Monotónnosť funkcie a lok. extrémy.

$f(x) = \frac{x^2+1}{x}$ $D(f) = R - \{0\}$

$x \neq 0$

$f'(x) = \frac{2x \cdot x - (x^2+1)}{x^2} = \frac{x^2-1}{x^2}$

$f'(x) = 0$

$\frac{x^2-1}{x^2} = 0 \iff x^2 - 1 = 0$
$x^2 = 1$
$|x| = 1$
$x = \pm 1$ SB

$f'(x)$: $+$ | -1 | $-$ | 0 | $-$ | 1 | $+$

$f(x)$: ↗ v $x=-1$ je lok. max ↘ X ↘ v $x=1$ je lok. min ↗

$f(-1) = -2$ $f(1) = 2$

fcia $f(x)$ je rastúca na $(-\infty, -1)$ a na $(1, \infty)$
klesajúca na $(-1, 0)$ a na $(0, 1)$

$f(x) = \frac{x^2}{x^2-4}$ $x^2 - 4 \neq 0$, $x^2 \neq 4$, $x \neq \pm 2$ $D(f) = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

$f'(x) = \frac{2x(x^2-4) - x^2 \cdot 2x}{(x^2-4)^2} = \frac{2x^3 - 8x - 2x^3}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$

$f'(x) = 0$

$\frac{-8x}{(x^2-4)^2} = 0 \iff x = 0$ SB

$f'(x)$: $+$ | -2 | $+$ | 0 | $-$ | 2 | $-$

$f(x)$: ↗ X ↗ v $x=0$ je l. MAX ↘ X ↘

$f(0) = 0$

$f(x)$ je rastúca na $(-\infty, -2)$ a na $(-2, 0)$
klesajúca na $(0, 2)$ a na $(2, \infty)$

$f(x) = \frac{x^2}{x^2+4}$ $D(f) = R$

$f'(x) = \frac{2x \cdot (x^2+4) - x^2 \cdot 2x}{(x^2+4)^2} = \frac{8x}{(x^2+4)^2}$

$f'(x) \iff 8x = 0 \iff x = 0$

$f'(x)$: $-$ | 0 | $+$

$f(x)$: ↘ v $x=0$ je l. MIN. ↗

$f(0) = 0$

fcia $f(x)$ je rastúca na $(0, \infty)$
klesajúca na $(-\infty, 0)$

$f(x) = x \cdot \ln x$ $D(f) = (0, \infty)$

$x > 0$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$f'(x) = 0 \iff \ln x + 1 = 0$
$\ln x = -1$
$e^{-1} = x$
$x = \frac{1}{e}$ SB

$f'(x)$: 0 | $-$ | $\frac{1}{e}$ | $+$

$f(x)$: ↘ v $x = \frac{1}{e}$ je l. MIN ↗

$f\left(\frac{1}{e}\right) = e^{-1} \cdot \ln e^{-1} = -\frac{1}{e}$

$f(x)$ je klesajúca na $\left(0, \frac{1}{e}\right)$
rastúca na $\left(\frac{1}{e}, \infty\right)$

$f(x) = \ln \frac{4-x}{4+x}$ $D(f) = (-4, 4)$

$\frac{4-x}{4+x} > 0$

$-$ | -4 | $+$ | 4 | $-$

$f'(x) = \frac{1}{\frac{4-x}{4+x}} \cdot \frac{-1(4+x) - (4-x) \cdot 1}{(4+x)^2} = \frac{4+x}{4-x} \cdot \frac{-8}{(4+x)^2} = \frac{-8}{(4-x)(4+x)} \neq 0$ SB neex.

$f'(x)$: -4 | $-$ | 4

$f(x)$: ↘ l. extrém neex.

$f(x)$ je na $D(f)$ klesajúca

Konkávnosť, konvexnosť

$f(x) = \frac{x^2}{x-4}$ $D(f) = R - \{4\}$

$x - 4 \neq 0$
$x \neq 4$

$f'(x) = \frac{2x(x-4) - x^2}{(x-4)^2} = \frac{x^2 - 8x}{(x-4)^2}$

$f''(x) = \frac{(2x-8)(x-4)^2 - (x^2-8x) \cdot 2(x-4)}{(x-4)^4} = \frac{(2x-8)(x-4) - 2(x^2-8x)}{(x-4)^3} =$

$= \frac{2x^2 - 8x - 8x + 32 - 2x^2 + 16x}{(x-4)^3} = \frac{32}{(x-4)^3} \neq 0$

$f(x)$ je konkávna na $(-\infty, 4)$
konvexná na $(4, \infty)$

$f''(x)$: $-$ | 4 | $+$

$f(x)$: ∩ X ∪ IB neex.

$f(x) = \frac{1}{x^2-4}$ $D(f) = R - \{2, -2\}$

$x^2 - 4 \neq 0$
$x \neq \pm 2$

$f'(x) = \frac{-2x}{(x^2-4)^2}$

$f''(x) = \frac{-2(x^2-4)^2 + 2x \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4} =$

$= \frac{-2x^2 + 8 + 8x^2}{(x^2-4)^3} = \frac{6x^2+8}{(x^2-4)^3} \neq 0$ ($6x^2+8 > 0$)

$f''(x)$: $+$ | -2 | $-$ | 2 | $+$

$f(x)$: ∪ X ∩ X ∪ IB neex.

$f(x)$ je konvexná na $(-\infty, -2)$ a na $(2, \infty)$
konkávna na $(-2, 2)$

$f(x) = \frac{1-x}{x+3}$ $D(f) = R - \{-3\}$

$x + 3 \neq 0$
$x \neq -3$

$f'(x) = \frac{-1(x+3) - (1-x)}{(x+3)^2} = \frac{-x - 3 - 1 + x}{(x+3)^2} = \frac{-4}{(x+3)^2}$

$f''(x) = \frac{4 \cdot 2(x+3)}{(x+3)^4} = \frac{8}{(x+3)^3} \neq 0$

$f''(x)$: $-$ | -3 | $+$

$f(x)$: ∩ X ∪

$f(x)$ je konkávne na $(-\infty, -3)$ IB neex.
konvexná na $(-3, \infty)$

$f(x) = \frac{x^3}{x^2-4}$ $D(f) = R - \{2, -2\}$ ↗, ↘, l. ext.
∩, ∪, IB

$f'(x) = \frac{3x^2(x^2-4) - x^3 \cdot 2x}{(x^2-4)^2} = \frac{3x^4 - 12x^2 - 2x^4}{(x^2-4)^2} = \frac{x^4 - 12x^2}{(x^2-4)^2}$

$\frac{x^4 - 12x^2}{(x^2-4)^2} = 0 \iff x^4 - 12x^2 = 0$ ($(x^2-4)^2 > 0$)

$x^2(x^2 - 12) = 0 \iff x = 0 \lor x^2 - 12 = 0$ ($x^2 > 0$)
$x^2 = 12$
$x = \pm\sqrt{12}$
$x = \pm 2\sqrt{3}$ SB

$f'(x)$: $+$ | $-2\sqrt{3}$ | $-$ | -2 | $-$ | 0 | $-$ | 2 | $-$ | $2\sqrt{3}$ | $+$

$f(x)$: ↗ ↘ X ↘ ↘ X ↘ ↗

v $x = -2\sqrt{3}$ je l. MAX; v $x = 0$ nie je l. extrém; v $x = 2\sqrt{3}$ je l. MIN

$f(x)$ je rastúca na $(-\infty, -2\sqrt{3})$ a na $(2\sqrt{3}, \infty)$
klesajúca na $(-2\sqrt{3}, -2)$ a na $(-2, 2)$ a na $(2, 2\sqrt{3})$

$f'(x) = \frac{x^4 - 12x^2}{(x^2-4)^2}$ $f''(x) = \frac{(4x^3 - 24x)(x^2-4)^2 - (x^4 - 12x^2) \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4} =$

$= \frac{(4x^3 - 24x)(x^2-4) - 4x(x^4 - 12x^2)}{(x^2-4)^3} =$

$= \frac{4x^5 - 16x^3 - 24x^3 + 96x - 4x^5 + 48x^3}{(x^2-4)^3} =$

$= \frac{8x^3 + 96x}{(x^2-4)^3} = \frac{8x(x^2+12)}{(x^2-4)^3}$

$f''(x) = 0 \iff$

$\iff \frac{8x^3 + 96x}{(x^2-4)^3} = 0 \iff$

$\iff 8x^3 + 96x = 0$

$8x(x^2+12) = 0 \iff x = 0$ ($x^2 + 12 > 0$)

$f''(x)$: $\ominus$ | -2 | $\oplus$ | 0 | $\ominus$ | 2 | $\oplus$

$f(x)$: ∩ X ∪ v $x=0$ je IB ∩ X ∪

$f(x)$ je konkávna na $(-\infty, -2)$ a na $(0, 2)$
konvexná na $(-2, 0)$ a na $(2, \infty)$

$f(x) = \frac{x^2 - 2x + 2}{x - 1}$ $D(f) = R - \{1\}$

$f'(x) = \frac{(2x-2)(x-1) - (x^2 - 2x + 2)}{(x-1)^2} = \frac{2x^2 - 2x - 2x + 2 - x^2 + 2x - 2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2} =$

$= \frac{x(x-2)}{(x-1)^2}$ $f'(x) = 0 \iff x(x-2) = 0 \iff x = 0 \lor x = 2$ SB

$f'(x)$: $+$ | 0 | $-$ | 1 | $-$ | 2 | $+$

$f(x)$: ↗ v $x=0$ je l. MAX ↘ X ↘ v $x=2$ je l. MIN ↗

$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$ $f''(x) = \frac{(2x-2)(x-1)^2 - (x^2 - 2x) \cdot 2(x-1)}{(x-1)^4} =$

$= \frac{2x^2 - 2x - 2x + 2 - 2x^2 + 4x}{(x-1)^3} = \frac{2}{(x-1)^3} \neq 0$

$f''(x)$: $-$ | 1 | $+$

$f(x)$: ∩ X ∪ IB neex.

$f(x)$ je konkávne na $(-\infty, 1)$
konvexná na $(1, \infty)$

c) $\left(\frac{f}{g}\right)' = \frac{\qquad}{\qquad}$