

$$\int \frac{x^3}{\sqrt{1-x^2}} dx = \left| \begin{array}{l} 1-x^2 = t \\ -2x dx = dt \\ x dx = -\frac{1}{2} dt \end{array} \right| = \int \frac{t^{\frac{3}{2}}}{\sqrt{t}} \cdot \left(-\frac{1}{2}\right) dt = -\frac{1}{2} \int t^{\frac{3}{2}} dt = \left[\int x^n dx = \frac{x^{n+1}}{n+1} + C \right]$$

$$= -\frac{1}{2} \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + C = -\frac{1}{2} \cdot \frac{2}{5} (1-x^2)^{\frac{5}{2}} + C = -\frac{1}{5} (1-x^2)^{\frac{5}{2}} + C$$

$$\int \frac{e^{\ln x}}{x} \cdot \ln x dx = \left| \begin{array}{l} 1 - \ln x = t \\ -\frac{1}{x} dx = dt \\ \ln x dx = -\frac{1}{2} dt \end{array} \right| = \int t^{\frac{1}{2}} \cdot \left(-\frac{1}{2}\right) dt = -\frac{1}{2} \int t^{\frac{1}{2}} dt$$

$$= -\frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{1}{3} t^{\frac{3}{2}} + C = -\frac{1}{3} (1 - \ln x)^{\frac{3}{2}} + C$$

$$\int \frac{\sin x}{\cos^2 x + 3} dx = \left| \begin{array}{l} \cos x = t \\ -\sin x dx = dt \\ \sin x dx = -dt \end{array} \right| = \int \frac{1}{t^2 + 3} dt = \left[\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C \right]$$

$$= -\ln |t + \sqrt{t^2 + 3}| + C = -\ln |\cos x + \sqrt{\cos^2 x + 3}| + C$$

$$\int \frac{\cos x}{\sin^2 x + 8 \sin x + 26} dx = \left| \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right| = \int \frac{1}{t^2 + 8t + 26} dt =$$

$$t^2 + 8t + 26 = (t + 4)^2 + 10 \quad D = b^2 - 4ac = 8^2 - 4 \cdot 1 \cdot 26 = 64 - 104 = -40$$

$$\sqrt{D} = \sqrt{-40} \neq \mathbb{R}$$

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$$\frac{t^2 + 8t + 26}{(t + 4)^2 + 10}$$

$$\frac{a^2 \pm 2ab + b^2}{(a \pm b)^2}$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$= \int \frac{1}{(t+4)^2 + 10} dt = \left| \begin{array}{l} t+4 = s \\ dt = ds \end{array} \right| = \int \frac{1}{s^2 + 10} ds = \frac{1}{\sqrt{10}} \arctan \frac{s}{\sqrt{10}} + C =$$

$$= \frac{1}{\sqrt{10}} \arctan \frac{t+4}{\sqrt{10}} + C = \frac{1}{\sqrt{10}} \arctan \frac{\sin x + 4}{\sqrt{10}} + C$$

$$\int \frac{dx}{\sqrt{8-6x-9x^2}} = \int \frac{dx}{\sqrt{9 - (3x+1)^2}} = \left| \begin{array}{l} 3x+1 = t \\ 3 dx = dt \\ dx = \frac{1}{3} dt \end{array} \right| = \int \frac{1}{\sqrt{9-t^2}} dt = \left[\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + C \right]$$

$$-9x^2 - 6x + 8 = -(9x^2 + 6x + 8)$$

$$= -(3x+1)^2 - 9$$

$$\frac{a^2 + 2ab + b^2}{(a+b)^2} \quad 2 \cdot 3x \cdot a = 6x$$

$$(3x+1)^2 = 9x^2 + 6x + 1$$

$$* = \int \frac{1}{\sqrt{9-t^2}} \cdot \frac{1}{3} dt = \frac{1}{3} \int \frac{1}{\sqrt{9-t^2}} dt = \frac{1}{3} \arcsin \frac{t}{3} + C =$$

$$= \frac{1}{3} \arcsin \frac{3x+1}{3} + C = \frac{1}{3} \arcsin \left(x + \frac{1}{3}\right) + C$$

$$\int \frac{1}{\sqrt{x}} dx = \left| \begin{array}{l} x = t^2 \\ dx = 2t dt \end{array} \right| = \int \frac{1}{t} \cdot 2t dt = 2 \int dt = 2t + C = 2\sqrt{x} + C$$

PER PARTES: $\int u(x) v'(x) dx = u(x)v(x) - \int u'(x) \cdot v(x) dx$

- | | | | |
|----------|----------|----------|---------------------------|
| $u(x)$ | $v'(x)$ | $v(x)$ | $u'(x)$ |
| $P_n(x)$ | e^x | $P_n(x)$ | $\ln x$ |
| $P_n(x)$ | a^x | $P_n(x)$ | $\arcsin x$ |
| $P_n(x)$ | $\sin x$ | $P_n(x)$ | $\arccos x$ |
| $P_n(x)$ | $\cos x$ | $P_n(x)$ | $\arctan x$ |
| | | $P_n(x)$ | $\operatorname{arccot} x$ |

$$* = 2 \int t e^t dt = \left| \begin{array}{l} u = t \\ u' = 1 \\ v = e^t \\ v' = e^t \end{array} \right| = 2 \left[t e^t - \int e^t dt \right] =$$

$$2(t e^t - e^t) + C = 2e^t (t - 1) + C$$

$$x = t^2$$

$$t = \sqrt{x}$$

$$\int \arcsin x dx = \left| \begin{array}{l} u = \arcsin x \\ u' = \frac{1}{\sqrt{1-x^2}} \\ v = x \\ v' = 1 \end{array} \right| =$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx = x \arcsin x + \sqrt{1-x^2} + C \quad C \in \mathbb{R}$$

$$\int \frac{x}{\sqrt{1-x^2}} dx = \left| \begin{array}{l} 1-x^2 = t \\ -2x dx = dt \\ x dx = -\frac{1}{2} dt \end{array} \right| = -\frac{1}{2} \int \frac{1}{\sqrt{t}} dt = -\frac{1}{2} \int t^{-\frac{1}{2}} dt =$$

$$= -\frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = -t^{\frac{1}{2}} + C = -\sqrt{1-x^2} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$