$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}}\cdot \ln x\,dx = \left| \begin{matrix} u = \ln x & v' = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \\ u' = \frac{1}{x} & v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} = 2\sqrt{x} \end{matrix} \right| \quad \left| \int x^n dx = \frac{x^{n+1}}{n+1} + c \right|$$

$$\boxed{\int u(x)\,v'(x) = u(x)\,v(x) - \int u'(x)\,v(x)\,dx}$$

$$= 2\sqrt{x}\ln x - \int \frac{2\sqrt{x}}{x}\,dx = 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\ln x - 2\cdot 2\sqrt{x} + c$$

$$x^{-\frac{1}{2}} \qquad = 2\sqrt{x}(\ln x - 2) + c$$

$$\int (2x^2+3x+1)\sin x\,dx = \left| \begin{matrix} u = 2x^2+3x+1 & v' = \sin x \\ u' = 4x+3 & v = -\cos x \end{matrix} \right| = \quad \left| \begin{matrix} \int \sin x\,dx \\ = -\cos x + c \end{matrix} \right.$$

$$= (2x^2+3x+1)\cdot(-\cos x) + \int (4x+3)\cos x\,dx = \left| \begin{matrix} u = 4x+3 & v' = \cos x \\ u' = 4 & v = \sin x \end{matrix} \right| \quad \left| \begin{matrix} \int \cos x\,dx = \\ = \sin x + c \end{matrix} \right.$$

OPÍŠEM

$$= -(2x^2+3x+1)(\cos x) + (4x+3)\sin x - 4\int \sin x\,dx = \quad \left| \begin{matrix} \int \sin x\,dx = \\ = -\cos x\,dx \end{matrix} \right.$$

$$= \boxed{-(2x^2+3x+1)\cos x} + (4x+3)\sin x + \boxed{4\cos x} + c =$$

$$= (3 - 3x - 2x^2)\cos x + (4x+3)\sin x + c$$

$$1 = x^0 \qquad \int x^n dx$$

$$\int 1\,dx = \int x^0\,dx = \frac{x^1}{1} + c = x + c$$

$$\int 4\,dx = 4\int 1\,dx = 4x$$

$$\int k\,dx = kx + c \quad k \in \mathbb{R}$$

$$\int (4x^3+3x^2+2x+1)\ln x\,dx = \left| \begin{matrix} u = \ln x & v' = 4x^3+3x^2+2x+1 \\ u' = \frac{1}{x} & v = 4\frac{x^4}{4} + 3\frac{x^3}{3} + 2\frac{x^2}{2} + x \end{matrix} \right|$$

$$= (x^4+x^3+x^2+x)\ln x - \int \frac{1}{x}(x^4+x^3+x^2+x)\,dx = \quad \left| \begin{matrix} \int x^n dx = \\ \frac{x^{n+1}}{n+1} + c \end{matrix} \right.$$

$$= (x^4+x^3+x^2+x)\ln x - \int (x^3+x^2+x+1)\,dx =$$

$$= (x^4+x^3+x^2+x)\ln x - \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x\right] + c =$$

$$= (x^4+x^3+x^2+x)\ln x - \frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$= (x^4+x^3+x^2+x)\ln x - \frac{3x^4+4x^3+6x+12}{12} + c$$

$$\int e^{kx}\,dx = \frac{e^{kx}}{k} + c$$

$$\int \cos kx\,dx = \frac{\sin kx}{k} + c$$

$$\int \sin kx\,dx = -\frac{\cos kx}{k} + c$$

$$\int (4x+3)e^{4x}\,dx = \left| \begin{matrix} u = 4x+3 & v' = e^{4x} \\ u' = 4 & v = \frac{e^{4x}}{4} \end{matrix} \right| = \quad \left| \begin{matrix} \int e^{kx}\,dx = \\ = \frac{e^{kx}}{k} + c \end{matrix} \right.$$

$$= (4x+3)\cdot\frac{e^{4x}}{4} - \int 4\cdot\frac{e^{4x}}{4}\,dx = (4x+3)\cdot\frac{e^{4x}}{4} - \int e^{4x}\,dx =$$

$$= (4x+3)\frac{e^{4x}}{4} - \frac{e^{4x}}{4} + c = \frac{e^{4x}}{4}(4x+2) + c = e^{4x}\left(x + \frac{1}{2}\right) + c$$

$$\int (7-3x)\cos 5x\,dx = \left| \begin{matrix} u = 7-3x & v' = \cos 5x \\ u' = -3 & v = \frac{\sin 5x}{5} \end{matrix} \right| = \quad \left| \begin{matrix} \int \cos kx\,dx = \\ = \frac{\sin kx}{k} + c \end{matrix} \right.$$

$$= (7-3x)\cdot\frac{\sin 5x}{5} - \int (-3)\frac{\sin 5x}{5}\,dx =$$

$$= (7-3x)\frac{\sin 5x}{5} + \frac{3}{5}\int \sin 5x\,dx = \quad \left| \begin{matrix} \int \sin kx\,dx = \\ -\frac{\cos kx}{k} + c \end{matrix} \right.$$

$$= (7-3x)\frac{\sin 5x}{5} + \frac{3}{5}\left(-\frac{\cos 5x}{5}\right) + c =$$

$$= (7-3x)\frac{\sin 5x}{5} - \frac{3}{25}\cos 5x + c = \frac{5(7-3x)\sin 5x - 3\cos 5x}{25} + c$$

$$\int \frac{x-3}{x^4-3x^3+5x^2-5x+2}\,dx = \text{ROZKLAD FUNKCIE NA PARCIÁLNE ZLOMKY} = \int\left(\frac{1}{x-1} - \frac{1}{(x-1)^2} - \frac{x-1}{x^2-x+2}\right)dx$$

→ 3. cv

$r \ln x = \ln x^r$

$\ln a - \ln b = \ln \frac{a}{b}$

$\ln|x-1| - \ln\sqrt[2]{|x^2-x+1|}$

$\to \frac{1}{2}\ln|x^2-x+1| = \ln|x^2-x+1|^{\frac{1}{2}} = \ln\sqrt{|x^2-x+1|}$

$$\int\frac{f'}{f}dx \quad = \int \frac{1}{x-1}\,dx - \int \frac{1}{(x-1)^2}\,dx - \int \frac{x-1}{x^2-x+2}\,dx =$$

$$\ln|x-1| + \frac{1}{x-1} - \left(\frac{1}{2}\ln|x^2-x+1| + \frac{1}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}}\right) + c = \frac{1}{x-1} + \ln\frac{|x-1|}{\sqrt{|x^2-x+1|}} - \frac{1}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c$$

$$\left| \begin{matrix} \int \frac{1}{x}\,dx = \\ \ln|x| + c \end{matrix} \right. \qquad \leftarrow \int \frac{1}{x-1}\,dx = \left| \begin{matrix} t = x-1 \\ dt = dx \end{matrix} \right| = \int \frac{1}{t}\,dt = \ln|t| + c = \ln|x-1| + c \qquad \int \frac{1}{x\pm a}\,dx = \ln|x\pm a| + c$$

$$\left| \begin{matrix} \int x^n dx = \\ \frac{x^{n+1}}{n+1} + c \end{matrix} \right. \qquad \leftarrow \int \frac{1}{(x-1)^2}\,dx = \left| \begin{matrix} t = x-1 \\ dt = dx \end{matrix} \right| = \int \frac{1}{t^2}\,dt = \frac{t^{-1}}{-1} + c = -\frac{1}{x-1} + c$$

$$\int \frac{x-1}{x^2-x+2}\cdot\frac{2}{2}\,dx = \int \frac{2x-2}{2(x^2-x+2)} = \frac{1}{2}\int \frac{2x-1-1}{x^2-x+2}\,dx = \frac{1}{2}\left[\int \frac{2x-1}{x^2-x+2}\,dx - \int \frac{1}{x^2-x+2}\,dx\right] =$$

$$\int \frac{f'}{f}\,dx + \int \frac{k}{x^2 \pm a^2}\,dx \qquad \to (x^2-x+2)' = 2x-1$$

$$= \boxed{\frac{1}{2}\ln|x^2+x+2| - \frac{1}{2}\frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c}$$

$$\int \frac{2x-1}{x^2-x+2}\,dx = \ln|x^2-x+2| + c \qquad \left\{ \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \right.$$

$f'(x)$, $f(x)$

$$\int \frac{1}{x^2-x+2}\,dx = \int \frac{1}{(x-\frac{1}{2})^2 + \frac{7}{4}}\,dx = \left| \begin{matrix} x - \frac{1}{2} = t \\ dx = dt \end{matrix} \right| = \int \frac{1}{t^2 + \frac{7}{4}}\,dx = \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2t}{\sqrt{7}} + c = * \quad \left| \begin{matrix} \int \frac{1}{a^2+x^2}\,dx = \\ = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + c \end{matrix} \right.$$

$r^2 + s^2$

$r = t \quad s = \frac{\sqrt{7}}{2}$

NEMÁ REÁLNE KORENE → ROZKLAD MENOVATEĽA NA ŠTVOREC: $x^2 - x + 2$

$(x - \frac{1}{2})^2 + \frac{7}{4}$

$x^2 - x + \frac{1}{4} + ?$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$a = x \qquad 2ab = -x$

$b = -\frac{1}{2} \qquad 2xb = -x$

$$* = \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2t}{\sqrt{7}} + c = \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2(x-\frac{1}{2})}{\sqrt{7}} + c = \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c$$