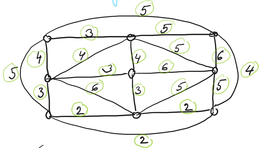
Grafové algoritmy

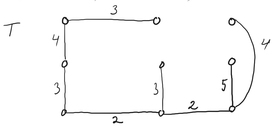
1. Minimálna kostra grafu

Kruskalov algoritmus



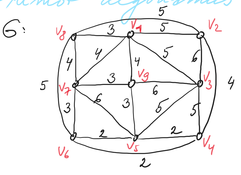
$|V| = 9$

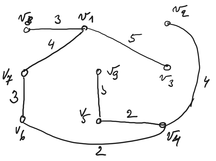
2 2 3 3 3 4 4 5 5 5 5 5 6 6 6



$w(T) = 4+3+4+5+5+5 = 26$

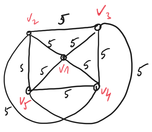
Primov algoritmus

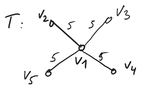




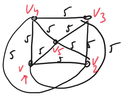
1. $\{v_1, v_8\}$
2. $\{v_1, v_7\}$
3. $\{v_7, v_6\}$
4. $\{v_6, v_5\}$
5. $\{v_4, v_5\}$
6. $\{v_5, v_3\}$
7. $\{v_8, v_2\}$
8. $\{v_3, v_2\}$

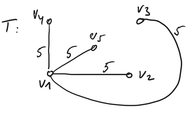
$w(T) = 26$





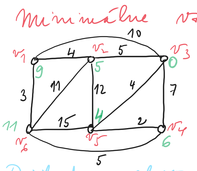
1. $\{v_1, v_2\}$
2. $\{v_1, v_3\}$
3. $\{v_1, v_4\}$
4. $\{v_1, v_5\}$





1. $\{v_1, v_5\}$
2. $\{v_1, v_3\}$
3. $\{v_1, v_4\}$
4. $\{v_1, v_2\}$

2. Minimálne vzdialenosti v grafe/v digrafe



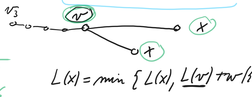
- vzdialenosť z vrchola v_3 do všetkých ostatných vrcholov

$d(v_3, v_i)$

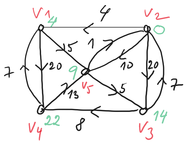
Dijkstrov algoritmus

v_1	v_2	v_3	v_4	v_5	v_6
∞	∞	0	∞	∞	∞
10	5		7	4	∞
10			6		19
9					16
9					11
					11

$d(v_3, v_3) = 0$
$d(v_3, v_5) = 4$
$d(v_3, v_2) = 5$
$d(v_3, v_4) = 6$
$d(v_3, v_1) = 9$
$d(v_3, v_6) = 11$



$L(x) = \min\{L(x), L(v) + w(\{v,x\})\}$



$\vec{d}(v_2, v_i) = ?$ pre vš. i

v_1	v_2	v_3	v_4	v_5
∞	0	∞	∞	∞
4		20	∞	10
		20	24	9
		14	24	
			22	

$\vec{d}(v_2, v_2) = 0$
$\vec{d}(v_2, v_1) = 4$
$\vec{d}(v_2, v_5) = 9$
$\vec{d}(v_2, v_3) = 14$
$\vec{d}(v_2, v_4) = 22$

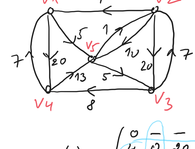
$\vec{d}(v_i, v_j) = ?$ pre vš. i, j

Floydov algoritmus

$\vec{G}$ $\quad |V| = n$

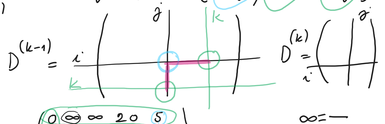
$W = D^{(0)}, D^{(1)}, D^{(2)}, \dots, D^{(n)}$

↳ cenová matica



$D^{(k)} = d_{ij}^{(k)}$

$d_{ij}^{(k)} = \min\{d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\}$



$\infty = —$

$$W = \begin{pmatrix} 0 & \infty & \infty & 20 & 5 \\ 4 & 0 & 20 & \infty & 10 \\ \infty & 7 & 0 & 8 & \infty \\ 7 & \infty & \infty & 0 & 13 \\ \infty & 1 & 5 & \infty & 0 \end{pmatrix} = D^{(0)}$$

$$D^{(1)} = \begin{pmatrix} 0 & — & — & 20 & 5 \\ 4 & 0 & 20 & 24 & 9 \\ — & 7 & 0 & 8 & — \\ 7 & — & — & 0 & 12 \\ — & 1 & 5 & — & 0 \end{pmatrix} \quad D^{(2)} = \begin{pmatrix} 0 & — & — & 20 & 5 \\ 4 & 0 & 20 & 24 & 9 \\ 11 & 7 & 0 & 8 & 16 \\ 7 & — & — & 0 & 12 \\ 5 & 1 & 5 & 25 & 0 \end{pmatrix} \quad D^{(3)} = \begin{pmatrix} 0 & — & — & 20 & 5 \\ 4 & 0 & 20 & 24 & 9 \\ 11 & 7 & 0 & 8 & 16 \\ 7 & — & — & 0 & 12 \\ 5 & 1 & 5 & 13 & 0 \end{pmatrix}$$

$$D^{(4)} = \begin{pmatrix} 0 & — & — & 20 & 5 \\ 4 & 0 & 20 & 24 & 9 \\ 11 & 7 & 0 & 8 & 16 \\ 7 & — & — & 0 & 12 \\ 5 & 1 & 5 & 13 & 0 \end{pmatrix} \quad D^{(5)} = \begin{pmatrix} 0 & 6 & 10 & 18 & 5 \\ 4 & 0 & 14 & 22 & 9 \\ 11 & 7 & 0 & 8 & 16 \\ 7 & 13 & 17 & 0 & 12 \\ 5 & 1 & 5 & 13 & 0 \end{pmatrix}$$

$= \vec{d}(v_2, v_4)$

$= \vec{d}(v_4, v_5)$

$\vec{d}(v_5, v_2)$