← matica

$$-5\begin{pmatrix}1 & -1 & 2\\ 3 & 5 & -2\\ 4 & 1 & -3\end{pmatrix} = \begin{pmatrix}-5 & 5 & -10\\ -15 & -25 & 10\\ -20 & -5 & 15\end{pmatrix}$$

$$-5\begin{vmatrix}1 & -1 & 2\\ 3 & 5 & -2\\ 4 & 1 & -3\end{vmatrix} = \begin{vmatrix}1 & -1 & 2\\ -15 & -25 & 10\\ 4 & 1 & -3\end{vmatrix}$$

← determinant

$$\begin{vmatrix}2 & -10\\ 4 & 6\end{vmatrix} = 2\cdot 2\begin{vmatrix}1 & -5\\ 2 & 3\end{vmatrix}$$

$$\begin{pmatrix}2 & -10\\ 4 & 6\end{pmatrix} = 2\begin{pmatrix}1 & -5\\ 2 & 3\end{pmatrix}$$

$$\begin{vmatrix}1 & -1 & 2\\ 3 & 5 & -2\\ 4 & 1 & -3\end{vmatrix} = -\begin{vmatrix}3 & 5 & -2\\ 1 & -1 & 2\\ 4 & 1 & -3\end{vmatrix}$$

1.) $A = \begin{pmatrix}1 & 3\\ -1 & 2\end{pmatrix}$ $B = \begin{pmatrix}3 & 3\\ 2 & -4\end{pmatrix}$

$$(A-B)\cdot 2B = \begin{pmatrix}-2 & 0\\ -3 & 6\end{pmatrix}\cdot\begin{pmatrix}6 & 6\\ 4 & -8\end{pmatrix} = \begin{pmatrix}-12 & -12\\ 6 & -66\end{pmatrix}$$

$$2B(A-B) \neq (A-B)\cdot 2B$$

2.)

$$\begin{vmatrix}1 & 1 & 1 & -2\\ 0 & 2 & 1 & 0\\ 1 & 3 & -2 & 1\\ 2 & 1 & 1 & 1\end{vmatrix}\begin{matrix}+2R_3\\ \\ \\ -R_3\end{matrix} = \begin{vmatrix}3 & 7 & -3 & 0\\ 0 & 2 & 1 & 0\\ 1 & 3 & -2 & 1\\ 1 & -2 & 3 & 0\end{vmatrix} = 1\cdot(-1)^{3+4}\begin{vmatrix}3 & 7 & -3\\ 0 & 2 & 1\\ 1 & -2 & 3\end{vmatrix} =$$

$$\begin{matrix}3 & 7 & -3\\ 0 & 2 & 1\end{matrix}$$

$$= -\left(18+7-(-6-6)\right) =$$
$$= \underline{\underline{-37}}$$

3.) Gaussova eliminačná metóda

$$\begin{aligned}2x_1 - x_2 + x_3 - x_4 &= 3\\ 4x_1 - 2x_2 - 2x_3 + 3x_4 &= 2\\ 2x_1 - x_2 + 5x_3 - 6x_4 &= 1\\ 2x_1 - x_2 - 3x_3 + 4x_4 &= 5\end{aligned}$$

$$\left(\begin{array}{cccc|c}2 & -1 & 1 & -1 & 3\\ 4 & -2 & -2 & 3 & 2\\ 2 & -1 & 5 & -6 & 1\\ 2 & -1 & -3 & 4 & 5\end{array}\right)\begin{matrix}\\ -2R_1\\ -R_1\\ -R_1\end{matrix} \sim$$

$$\left(\begin{array}{cccc|c}2 & -1 & 1 & -1 & 3\\ 0 & 0 & -4 & 5 & -4\\ 0 & 0 & 4 & -5 & -2\\ 0 & 0 & -4 & 5 & 2\end{array}\right)\begin{matrix}\\ \\ +R_2\\ -R_2\end{matrix} \sim \left(\begin{array}{cccc|c}2 & -1 & 1 & -1 & 3\\ 0 & 0 & -4 & 5 & -4\\ 0 & 0 & 0 & 0 & -6\\ 0 & 0 & 0 & 0 & 6\end{array}\right)\begin{matrix}\\ \\ \\ +R_3\end{matrix} \sim \left(\begin{array}{cccc|c}2 & -1 & 1 & -1 & 3\\ 0 & 0 & -4 & 5 & -4\\ 0 & 0 & 0 & 0 & -6\\ 0 & 0 & 0 & 0 & 0\end{array}\right)$$

$h(A) = 2$

$h(A') = 3$

$0 = -6$ nepravda

$h(A) \neq h(A') \Rightarrow$ neexistuje riešenie

4.) Cramerovo pravidlo

$$\begin{aligned}x_1 - 2x_2 + 3x_3 &= 4\\ 2x_1 - 2x_2 + x_3 &= 0\\ x_1 + 3x_2 - x_3 &= 1\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & PS\\ 1 & -2 & 3 & 4\\ 2 & -2 & 1 & 0\\ 1 & 3 & -1 & 1\end{array}$$

$$D = \begin{vmatrix}1 & -2 & 3\\ 2 & -2 & 1\\ 1 & 3 & -1\end{vmatrix} = (2+18-2)-(-6+3+4) = 17$$

$$\begin{matrix}1 & -2 & 3\\ 2 & -2 & 1\end{matrix}$$

$D \neq 0$

$$D_1 = \begin{vmatrix}4 & -2 & 3\\ 0 & -2 & 1\\ 1 & 3 & -1\end{vmatrix} = (8-2)-(-6+12) = 0$$

$$\begin{matrix}4 & -2 & 3\\ 0 & -2 & 1\end{matrix}$$

$$D_2 = \begin{vmatrix}1 & 4 & 3\\ 2 & 0 & 1\\ 1 & 1 & -1\end{vmatrix} = (6+4)-(1-8) = 17$$

$$\begin{matrix}1 & 4 & 3\\ 2 & 0 & 1\end{matrix}$$

$$D_3 = \begin{vmatrix}1 & -2 & 4\\ 2 & -2 & 0\\ 1 & 3 & 1\end{vmatrix} = (-2+24)-(-8-4) = 34$$

$$\begin{matrix}1 & -2 & 4\\ 2 & -2 & 0\end{matrix}$$

$$x_1 = \frac{D_1}{D} = \frac{0}{17} = 0$$
$$x_2 = \frac{D_2}{D} = \frac{17}{17} = 1$$
$$x_3 = \frac{D_3}{D} = \frac{34}{17} = 2$$

$$x = (x_1, x_2, x_3)^T = (0, 1, 2)^T = \begin{pmatrix}0\\ 1\\ 2\end{pmatrix}$$

$$\begin{pmatrix}1 & -2 & 3\\ 2 & -2 & 1\\ 1 & 3 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}4\\ 0\\ 1\end{pmatrix}$$ ← sústava lin. rovníc zo zadania

$$A\cdot x = b$$

5.) $f(x) = \dfrac{x^2}{x-2}$

$x - 2 \neq 0$

$x \neq 2$

$D(f) = \mathbb{R} - \{2\} \Rightarrow$ nie je párna ani nepárna

- def. obor
- párnosť, nepárnosť
- $\nearrow, \searrow$ (monotónnosť)
- lokálne extrémy

$$f'(x) = \frac{2x(x-2) - x^2\cdot 1}{(x-2)^2} = \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2}$$

$f'(x) = 0 \iff x^2 - 4x = 0$

$x(x-4) = 0 \iff x = 0 \vee x = 4$ alebo

$\vee$ alebo

$\wedge$ a zároveň

$$\begin{array}{c|ccccccc} f'(x) & + & 0 & - & 2 & - & 4 & + \\ f(x) & \nearrow & & \searrow & \times & \searrow & & \nearrow \end{array}$$

v $x = 0$ je l. MAX

v $x = 4$ je l. MIN.

na $(-\infty, 0)$, $(4, \infty)$ je rastúca

na $(0, 2)$, $(2, 4)$ je klesajúca

6.) $f(x) = \dfrac{x^2+4}{2x}$

$2x \neq 0$

$x \neq 0$

$D(f) = \mathbb{R} - \{0\}$

- def. obor
- párnosť, nepárnosť
- $\cap, \cup$
- inflexné body

$\forall x \in D(f)$ $\exists (-x) \in D(f)$: $f(-x) = \dfrac{(-x)^2+4}{2(-x)} =$

$$= \frac{x^2+4}{-2x} = -\frac{x^2+4}{2x} = -f(x)$$

$\Downarrow$

$f(x)$ je nepárna fcia

$$\boxed{f'(x)} = \frac{2x\cdot \cancel{2}x - (x^2+4)\cdot\cancel{2}}{\cancel{4}_{2}x^2} = \frac{2x^2 - x^2 - 4}{2x^2} = \boxed{\frac{x^2-4}{2x^2}}$$

$$f''(x) = \frac{\cancel{2x}\cdot 2x^2 - (x^2-4)\cancel{4x}}{\cancel{4}x^{\cancel{4}\,3}} = \frac{x^2 - x^2 + 4}{x^3} = \frac{4}{x^3}$$

$f''(x) \neq 0$

$$\begin{array}{c|ccc} f''(x) & - & 0 & + \\ f(x) & \cap & \times & \cup \end{array}$$

IB neexistuje

na $(-\infty, 0)$ je konkávna

na $(0, \infty)$ je konvexná