Optimalizácia funkcií ekonomickej analýzy

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OPTIMALIZÁCIA PRIEMERNÝCH NÁKLADOV (PRÍJMOV)

PR1 $C(q) = 3q^2 + q + 48$

1) $AC(q)$ MINIMÁLNE
2) $AC(q) \overset{?}{=} MC(q)$
3) GRAFY $AC(q)$, $MC(q)$

1) $AC(q) = \dfrac{C(q)}{q} = \dfrac{3q^2+q+48}{q}$

$$AC'(q) = \frac{(6q+1)\cdot q - (3q^2+q+48)\cdot 1}{q^2} =$$

$$= \frac{6q^2+q-3q^2-q-48}{q^2} =$$

$$= \frac{3q^2-48}{q^2}$$

S.B. $AC'(q) = 0$

$3q^2 - 48 = 0$ RESP. $3(q^2-16)=0$

$q^2 = 16$ $(q-4)(q+4)=0$

$q = \pm 4$

$q > 0$ $q = 4$ STACIONÁRNY BOD

URČENIE EXTRÉMU V STACIONÁRNOM BODE

1. SPÔSOB: $AC''(q) = \dfrac{6q\cdot q^2 - (3q^2-48)2q}{q^4}$

$$= \frac{\cancel{q}(6q^2-6q^2+96)}{q^{\cancel{4}\,3}} = \frac{96}{q^3}$$

$AC''(4) = \dfrac{96}{4^3} = \dfrac{96}{64} > 0$ $\Rightarrow$ LOKÁLNE MINIMUM

II. SPÔSOB $AC'(q) = \dfrac{3q^2-48}{q^2} = \dfrac{3(q^2-16)}{q^2} = \dfrac{3(q+4)(q-4)}{q^2}$

$AC'(q)$:	+	-4	−	0	−	4	+
$AC(q)$:	↗		↘		↘		↗

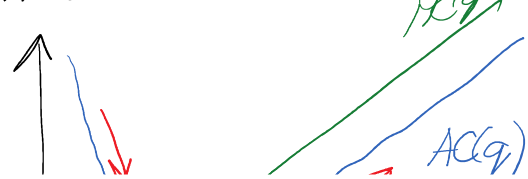
↑ BOD LOKÁLNEHO MINIMA

FUNKCIA $AC(q)$ NADOBÚDA V BODE $q = 4$
LOKÁLNE MINIMUM $AC(4) = 25$

2) $AC(q) = MC(q)$

$\dfrac{C(q)}{q} = C'(q)$

3)



2) $AC(q) = MC(q)$

$$\frac{C(q)}{q} = C'(q)$$

$$\frac{3q^2+q+48}{q} = 6q+1$$

$$3q^2+q+48 = 6q^2+q$$

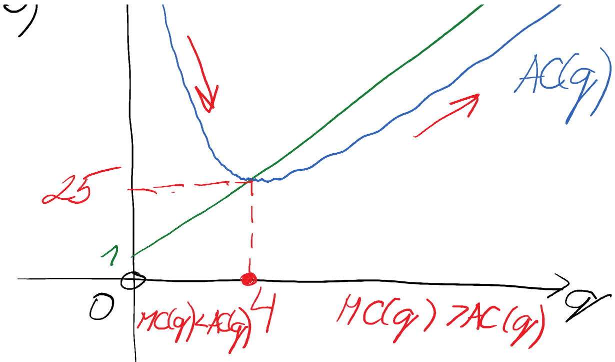
$$3q^2 = 48$$

$$q^2 = 16$$

$$q = \pm 4$$

$$q > 0 \quad q = 4$$

3)



$q > 0$

PR2 $\quad R(q) = -2q^2+68q-128$

1) $AR(q) \overset{?}{=} MR(q)$ ✓

2) MONOTÓNNOSŤ $AR(q)$

P 3) GRAFY $AR(q)$, $MR(q)$

$$AR(q) = \frac{R(q)}{q} = \frac{-2q^2+68q-128}{q}$$

$$MR(q) = R'(q) = -4q+68$$

$$AR(q) = MR(q)$$

$$\frac{-2q^2+68q-128}{q} = -4q+68 \quad / \cdot q$$

$$-2q^2+\cancel{68q}-128 = -4q^2+\cancel{68q} \quad / +4q^2$$

$$2q^2-128 = 0 \quad / :2$$

$$q^2 = 64$$

$$q = \pm 8$$

$$q > 0 \qquad \underline{\underline{q = 8}}$$

2) $$AR(q) = \frac{-2q^2+68q-128}{q}$$

$$AR'(q) = \frac{(-4q+68)q-(-2q^2+68q-128)\cdot 1}{q^2} =$$

$$= \frac{-4q^2+\cancel{68q}+2q^2-\cancel{68q}+128}{q^2} =$$

$$= \frac{-2q^2+128}{q^2} = \frac{-2(q^2-64)}{q^2}$$

q^2 $\qquad$ q^2

S.B. $AR'(q) = 0$

$-2q^2 + 128 = 0$

$q^2 = 64$

$q > 0 \qquad q = \pm 8$

$q = 8$

$AR'(q)$: 0 ... + ... 8 ... −

$AR(q)$: 0 ↗ 8 ↘

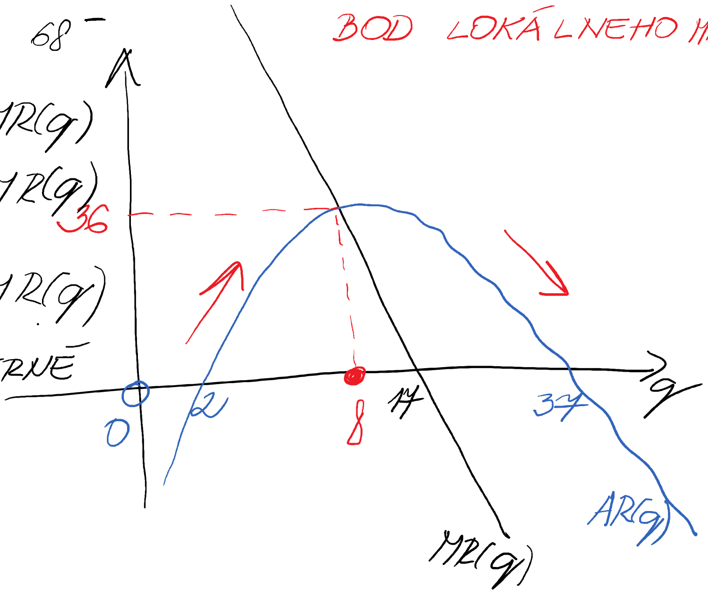
8 — BOD LOKÁLNEHO MAXIMA

$0 < q < 8 \qquad AR(q) < MR(q)$

$8 < q < \infty \ (37) \qquad AR(q) > MR(q)$

$q = 8 \qquad AR(q) = MR(q)$

MAXIMÁLNE PRIEMERNÉ PRÍJMY

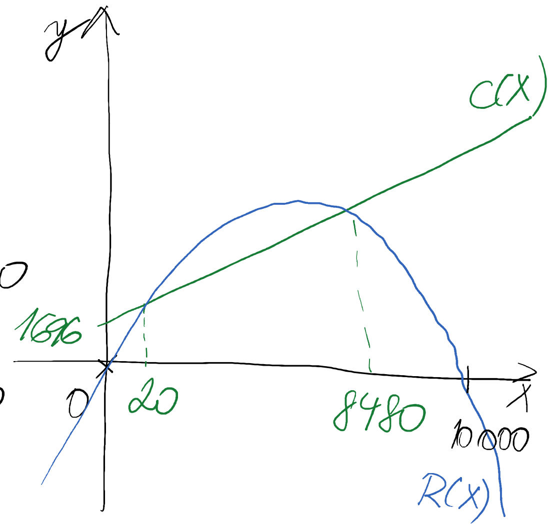


OPTIMALIZÁCIA CELKOVÝCH VELIČÍN

PR 1 $\quad C(x) = 15x + 1696$

$R(x) = 100x - 0{,}01x^2 = x(100 - 0{,}01x)$

1) GRAFY
2) ZISKOVÁ VÝROBA
3) MAXIMÁLNY ZISK



1) $C(x) = R(x) \qquad \underbrace{R(x) - C(x)}_{= P(x)} = 0$

$15x + 1696 = 100x - 0{,}01x^2$

$0{,}01x^2 - 85x + 1696 = 0 \quad / \cdot 100$

$x^2 - 8500x + 169\,600 = 0$

$(x - 20)(x - 8480) = 0$

$x_1 = 20 \qquad x_2 = 8480$

2) $R(x) > C(x)$

$x \in (20, 8480)$ $\quad$ VÝROBA ZISKOVÁ

$x \in \langle 20, 8480 \rangle$ $\quad$ NA TOMTO INTERVALE HĽADÁME ABSOLÚTNE MAXIMUM

3) $P(x) = R(x) - C(x) =$

$= 100x - 0{,}01x^2 - (15x + 1696)$

$= 100x - 0{,}01x^2 - 15x - 1696$

$= -0{,}01x^2 + 85x - 1696$

$P'(x) = -0{,}02x + 85$

S.B. $P'(x) = 0$

$-0{,}02x + 85 = 0$

$0{,}02x = 85 \quad / \cdot \frac{100}{2}$

$x = 4250 \in \langle 20, 8480 \rangle$

$P''(x) = -0{,}02$

$P''(4250) = -0{,}02 < 0$ LOKÁLNE MAXIMUM V $x = 4250$

$P(4250) = 178\,929$

GLOBÁLNE MAXIMUM NA $\langle a, b \rangle = \langle 20, 8480 \rangle$

$\max\limits_{\langle 20, 8480 \rangle} P(x) = \max \{P(4250), P(20), P(8480)\} =$

$= \max \{178\,929, 0, 0\} = 178\,929$

ÚROVEŇ PRODUKCIE, PRI KTOREJ FIRMA DOSAHUJE MAXIMÁLNY ZISK 178 929, JE 4250 KOSOV.

PR3: $K = 360$

$V(q) = (0{,}2q + 10) \cdot q$

$p = 50 - 0{,}2q$

1) $R(q) > C(q)$

2) $R(q)$ MAXIMÁLNE

3) $P(q)$ MAXIMÁLNE

1) $C(q) = V(q) + K = (0{,}2q + 10)q + 360 = 0{,}2q^2 + 10q + 360$

$R(q) = p \cdot q = (50 - 0{,}2q)q = 50q - 0{,}2q^2$

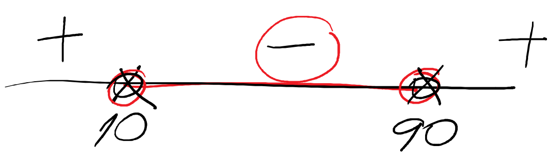
$R(q) > C(q)$

$50q - 0{,}2q^2 > 0{,}2q^2 + 10q + 360$

$0 > 0{,}4q^2 - 40q + 360 \quad / \cdot \frac{10}{4}$

$q^2 - 100q + 900 < 0$

$(q - 10)(q - 90) < 0$



ziskná $x \in (10, 90)$

ziskova $x \in (10,90)$

2) $R(q) = 50q - 0{,}2q^2$

$R'(q) = 50 - 0{,}4q$

S.B. $R'(q) = 0$

$50 - 0{,}4q = 0$

$0{,}4q = 50 \quad / \cdot \frac{10}{4}$

$q = 125 \notin (10,90)$ (NIE JE ZISKOVÁ)

$R''(q) = -0{,}4 < 0$ LOKÁLNE MAXIMUM

PRÍJMY SÚ MAXIMÁLNE PRE $q = 125$ KUSOV

$R(125) = 3125$

3) $P(q) = R(q) - C(q) = -0{,}4q^2 + 40q - 360$

$P'(q) = -0{,}8q + 40$

S.B. $P'(q) = 0$

$-0{,}8q + 40 = 0$

$0{,}8q = 40 \quad / \cdot \frac{10}{8}$

$q = 50 \in (10,90)$

$P''(q) = -0{,}8 < 0$ LOKÁLNE MAXIMUM

ZISK JE MAXIMÁLNY PRI POČTE $q = 50$ KS $P(50) = 640$

(LOKÁLNY AJ GLOBÁLNY EXTRÉM NA $\langle 10,90 \rangle$)

PR 4

	STARÉ	NOVÉ	
R:	$p = 6$	$p = 6 + x \cdot 1$	
	$q = 3\,000$	$q = 3000 - x \cdot 1000$	
C:	$p = 4$	$p = 4$	P: $p = 6 + x - 4 = 2 + x$
	$q = 3000$	$q = 3000 - 1000x$	$q = 3000 - 1000x$

x … POČET ZVÝŠENÍ CENY O 1\$

$P(x)$ MAXIMÁLNE

$R(x) = p \cdot q = (6 + x)(3000 - 1000x)$

$C(x) = 4(3000-1000x)$

$P(x) = R(x) - C(x) =$

$= (6+x)(3000-1000x) - 4(3000-1000x) =$

$= (6+x-4)(3000-1000x) = (2+x)(3000-1000x) =$

$= 1000(2+x)(3-x)$

$P'(x) = 1000\,[1\cdot(3-x) + (2+x)\cdot(-1)] =$

$= 1000(3-x-2-x) = 1000(1-2x)$

S.B. $P'(x) = 0$

$1000(1-2x) = 0$

$1-2x = 0$

$x = \frac{1}{2}$

$P''(x) = 1000\cdot(-2) = -2000 < 0$ LOKÁLNE MAXIMUM

$p = 6 + x = 6 + \frac{1}{2} = 6{,}5\ \$$

PRI CENE $p = 6{,}5\ \$$ ZA KUS DOSIAHNE MAXIMÁLNY ZISK $P(6{,}5)$

PR5

	STARÉ	NOVÉ
U:	$q = 60$	$q = 60+x$
	$p = 400$	$p = 400-4x$

x ... POČET DODATOČNE ZASADENÝCH STROMOV

U MAXIMÁLNA

$U(x) = p\cdot q = (60+x)(400-4x)$

$U'(x) = 1\cdot(400-4x) + (60+x)\cdot(-4) =$

$= 400-4x-240-4x =$

$= 160-8x$

S.B. $U'(x) = 0$

$160-8x = 0$

$8x = 160$

$x = 20$

$U''(x) = -8 < 0$ LOKÁLNE MAXIMUM

ÚRODA JE MAXIMÁLNA, AK DODATOČNE ZASADÍ 20 STROMOV

PR 2)

$C(q) = 5q$

$p = 25 - 2q$ (PREDAJNÁ CENA)

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1) $P(q)$ MAXIMÁLNY

2) $P(q)$ S DAŇOU t DOLÁROV ZA KUS

3) PRI q_0: $P(q_0)$ JE MAXIMÁLNE
MAXIMALIZOVAŤ VÝBER DANE

1) $R(q) = p \cdot q = (25 - 2q) \cdot q = 25q - 2q^2$

$C(q) = 5q$

$P(q) = R(q) - C(q) = 25q - 2q^2 - 5q = 20q - 2q^2$

$20q - 2q^2 > 0$

$10q - q^2 > 0$

$q(10 - q) > 0$

$P(q)$: $-$ $\;0\;$ $+$ $\;10\;$ $-$

$(0, 10)$ ZISKOVÁ

$P'(q) = 20 - 4q$

S.B. $P'(q) = 0$

$20 - 4q = 0$

$q = 5 \in (0, 10)$

$P''(q) = -4 < 0$ LOKÁLNE MAXIMUM

ZISK SPOLOČNOSTI JE MAXIMÁLNY PRI POČTE $q = 5$ KUSOV

2) $f(t) = t \cdot q$ (DAŇ)

$P(q) = 20q - 2q^2 - t \cdot q$

$P'(q) = 20 - 4q - t$

S.B. $P'(q) = 0$

$20 - 4q - t = 0$

$4q = 20 - t$

$q = 5 - \frac{t}{4} > 0 \qquad t \leq 20$

$P''(t) = -4 < 0$ LOKÁLNE MAXIMUM

PRI OBJEME PRODUKCIE $q = 5 - \frac{t}{4}$ KUSOV JE ZISK SPOLOČNOSTI MAXIMÁLNY, AK JE KAŽDÝ KUS ZDANENÝ SUMOU t DOLÁROV.

3) $f(t) = t \cdot q = t\left(5 - \frac{t}{4}\right) = 5t - \frac{1}{4}t^2$

$f'(t) = 5 - \frac{1}{4} \cdot 2t = 5 - \frac{1}{2}t$

S.B $\quad f'(t) = 0$

$5 - \frac{1}{2}t = 0$

$\frac{1}{2}t = 5$

$t = 10 \quad \leq 20$

$f''(t) = -\frac{1}{2} < 0$ LOKÁLNE MAXIMUM

PRÍJEM Z DANE JE MAXIMÁLNY PRI DANI 10 \$ ZA KUS