

Gaussova eliminační metoda

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ 3x_1 + 2x_2 - 2x_3 = 5 \\ 4x_1 - 3x_2 + 2x_3 = -1 \end{cases} \quad (m=3)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 3 & 2 & -2 & 5 \\ 4 & -3 & 2 & -1 \end{array} \right) \xrightarrow{-3R_1} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & -7 & 5 & -9 \end{array} \right) \xrightarrow{-4R_1} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & -7 & 5 & -9 \end{array} \right) \xrightarrow{-7R_2} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -2 & -2 \end{array} \right) \xrightarrow{(-1) \cdot (-1)} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -2 & -2 \end{array} \right) \xrightarrow{1 \cdot (-1)} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$x_1 + x_2 - x_3 = 2$
 $x_2 - x_3 = 1$
 $x_3 = 2$

$x_1 + 3 - 2 = 2 \Rightarrow x_1 = 1$
 $x_2 - 2 = 1 \Rightarrow x_2 = 3$

$m=3 \quad h(A)=3 \quad h(A')=3 = m$
 $(x_1, x_2, x_3)^T = (1, 3, 2)^T$
PRÁVE 1 RIEŠENIE

$$\begin{cases} x - 2y = -1 \\ -x + y + z = 5 \\ y - z = -1 \end{cases} \quad (m=3)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 1 & 1 & 5 \\ 0 & 1 & -1 & -1 \end{array} \right) \xrightarrow{+R_1} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & -1 & 1 & 4 \\ 0 & 1 & -1 & -1 \end{array} \right) \xrightarrow{+R_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 3 \end{array} \right)$$

$0 \neq 3$

NEEXISTUJE RIEŠENIE

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 1 \\ -x_1 + 2x_3 + x_4 = 2 \\ 2x_2 + x_3 + 2x_4 = 3 \\ x_1 + 4x_2 + 3x_4 = 4 \end{cases} \quad (m=4)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ -1 & 0 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 3 \\ 1 & 4 & 0 & 3 & 4 \end{array} \right) \xrightarrow{+R_1} \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 2 & 1 & 2 & 3 \end{array} \right) \xrightarrow{-R_2, -R_3, -R_4} \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$x_1 = 1 - 2x_2 + x_3 - x_4$
 $2x_2 + x_3 + 2x_4 = 3$
 $x_3 = 3 - 2x_2 - 2x_4$
 $x_4 = 3 - 2x_2 - 2x_4 \Rightarrow x_4 = 3 - 2x_2 - 2(3 - 2x_2 - 2x_4) \Rightarrow x_4 = 3 - 2x_2 - 6 + 4x_2 + 4x_4 \Rightarrow 3x_4 = -3 + 2x_2 + 4x_4 \Rightarrow 0 = -3 + 2x_2 + x_4 \Rightarrow x_4 = 3 - 2x_2$

x_2 - ľub. $x_2 = t$
 x_4 - ľub. $x_4 = \Delta$, $t, \Delta \in \mathbb{R}$

$h(A)=2, h(A')=2, m=4, 2 \neq 4 \Rightarrow$ **EXISTUJE NEKONEČNE VEĽA RIEŠENÍ**
 $(x_1, x_2, x_3, x_4)^T = (4 - 4t - 3\Delta, t, 2 - 2t - 2\Delta, \Delta)$
 $\Delta, t \in \mathbb{R}$

$h(A) \neq h(A') \Rightarrow$ neex. rieš.
 $h(A) = h(A') = m \Rightarrow$ ex. 1 rieš.
 $h(A) = h(A') \neq m \Rightarrow$ ex. ∞ rieš.
 ↓ počet neznámych
 $(m - h(A))$ - počet premenných, ktoré volíme ľubovoľne

$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ 2x_1 + 3x_2 + 4x_3 = 3 \\ 3x_1 + 2x_2 + x_3 = 7 \end{cases} \quad (m=3)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \\ 3 & 2 & 1 & 7 \end{array} \right) \xrightarrow{-2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & -2 & 3 \end{array} \right) \xrightarrow{-3R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -5 & 1 \end{array} \right) \xrightarrow{+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{array} \right)$$

$h(A)=2, h(A')=2, m=3, 2 \neq 3 \Rightarrow$ ex. ∞ riešenií
 $m - h(A) = 3 - 2 = 1$ neznámu volíme ľubovoľne
 ľubovoľne $x_3 = t, t \in \mathbb{R}$

$x_2 + 2x_3 = -1 \Rightarrow x_2 = -1 - 2x_3$
 $x_2 = -1 - 2t$
 $x_1 = 2 - x_2 - x_3 = 2 - (-1 - 2t) - t = 3 + t$
 $(x_1, x_2, x_3)^T = (3 + t, -1 - 2t, t)^T, t \in \mathbb{R}$

$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ x_2 + 2x_3 = -1 \end{cases} \Rightarrow x_3 = \frac{-1-t}{2}$$

mech. $x_2 = t$
 $(x_1, x_2, x_3)^T = (\frac{5-t}{2}, t, \frac{-1-t}{2})^T$

$x_1 + x_2 + x_3 = 2$
 $x_2 + 2x_3 = -1$
 mech. $x_1 = t$

$t + x_2 + x_3 = 2$
 $x_2 + 2x_3 = -1$
 $x_2 = \dots$
 $x_3 = \dots$

$x_1 + x_2 + x_3 = 5$
 $(1 \ 1 \ 1 \ 5)$
 $x_1 = t, x_2 = 5 - t - 5 = -t$

$$\begin{cases} 3x_1 + 2x_2 + 4x_3 + x_4 + 2x_5 = 3 \\ 6x_1 + 4x_2 + 5x_3 + 2x_4 + 3x_5 = 1 \\ 3x_1 + 2x_2 - 2x_3 + x_4 = -7 \\ 9x_1 + 6x_2 + x_3 + 3x_4 + 2x_5 = 2 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 3 & 2 & 4 & 1 & 2 & 3 \\ 6 & 4 & 5 & 2 & 3 & 1 \\ 3 & 2 & -2 & 1 & 0 & -7 \\ 9 & 6 & 1 & 3 & 2 & 2 \end{array} \right) \xrightarrow{-2R_1} \left(\begin{array}{ccccc|c} 3 & 2 & 4 & 1 & 2 & 3 \\ 0 & 0 & -3 & 0 & -1 & -5 \\ 0 & 0 & -6 & 0 & -2 & -10 \\ 0 & 0 & -11 & 0 & -4 & -7 \end{array} \right) \xrightarrow{-2R_2} \left(\begin{array}{ccccc|c} 3 & 2 & 4 & 1 & 2 & 3 \\ 0 & 0 & -3 & 0 & -1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -11 & 0 & -4 & -7 \end{array} \right) \xrightarrow{-4R_2} \left(\begin{array}{ccccc|c} 3 & 2 & 4 & 1 & 2 & 3 \\ 0 & 0 & -3 & 0 & -1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -11 & 0 & -4 & -7 \end{array} \right)$$

$h(A)=3, h(A')=3, m=5, 5-3=2$ neznáme volíme ľubovoľne
 $3x_1 + 2x_2 + 4x_3 + x_4 + 2x_5 = 3$
 $3x_1 + 2x_2 + 5x_3 + 2x_4 - 6x_5 = 3$
 $(x_1, x_2, x_3, x_4, x_5)^T = (t, \Delta, 13, 19 - 3t - 2\Delta, -3t)^T, \Delta, t \in \mathbb{R}$

$3x_1 + 2x_2 + x_4 = 19$
 $x_1 = t, x_2 = \Delta, x_4 = 19 - 3t - 2\Delta$

$$\begin{cases} x_1 + x_3 - x_4 = 2 \\ x_2 - 2x_3 + 6x_4 = -2 \\ -5x_3 + 16x_4 = -1 \\ x_4 = 1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 2 \\ 0 & 1 & -2 & 6 & -2 \\ 0 & 0 & -5 & 16 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{+R_4} \left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 2 \\ 0 & 1 & -2 & 6 & -2 \\ 0 & 0 & -5 & 16 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{-R_4} \left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 2 \\ 0 & 1 & -2 & 6 & -2 \\ 0 & 0 & -5 & 16 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$h(A)=4, h(A')=4, m=4 \Rightarrow$ 1 rieš.
 $m=4$

$-5x_3 + 16 = -1 \Rightarrow -5x_3 = -17 \Rightarrow x_3 = \frac{17}{5}$
 $x_2 - 2 \cdot \frac{17}{5} + 6 = -2 \Rightarrow x_2 - \frac{34}{5} + 6 = -2 \Rightarrow x_2 = -8 + \frac{34}{5} = \frac{-40 + 34}{5} = \frac{-6}{5}$
 $x_1 + \frac{17}{5} - 1 = 2 \Rightarrow x_1 = 3 - \frac{17}{5} = \frac{15 - 17}{5} = \frac{-2}{5}$

$(x_1, x_2, x_3, x_4)^T = (\frac{-2}{5}, \frac{-6}{5}, \frac{17}{5}, 1)^T$

Determinanty

$|A| = \begin{vmatrix} 3 & 4 \\ 2 & -5 \end{vmatrix} = -15 - 8 = -23$

$|A| = \begin{vmatrix} -1 & 2 \\ -3 & -10 \end{vmatrix} = 10 + 6 = 16$

$|A| = \begin{vmatrix} 3 & 10 \\ 4 & 0 \end{vmatrix} = 0 - 40 = -40$

$|A| = \begin{vmatrix} 6 & 2 & -2 \\ 1 & 5 & 2 \\ 2 & 9 & 1 \end{vmatrix} = (30 - 18 + 8) - (-20 + 108 + 2) = 20 - 90 = -70$

$|A| = \begin{vmatrix} -1 & 2 & 3 \\ 5 & 0 & 2 \\ -2 & -3 & 0 \end{vmatrix} = (0 - 45 - 8) - (0 + 6 + 0) = -53 - 6 = -59$

$|A| = \begin{vmatrix} 1 & 2 & -3 \\ -4 & -1 & -2 \\ 3 & 2 & 1 \end{vmatrix} = (-1 + 24 - 12) - (9 - 4 - 8) = 11 + 3 = 14$

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$
 $(1 \ 2 \ 3)^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
 $A: m \times n, A^T: n \times m$