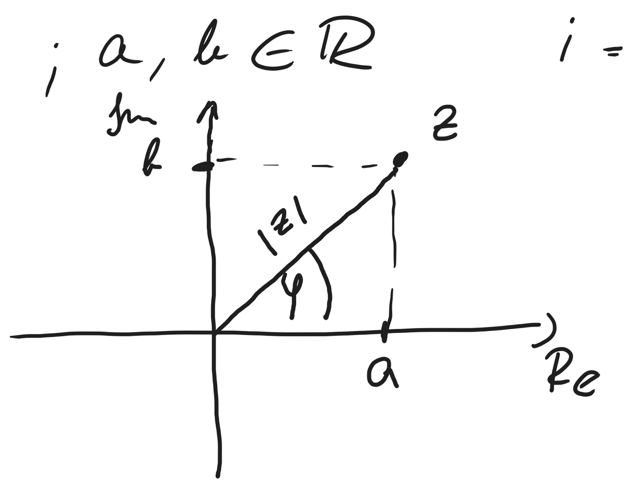
ALGEBRAICKÝ TVAR

$\boxed{z = a + bi}$ $\rightarrow$ ; $a, b \in \mathbb{R}$   $i = \sqrt{-1}$   $\boxed{i^2 = -1}$

$Re(z) = a$

$Im(z) = b$




$z = a + bi$

$z_1 = 5 - 5\sqrt{3}i$   $z_2 = \sqrt{3} + i$

$z_1 + z_2 = (5 - 5\sqrt{3}i) + (\sqrt{3} + 1i) = \underbrace{(5 + \sqrt{3})}_{Re} + \underbrace{(-5\sqrt{3} + 1)}_{Im} i$

$z_1 - z_2 = (5 - 5\sqrt{3}i) - (\sqrt{3} + i) = 5 - 5\sqrt{3}i - \sqrt{3} - i = \underbrace{(5 - \sqrt{3})}_{Re} + \underbrace{(-5\sqrt{3} - 1)}_{Im} i$

$(5 - \sqrt{3}) - \underbrace{(5\sqrt{3} + 1)}_{Im} i$

$z_1 \cdot z_2 = (5 - 5\sqrt{3}i) \cdot (\sqrt{3} + i) = 5\sqrt{3} \underbrace{- 5 \cdot 3}_{-15} i + 5i \underbrace{- 5\sqrt{3} i^2}_{+5\sqrt{3}} =$   $i^2 = -1$

$z_1 \cdot z_2 = 10\sqrt{3} - 10i$   $i^2 = -1$

$\frac{z_2}{z_1} = \frac{\sqrt{3} + i}{5 - 5\sqrt{3}i} \cdot \frac{5 + 5\sqrt{3}i}{5 + 5\sqrt{3}i} =$

$= \frac{(\sqrt{3} + i)(5 + 5\sqrt{3}i)}{(5 - 5\sqrt{3}i) \cdot (5 + 5\sqrt{3}i)} =$

$= \frac{5\sqrt{3} + 5i + 5 \cdot 3 i + 5\sqrt{3} i^2}{5^2 - (5\sqrt{3}i)^2} =$   $i^2 = -1$   $= -5\sqrt{3}$

$= \frac{0 + 20i}{25 - (-75)} = \frac{20i}{100} = \boxed{0{,}2i} = \underbrace{0}_{Re} + \underbrace{\boxed{0{,}2}}_{Im} i$

$1 = \frac{2}{2} = \frac{-7}{-7} = \frac{x-3}{x-3} = \frac{♡}{♡}$

$\boxed{r^2 - s^2 = (r - s)(r\ s)}$

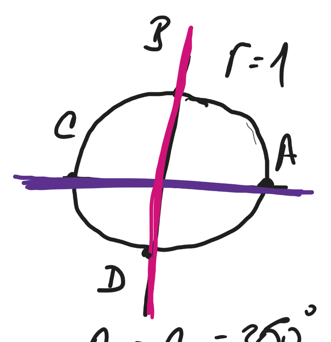
$z = a + bi$

$\bar{z} = a - bi$

$(5\sqrt{3}i)^2 = 25 \cdot 3 i^2 = -75$   $i^2 = -1$

$z = \underbrace{a + bi}_{\text{ALGEBRAICKÝ}} = \underbrace{|z|(\cos\varphi + i\sin\varphi)}_{\text{GONIOMETRICKÝ}} = \underbrace{|z| e^{i\varphi}}_{\text{EXPONENCIÁLNY}}$

| | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|
| | $0°$ | $30°$ | $45°$ | $60°$ | $90°$ | $180°$ | $270°$ | $360°$ |
| $\sin\varphi$ | $\frac{\sqrt{0}}{2} = 0$ | $\frac{\sqrt{1}}{2} = \frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2} = 1$ | $0$ | $-1$ | $0$ |
| $\cos\varphi$ | $\frac{\sqrt{4}}{2} = 1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2} = \frac{1}{2}$ | $\frac{\sqrt{0}}{2} = 0$ | $-1$ | $0$ | $1$ |



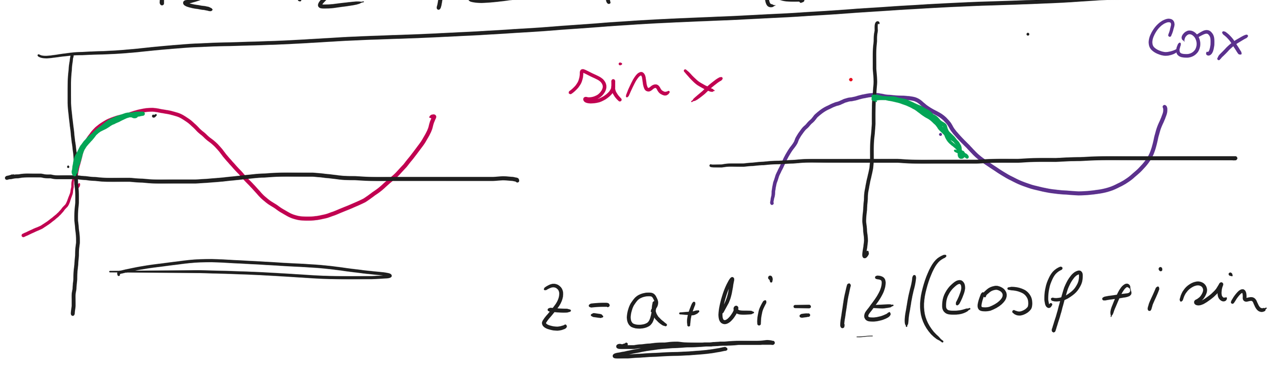

$A \to A = 360°$

$2\pi r = 2\pi$

$A \to C \to 180°$

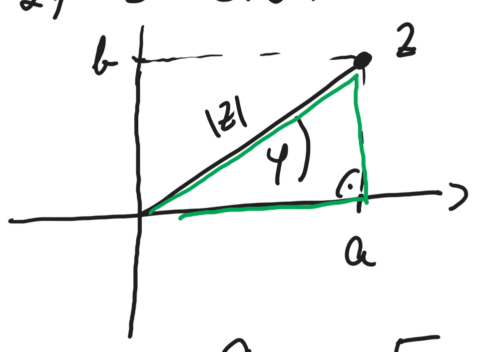
$A \to B \to 90° \to \frac{\pi}{2}$

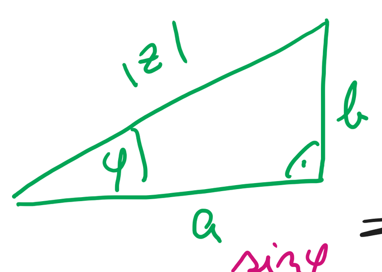
$A \to D \to 270°$




$z = a + bi = |z|(\cos\varphi + i\sin\varphi)$

$z_1 = 5 - 5\sqrt{3}i$



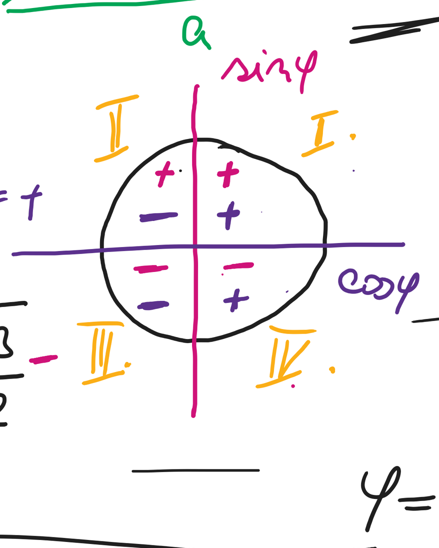

$|z| = \sqrt{a^2 + b^2}$

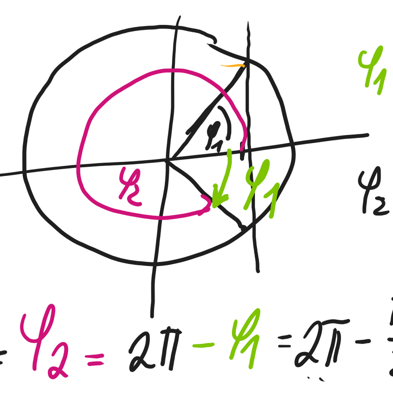
$|z_1| = \sqrt{5^2 + (-5\sqrt{3})^2} =$

$= \sqrt{25 + 25 \cdot 3} = \sqrt{100} = 10$

$\cos\varphi = \frac{a}{|z|} = \frac{5}{10} = \frac{1}{2} = +$

$\sin\varphi = \frac{b}{|z|} = \frac{-5\sqrt{3}}{10} = -\frac{\sqrt{3}}{2} -$




$\varphi_1 = 60° = \frac{\pi}{3}$

$\varphi_2 = 300°$

$\varphi = \varphi_2 = 2\pi - \varphi_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$z_1 = 5 - 5\sqrt{3}i = \boxed{10\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)}$

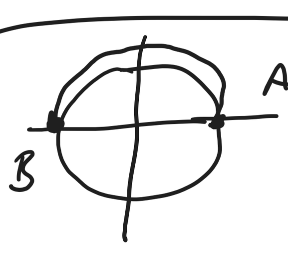
$z_1^9 = 10^9\left(\cos \cancel{9}^{3} \cdot \frac{5\pi}{\cancel{3}_1} + i\sin \cancel{9}^{3} \cdot \frac{5\pi}{\cancel{3}_1}\right) =$

$= 10^9(\cos 15\pi + i\sin 15\pi) \approx$

$\approx 10^9(\underbrace{\cos\pi}_{-1} + i\underbrace{\sin\pi}_{0}) = 10^9(-1 + 0i)$

$z = |z|(\cos\varphi + i\sin\varphi)$

$z^n = |z|^n(\cos n\varphi + i\sin n\varphi)$

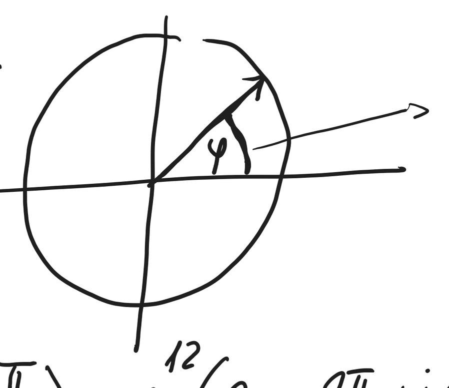



$z_1^9 = (5 - 5\sqrt{3}i)^9 = -10^9$

$z_2 = \sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$   $a + bi = |z|(\cos\varphi + i\sin\varphi)$

$|z_2| = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3 + 1} = \sqrt{4} = 2$

$\left.\begin{array}{l} \cos\varphi = \frac{a}{|z|} = \frac{\sqrt{3}}{2} = + \\ \sin\varphi = \frac{b}{|z|} = \frac{1}{2} = + \end{array}\right\} \varphi = \frac{\pi}{6}$




$\varphi = \frac{\pi}{6}$

$z_2^{12} = 2^{12}\left(\cos \cancel{12}^{2} \cdot \frac{\pi}{\cancel{6}_1} + i\sin \cancel{12}^{2} \frac{\pi}{\cancel{6}_1}\right) = 2^{12}(\underbrace{\cos 2\pi}_{1} + i\underbrace{\sin 2\pi}_{0})$

$z_2^{12} = (\sqrt{3} + i)^{12} = 2^{12}$