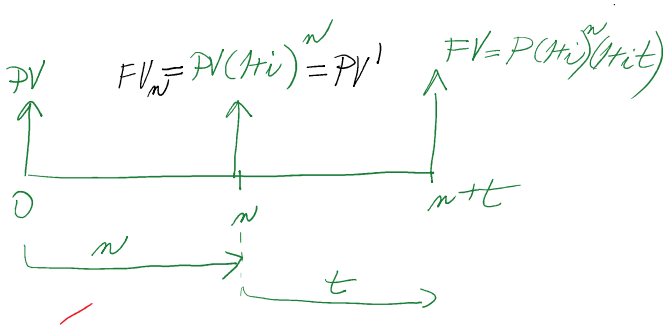
Zmiešané úrokovanie, spojité úrokovanie

streda 28. februára 2024 10:00

ZMIEŠANÉ ÚROKOVANIE

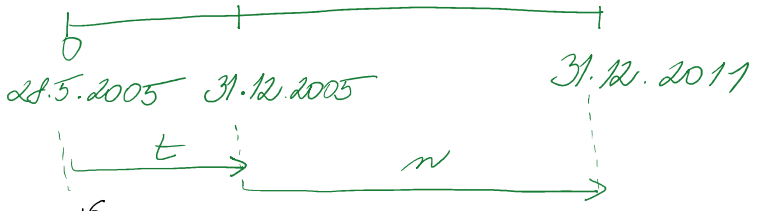
$$FV = \underbrace{PV(1+i)^n}_{PV'}(1+it)$$



ÚROČENIE

PR1

$PV = 1500$
$i = 0{,}05$
$t = \frac{212}{360}$
$n = 6$
$FV = ?$

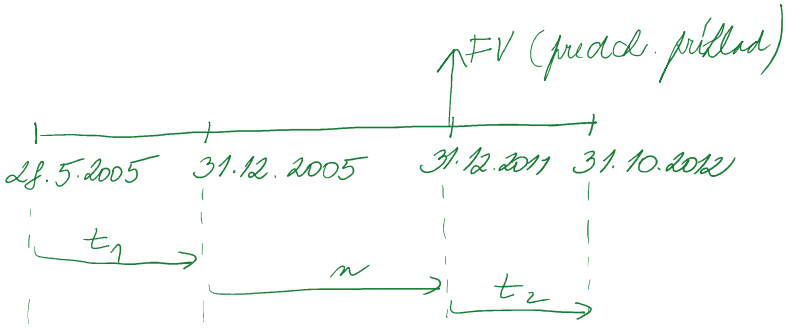


$$FV = PV(1+it)(1+i)^n$$
$$FV = 1500\left(1+0{,}05\cdot\frac{212}{360}\right)(1+0{,}05)^6$$
$$FV = 2069{,}331$$

$$t = \frac{30-28+7\cdot 30}{360} = \frac{2+210}{360} = \frac{212}{360}$$
$n = 6\ (= 2011-2005)$

PR2

$PV = 1500$
$i = 0{,}05$
$t_1 = \frac{212}{360}$
$n = 6$
$t_2 = \frac{10}{12}$
$FV = ?$

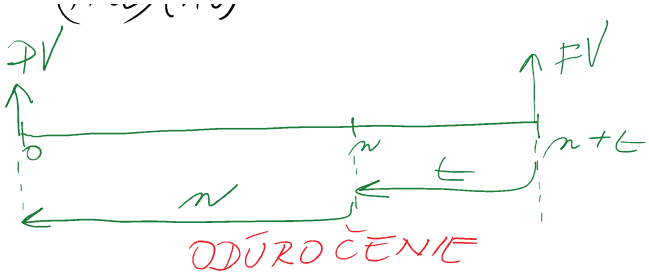


$$FV = PV(1+it_1)(1+i)^n(1+it_2)$$
$$FV = \underbrace{1500\left(1+0{,}05\cdot\frac{212}{360}\right)(1+0{,}05)^6}_{2069{,}331\ (\text{viď predošlý príklad})}\left(1+0{,}05\cdot\frac{10}{12}\right)$$
$$FV = 2155{,}553$$

$$FV = PV(1+it)(1+i)^n \;/\cdot\frac{1}{(1+it)(1+i)^n}$$

PV FV

$$PV = \frac{FV}{(1+it)(1+i)^n}$$



PR 3 $FV = 3200$

$i = 0{,}04$

$n = 3$

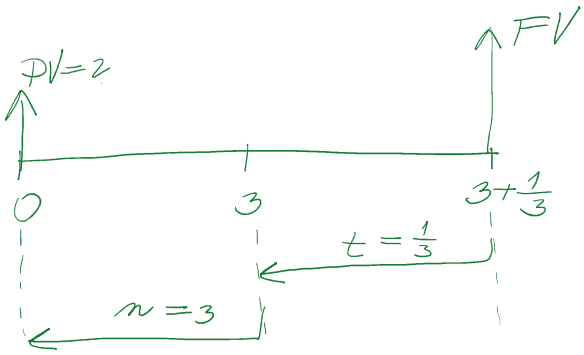
$t = \frac{4}{12} = \frac{1}{3}$

$PV = ?$

$$PV = \frac{FV}{(1+i)^n(1+it)}$$

$$PV = \frac{3200}{(1+0{,}04)^3\left(1+0{,}04\cdot\frac{1}{3}\right)}$$

$$PV = 2807{,}35\ €$$



SPOJITÉ ÚROKOVANIE

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k \quad \text{resp.} \quad \lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = e^k$$

$(x^a)^b = x^{a\cdot b}$

$$FV_n = PV\cdot\left(1+\frac{j}{m}\right)^{m\cdot n}$$

$$\lim_{m\to\infty}\left(1+\frac{j}{m}\right)^{m\cdot n} = \lim_{m\to\infty}\left[\left(1+\frac{j}{m}\right)^m\right]^n = \left[\lim_{m\to\infty}\left(1+\frac{j}{m}\right)^m\right]^n = [e^j]^n = e^{j\cdot n}$$

e^j

$$FV_t = PV\, e^{jt}$$

PR 1

1. $j = 90\%$, $FV_n = 2PV$, $m = 12$, $n = ?$

2. $j = 90\%$, $FV_t = 2PV$, $m \to \infty$, $t = ?$

1. $FV_n = PV\left(1+\frac{j}{m}\right)^{m\cdot n}$ $/\cdot\frac{1}{PV}$ 2 $FV_t = PV\, e^{jt}$ $/\cdot\frac{1}{PV}$

1. $FV_n = PV\left(1+\frac{j}{m}\right)^{m\cdot n}$ $/\cdot\frac{1}{PV}$

$2PV = PV\left(1+\frac{0{,}08}{12}\right)^{12\cdot n}$

$\left(1+\frac{0{,}08}{12}\right)^{12n} = 2$ $/\ln(\,)$

$\ln\left(1+\frac{0{,}08}{12}\right)^{12n} = \ln 2$

$12n\cdot\ln\left(1+\frac{0{,}08}{12}\right) = \ln 2$ $/\ \frac{1}{12\ln\left(1+\frac{0{,}08}{12}\right)}$

$n = \frac{\ln 2}{12\ln\left(1+\frac{0{,}08}{12}\right)}$

$n = 8{,}69$ rokov

2. $FV_t = PV e^{jt}$ $/\cdot\frac{1}{PV}$

$2PV = PVe^{0{,}08t}$

$e^{0{,}08t} = 2$ $/\ln(\,)$

$\ln e^{0{,}08t} = \ln 2$

$0{,}08t\cdot\ln e = \ln 2$ $\quad \log_a a = 1$

$\ln e = 1$

$0{,}08t = \ln 2$ $/\ \frac{1}{0{,}08}$

$t = \frac{\ln 2}{0{,}08}$

$t = 8{,}66$ rokov

PR2 $PV = 16\,000$

$FV_4 = 21\,597{,}74$

$t = 4$

$m \to \infty$

$j = ?$

$FV_t = PV\cdot e^{jt}$

$21\,597{,}74 = 16\,000\cdot e^{j\cdot 4}$ $/:16\,000$

$e^{4j} = \frac{21\,597{,}74}{16\,000}$ $/\ln(\,)$

$\ln e^{4j} = \ln\frac{21\,597{,}74}{16\,000}$

$4j\ln e = \ln\frac{21\,597{,}74}{16\,000}$ $/:4$

$j = \frac{\ln\frac{21\,597{,}74}{16\,000}}{4}$

$j = 0{,}075$

$j\cdot 100\% = 7{,}5\%$

PR3 $FV_{10} = 50\,000$

$j = 0{,}05$

TKJ $FV_{10} = 50\,000$

$j = 0{,}05$

$t = 10$

$m \to \infty$

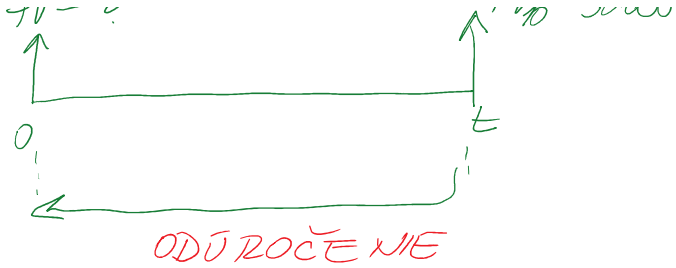
$PV = ?$

$FV_t = PV\, e^{jt}$

$PV = \frac{FV_t}{e^{jt}}$

$PV = \frac{50\,000}{e^{0{,}05 \cdot 10}}$

$PV = 30\,326{,}53$



OPTIMÁLNA DOBA VLASTNENIA

$\frac{V'(t)}{V(t)}$ ← zmena

… relatívna miera (zmeny)

$V(t)$ ← pôvodná hodnota

PR $V(t) = 10\,000\, e^{\sqrt{t}}$

$j = 0{,}05$

$m \to \infty$

$t = ?$

$\frac{V'(t)}{V(t)} = \frac{10000 \cdot e^{\sqrt{t}} \cdot (\sqrt{t})'}{10000\, e^{\sqrt{t}}} = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$ (klesajúca)

$(t^{\frac{1}{2}})'$

$\frac{1}{2\sqrt{t}} = 0{,}05$

$\frac{1}{2\sqrt{t}} = \frac{5}{100}$ / $\frac{1}{(\,)}$

$2\sqrt{t} = \frac{100}{5}$ / 1

$2\sqrt{t} = 20$ /:2

$\sqrt{t} = 10$ /()²

$t = 100$

OPLATÍ SA PREDAŤ PO 100 ROKOCH