\mu) > 0 \quad 240 - 4\mu > 0$
$240 > 4\mu$
$\mu < 60$

$R'(\mu) < 0 \quad 240 - 4\mu < 0$
$240 < 4\mu$
$60 < \mu$

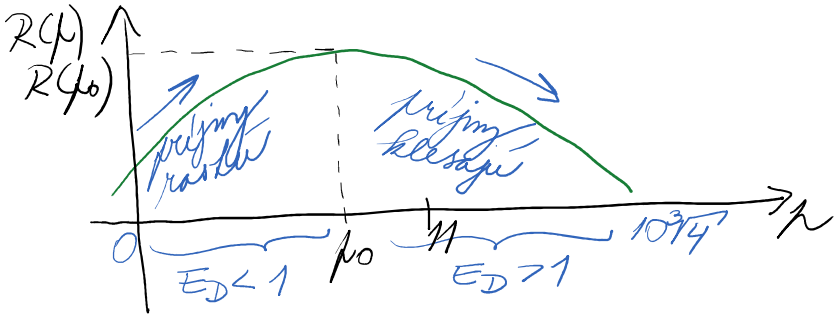
PR. $D(\mu) = 4000 - \mu^3$
$0 < \mu < 10^3 \sqrt{7}$
$\mu_0 = 11$

$E_D(\mu_0) = ?$

$$E_D(\mu) = -\frac{D'(\mu)}{D(\mu)} \cdot \mu = -\frac{-3\mu^2}{4000 - \mu^3} \cdot \mu = \frac{3\mu^3}{4000 - \mu^3}$$

$$E_D(11) = \frac{3 \cdot 11^3}{4000 - 11^3} = 1{,}49 > 1$$ elasticky dopyt

ODPORÚČAME ZNÍŽIŤ CENU (NA HODNOTU, KEĎ $E_D = 1$)



$E_D = 1$

$$\frac{3p^3}{4000-p^3} = 1$$

$$3p^3 = 4000 - p^3$$

$$4p^3 = 4000$$

$$p^3 = 1000$$

$$p_0 = 10$$

JEDNOTKOVÁ ELASTICITA
(PRÍJMY MAXIMÁLNE)