VYŠETRITE PRIEBEH A NAKRESLITE GRAF FUNKCIE

PRÍKLAD 2)

$f(x) = x^5 - 5x^4 \qquad (= x^4(x-5))$

1) $D(f) = \mathbb{R}$

2) $\sigma_y: \; x = 0 \Rightarrow y = f(0) = 0 \qquad P_1[0,0]$ ✓

$\sigma_x: \; y = 0 \Rightarrow x^5 - 5x^4 = 0$

$x^4 \cdot (x-5) = 0 \qquad P_2[5,0]$ ✓

$x_1 = 0 \quad x_2 = 5$

$f(-x) = f(x)$ P
$f(-x) = -f(x)$ N

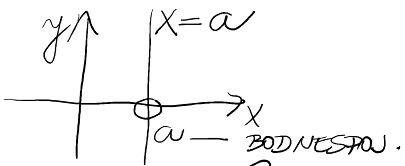
3) $\forall x \in D(f) \; -x \in D(f)$

$f(-x) = (-x)^5 - 5(-x)^4 = -x^5 - 5x^4 \neq f(x) \Rightarrow$ NIE JE PÁRNA

$-f(x) = -(x^5 - 5x^4) = -x^5 + 5x^4 \neq f(-x) \Rightarrow$ NIE JE NEPÁRNA

4) NEPERIODICKÁ

5) BODY NESPOJITOSTI (BN) NEMÁ $D(f) = \mathbb{R}$



6) AS:

ABS NEMÁ, PRETOŽE NEMÁ BN

ASS: $y = kx + q \qquad x \to \pm\infty$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{x^5 - 5x^4}{x} = \lim_{x\to\pm\infty} \frac{x^{\cancel{4}\,3}(x-5)}{\cancel{x}} = \infty$

NIE JE KONEČNÉ

$\Rightarrow \nexists$ ASS $\quad x \to \pm\infty$

7) $f'(x) = 5x^4 - 20x^3$

S.B.: $f'(x) = 0$

$5x^4 - 20x^3 = 0$

$5x^3(x-4) = 0$

$x_1 = 0 \quad x_2 = 4$

	$(-\infty,0)$	0	$(0,4)$	4	$(4,\infty)$
f'	+	0	−	0	+
f	↗	LOK MAX	↘	LOK. MIN	↗

f': + (0) − (4) +

$P_3 = [4, -256]$

8) FUNKCIA MÁ V BODE

$x = 0$ LOK. MAX. $f(0) = 0$
$x = 4$ LOK. MIN. $f(4) = -256$

9) $f''(x) = 20x^3 - 60x^2$

N.B. $f''(x) = 0$

$20x^3 - 60x^2 = 0$

	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f''	−	0	−	0	+

N.B. $f''(x)=0$

$20x^3-60x^2=0$

$20x^2(x-3)^1=0$

$x_1=0 \quad x_2=3$

f'': $-$ (0) $-$ (3) $+$

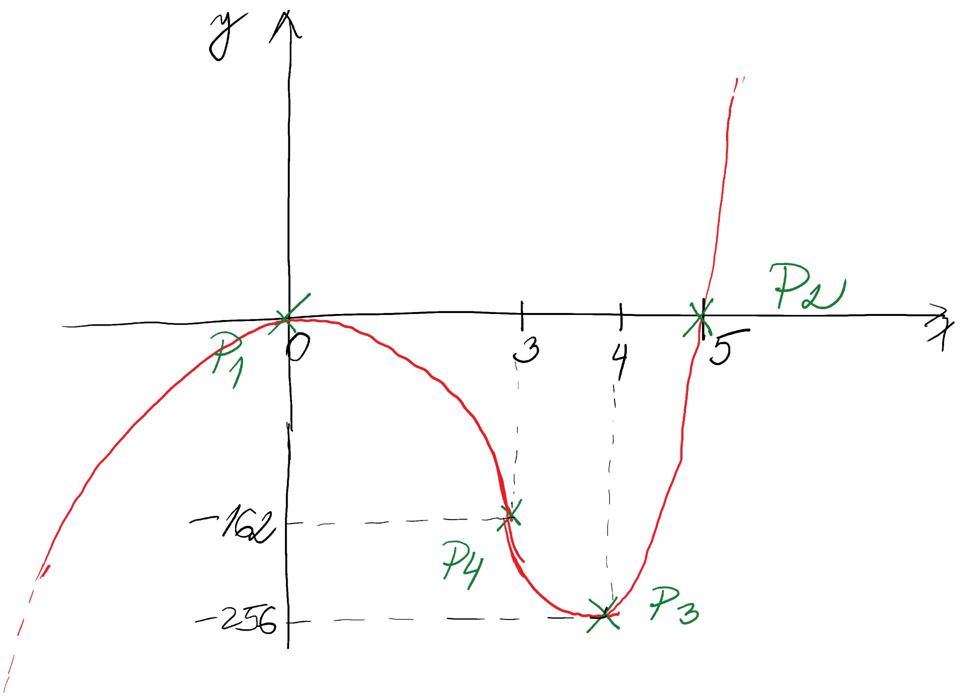
	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f''	$-$	0	$-$	0	$+$
f	$\cap$	—	$\cap$	I.B.	$\cup$

nie je I.B.

$= f(3)$

$P_4=[3,-162]$

10) $x=3$ JE INFLEXNÝ BOD



PRÍKLAD 3

$$f(x)=\frac{-x^3+x^2+4}{x^2}$$

1) $D(f)$: $x^2\neq 0 \qquad D(f)=\mathbb{R}-\{0\} = (-\infty,0)\cup(0,\infty)$

2) o_y: $x=0 \notin D(f)$

o_x: $y=0 \Rightarrow \frac{-x^3+x^2+4}{x^2}=0$

$-x^3+x^2+4=0$

$x^3-x^2-4=0$

DELITELE -4

	1	-1	0	-4
2		2	2	4
	1	1	2	$\boxed{0}$

$\checkmark\; x^3-8-x^2+4=0$

$(x^3-8)-(x^2-4)=0$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^2-b^2=(a-b)(a+b)$

$(x-2)(x^2+2x+4)-(x-2)(x+2)=0$

$(x-2)\left[x^2+2x+4-(x+2)\right]=0$

$(x-2)\,(\underbrace{x^2+x+2}_{\neq 0})=0 \qquad x=2 \qquad t_1[2,0]$

3) $\forall x\in D(f)\ -x\in D(f)$

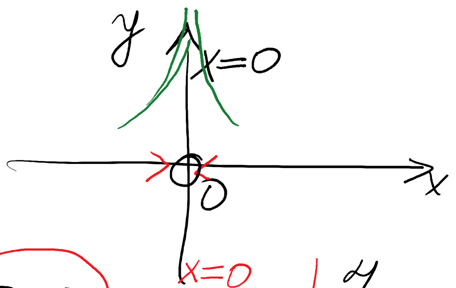
$$f(-x)=\frac{-(-x)^3+(-x)^2+4}{(-x)^2}=\frac{x^3+x^2+4}{x^2}\neq f(x)\ \cancel{P}$$

$$-f(x)=-\frac{-x^3+x^2+4}{x^2}=\frac{+x^3-x^2-4}{x^2}\neq f(-x)\ \cancel{N}$$

4) NEPERIODICKÁ

5) BN: $x=0$

6) ABS: $x=0$ (PRIAMKA $\parallel o_y$)



$$\lim_{x\to 0}\frac{-x^3+x^2+4}{x^2}=\infty \Rightarrow \quad \boxed{x=0 \text{ JE ABS}}$$

$$\frac{4}{\frac{1}{10}}=40 \qquad \frac{4}{\frac{1}{100}}=400$$

ASS: $y=kx+q$

$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{\frac{-x^3+x^2+4}{x^2}}{\frac{x}{1}}=$$

$$=\lim_{x\to\infty}\frac{-1x^3+x^2+4}{1x^3}\overset{-\frac{\infty}{\infty}}{\underset{L'H}{=}}\lim_{x\to\infty}\frac{-3x^2+2x}{3x^2}\overset{-\frac{\infty}{\infty}}{\underset{L'H}{=}}$$

$$=\lim_{x\to\infty}\frac{-6x+2}{6x}\overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}}\lim_{x\to\infty}\frac{-6}{6}=\boxed{-1}\ \checkmark$$

$$q=\lim_{x\to\infty}(f(x)-k\cdot x)=\lim_{x\to\infty}\left(\frac{-x^3+x^2+4}{x^2}-(-1)\cdot x\right)=$$

$$=\lim_{x\to\infty}\frac{\cancel{-x^3}+x^2+4\ \cancel{+x\cdot x^2}}{x^2}=\lim_{x\to\infty}\frac{x^2+4}{x^2}\overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}\lim_{x\to\infty}\frac{2x}{2x}$$

$$=\boxed{1}\ \checkmark$$

$y=-1\cdot x+1$ ASS $x\to\pm\infty$

7) $$f'(x)=\frac{(-3x^2+2x)\cdot x^2-(-x^3+x^2+4)\cdot 2x}{x^4}=$$

$$=\frac{\cancel{x}\left[-3x^3+\cancel{2x^2}+2x^3-\cancel{2x^2}-8\right]}{x^{\cancel{4}\,3}}=$$

$$=\frac{-x^3-8}{x^3}=\frac{-(x^3+8)}{x^3}\quad\left(=-1-\frac{8}{x^3}\right)$$

$$= \frac{-x^3 - 8}{x^3} = \frac{-(x^3+8)}{x^3} \quad \left(= -1 - \frac{8}{x^3}\right)$$

S.B. $f'(x) = 0$

$$-\frac{(x^3+8)}{x^3} = 0$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$x^3 + 8 = 0$$

$$(x+2)\underbrace{(x^2 - 2x + 4)}_{\neq 0} = 0$$

$$x_1 = -2$$

f': $-$ on $(-\infty,-2)$, $+$ on $(-2,0)$, $-$ on $(0,\infty)$; points -2, 0

	$(-\infty,-2)$	-2	$(-2,0)$	$(0,\infty)$
f'	$-$	0	$+$	$-$
f	↘	LOK. MIN	↗	↘

$P_2[-2,4]$

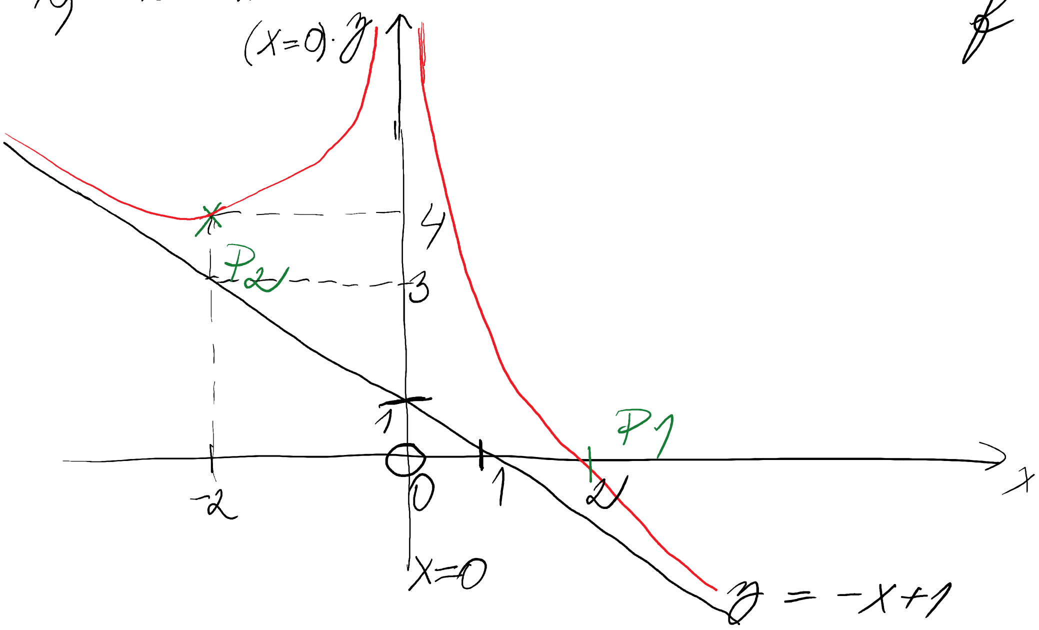
8) FUNKCIA MÁ V BODE $x=-2$ LOK. MIN. $f(-2) = 4$

9) $$f''(x) = \left[\frac{-(x^3+8)}{x^3}\right]' = -\left[\frac{x^3+8}{x^3}\right]' = -\left(1 + \frac{8}{x^3}\right)' =$$ ($8x^{-3}$)

$$= -\left(8 \cdot (-3) \cdot x^{-4}\right) = \frac{24}{x^4}$$

	$(-\infty,0)$	$(0,\infty)$
f''	$+$	$+$
f	∪	∪

10) NEMÁ INFLEXNÉ BODY



$$y = -x + 1$$

$$x + y = 1$$

$$y = -(-2) + 1 = 3$$

PRÍKLAD 4

$$f(x) = x \cdot \ln x$$

1) $D(f): x>0 \qquad D(f) = (0,\infty) = R^+$

2) $\sigma_y: x=0 \notin D(f)$

$\sigma_x: y=0 \qquad x\cdot \ln x = 0$

$x=0 \vee \ln x = 0$

$\notin D(f) \qquad x=1$

$P_1 = [1, 0]$

3) $\forall x \in D(f)\ -x\in D(f)$ NEPLATÍ, ALE PLATÍ NEGÁCIA:

$\exists x \in D(f)\ -x \notin D(f) \implies$ ANI $\cancel{P}$ ANI $\cancel{N}$

4) NEPERIODICKÁ

5) BODY NESPOJITOSTI NEMÁ

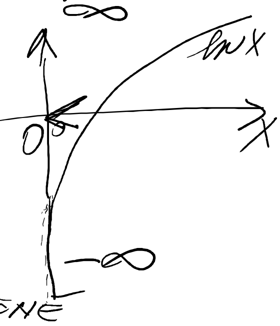
6) ABS $\quad x=0$

$D(f) = (0,\infty)$

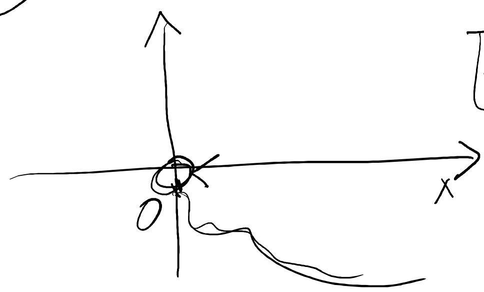
$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \underbrace{x}_{\to 0^+}\cdot \underbrace{\ln x}_{\to -\infty} = \lim\limits_{x\to 0^+} \dfrac{\overbrace{\ln x}^{-\infty}}{\underbrace{\frac{1}{x}}_{\to\infty}} \overset{\frac{-\infty}{\infty}}{=}$

$\underset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+} \underbrace{\dfrac{-x^2}{x}}_{-x} = 0$

KONEČNÉ



$\boxed{\implies x=0 \text{ NIE JE ABS}}$



ASS $\quad y = kx + q \qquad x \to \infty$

$x \to -\infty$ NEMÔŽE $\quad D(f) = (0,\infty)$

$k = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{\cancel{x}\overbrace{\ln x}^{\to\infty}}{\cancel{x}} = \infty$ NIE JE KONEČNÉ

$\implies$ ASS $\nexists$

7) $f'(x) = 1\ln x + x\cdot\frac{1}{x} = \ln x + 1$

S.B.: $\quad f'(x) = 0$

$\ln x + 1 = 0$

$\ln x = -1 \ /e^{()}$

$x = e^{-1} = \frac{1}{e}$

	$(0, e^{-1})$	e^{-1}	(e^{-1}, ∞)
f'	$-$	0	$+$
f	↘	LOK. MIN.	↗

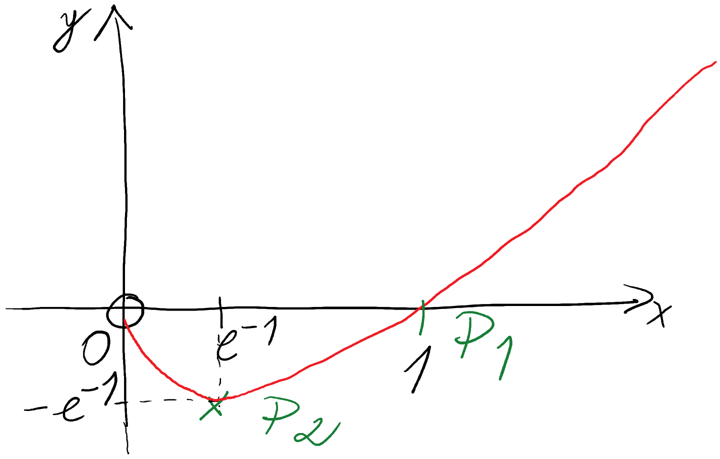
$P_2 [e^{-1}, -e^{-1}]$

$\Rightarrow$ FUNKCIA MÁ V BODE $x = e^{-1}$ LOK. MIN. $f(e^{-1}) = -e^{-1}$

f) FUNKCIA MÁ V BODE $x=e^{-1}$ LOK. MIN.. $f(e^{-1})=-e^{-1}$

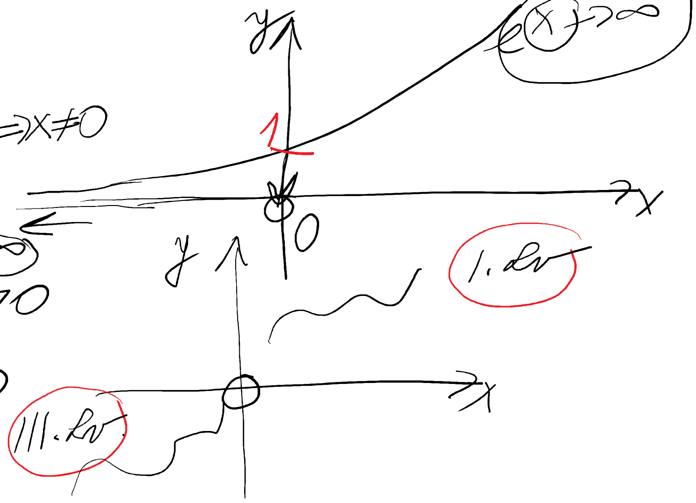
g) $f''(x)=(\ln x+1)'=\frac{1}{x}>0 \quad \forall x\in D(f)=(0,\infty)$ $\cup$

h) NEMÁ INFLEXNÝ BOD



PRÍKLAD 5

$f(x)=x\cdot e^{\frac{1}{x}}$ $\quad \frac{1}{x}\Rightarrow x\neq 0$



1) $D(f)=R-\{0\}$

2) $\sigma_y: \; x=0\notin D(f)$

$\sigma_x: \; y=0 \quad x\cdot e^{\frac{1}{x}}=0$

$x=0\notin D(f) \vee e^{\frac{1}{x}}=0$

$\emptyset \quad \emptyset$

NEMÁ PRIESEKY S OSAMI

3) $\forall x\in D(f) \; \exists -x\in D(f)$

$f(-x)=-x\,e^{-\frac{1}{x}}\neq f(x)$ ANI ~~P~~ ANI ~~N~~

$-f(x)=-x\,e^{\frac{1}{x}}\neq f(-x)$

4) NEPERIODICKÁ

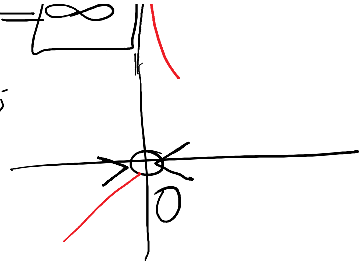
5) BN $x=0$

6) ABS $x=0$

$$\lim_{x\to 0^+} x\cdot e^{\frac{1}{x}} = \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}}{\frac{1}{x}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}\cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \boxed{\infty}$$

$\Longrightarrow$ $x=0$ JE ABS.

$\Longrightarrow x=0$ JE ABS ;



$$\lim_{x\to 0^-} \underbrace{x}_{\to 0}\,\underbrace{e^{\frac{1}{x}}}_{\to 0} = 0\cdot 0 = \underline{\underline{0}}$$

ASS $\quad y = kx+q$

$$k = \lim_{x\to\infty} \frac{\cancel{x}\, e^{\frac{1}{x}}}{\cancel{x}} = e^0 = \boxed{1}$$

$$q = \lim_{x\to\infty}\left(xe^{\frac{1}{x}} - 1\cdot x\right) = \lim_{x\to\infty} \underbrace{x}_{\to\infty}\underbrace{\left(e^{\frac{1}{x}}-1\right)}_{\to 0} =$$

$$= \lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{e^{\frac{1}{x}}\cdot\cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \boxed{1}$$

$$y = 1\cdot x + 1 \qquad x\to\pm\infty$$

7) $$f'(x) = e^{\frac{1}{x}} + \cancel{x}\cdot e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^{\cancel{2}}}\right) = e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x} =$$

$$= e^{\frac{1}{x}}\left(1-\frac{1}{x}\right) = \underbrace{e^{\frac{1}{x}}}_{>0}\left(\frac{x-1}{x}\right) \quad \neq 0$$

S.B. $f'(x)=0$

$\frac{x-1}{x}=0$

$x-1=0$

$x=1$

	$(-\infty,0)$	$(0,1)$	1	$(1,\infty)$
f'	$+$	$-$	0	$+$
f	$\nearrow$	$\searrow$	LOK MIN	$\nearrow$

8) FUNKCIA MÁ V BODE $x=1$ LOK. MIN. $f(1)=1\cdot e^{\frac{1}{1}}=e$

$P_1=[1,e]$

$1-x^{-1}$

9) $$f''(x) = \left[\left(1-\frac{1}{x}\right)e^{\frac{1}{x}}\right]' = \frac{1}{x^2}e^{\frac{1}{x}} + \left(1-\frac{1}{x}\right)e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right) =$$

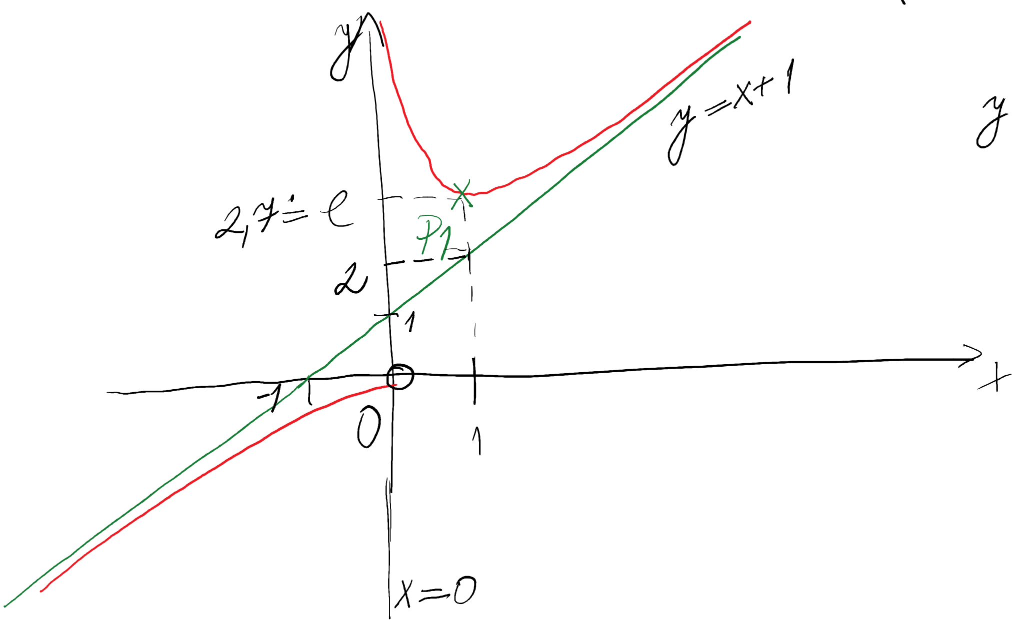
$$= \frac{1}{x^2}e^{\frac{1}{x}}\left(1-1+\frac{1}{x}\right) = \frac{e^{\frac{1}{x}}}{x^3} \qquad e^{\frac{1}{x}} > 0 \quad (\neq 0)$$

$\nexists$ NB

| $(-\infty,0)$ | $(0,\infty)$ |

10) NEMÁ INFLEXNÉ BODY

	$(-\infty,0)$	$(0,\infty)$
f''	$-$	$+$
f	$\cap$	$\cup$



$y = 1+1 = 2$