DOBRÉ RÁNKO :)

Inverzná matica (existuje k A, ak $|A| \neq 0$)

A^{-1} ; $A \cdot A^{-1} = A^{-1} \cdot A = E$

2. spôsob nájdenia inverznej matice:

$$\boxed{A^{-1} = \frac{1}{|A|} \operatorname{adj} A} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{12} \dots & A_{1m} \\ A_{21} & A_{22} \dots & A_{2n} \\ \vdots & & \\ A_{n1} & A_{n2} & A_{mm}\end{pmatrix}^T = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & & & \\ \vdots & & \vdots & \\ A_{1m} & & & A_{mm}\end{pmatrix}$$

$$A_{ij} = \underline{(-1)^{i+j} \cdot D_{ij}}$$

algebraický doplnok k a_{ij}

$A = (a_{ij})$

príklad:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{vmatrix} \begin{matrix} 1 & 1 & 1 \\ 6 & 5 & 4 \end{matrix} = \cancel{40} + 60 + 52 - 65 - \cancel{40} - 48 = $$

$-5 \quad +4$

$$= -1 \neq 0$$

$$A_{11} = (-1)^{1+1}\underbrace{\begin{vmatrix} 5 & 4 \\ 10 & 8 \end{vmatrix}}_{D_{11}} = 0$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 6 & 4 \\ 13 & 8 \end{vmatrix} = -1 \cdot (48-52) = 4$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 6 & 5 \\ 13 & 10 \end{vmatrix} = 60-65 = -5$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix} = -1 \cdot (8-10) = 2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 13 & 8 \end{vmatrix} = 8-13 = -5$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 13 & 10 \end{vmatrix} = -1 \cdot (10-13) = 3$$

$$A_{31} = (-1)^{3+1} \cdot | \quad | = -1$$

$$A_{32} = 2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 6 & 5 \end{vmatrix} = -1$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a.d - c.b$$

$$\operatorname{adj} A = \begin{pmatrix} 0 & 2 & -1 \\ 4 & -5 & 2 \\ -5 & 3 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A = -1 \cdot \operatorname{adj} A = \begin{pmatrix} 0 & -2 & 1 \\ -4 & 5 & -2 \\ 5 & -3 & +1 \end{pmatrix}$$

¿NAČO? riešenie sústav lineárnych rovníc: pomocou A^{-1} (ak existuje)

$$A\vec{x} = \vec{b} \quad | \; A^{-1}$$

$$A^{-1} \cdot A \cdot \vec{x} = A^{-1} \cdot \vec{b}$$

$$E \cdot \vec{x}$$

$$\boxed{\vec{x} = A^{-1} \cdot \vec{b}}$$

príklad:

$$x_1 + x_2 + x_3 = 0$$
$$6x_1 + 5x_2 + 4x_3 = 1$$
$$13x_1 + 10x_2 + 8x_3 = 1$$

riešme pomocou A^{-1} $\quad (|A| \neq 0)$

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$A \cdot \vec{x} = \vec{b}$$

$$\vec{x} = \boxed{A^{-1} \cdot \vec{b}}$$

$$A^{-1} = \begin{pmatrix} 0 & -2 & 1 \\ -4 & 5 & -2 \\ 5 & -3 & 1 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 0 & -2 & 1 \\ -4 & 5 & -2 \\ 5 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0-2+1 \\ 0+5-2 \\ 0-3+1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$$

$$(x_1, x_2, x_3)^T = (-1, 3, -2)^T$$