$$\int \frac{7x^5+14x^4+41x^3+72x^2+20x-6}{x^4+2x^3+6x^2+10x+5}\,dx = \dots = \quad =$$

ROZKLAD FCIE NA PARCIÁLNE ZLOMKY

$$= \int \left(7x - \frac{3}{(x+1)} + \frac{2}{(x+1)^2} + \frac{2x-1}{x^2+5}\right) dx =$$

$$= 7\int x\,dx - 3\int \frac{1}{x+1}\,dx + 2\int \frac{1}{(x+1)^2}\,dx + \int \frac{2x}{x^2+5}\,dx - \int \frac{1}{x^2+5}\,dx$$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ $\quad$ $\int \frac{f'(x)}{f(x)}\,dx$ $\quad$ subst.: $x+1 = t$, $dx = dt$ $\Rightarrow \int \frac{1}{t^2}\,dt$ $\quad$ $\int t^{-2}dt = \frac{t^{-1}}{-1}$ $\quad$ $\int \frac{f'(x)}{f(x)}\,dx$ $\quad$ $a^2 = 5$, $a = \sqrt{5}$ $\quad$ $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + c$

$$= 7\,\frac{x^2}{2} - 3\ln|x+1| - 2\,\frac{1}{x+1} + \ln|x^2+5| - \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{x}{\sqrt{5}} + c$$

$$= \frac{7}{2}x^2 - \frac{2}{x+1} + \ln\frac{x^2+5}{|x+1|^3} - \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{x}{\sqrt{5}} + c$$

$\ln a + \ln b = \ln(ab)$
$\ln a - \ln b = \ln\frac{a}{b}$
$r\ln a = \ln(a^r)$

$$\int \frac{\sqrt[3]{x}}{x(\sqrt{x}+\sqrt[3]{x})}\,dx = \left| \begin{array}{l} x = t^6 \\ \sqrt[6]{x} = t \\ dx = 6t^5dt \end{array} \right| =$$

$\sqrt[r]{a^s} = a^{\frac{s}{r}}$

$$= \int \frac{t^2}{t^6(t^3+t^2)}\,6t^5dt = 6\int \frac{\cancel{t^2}\cdot \cancel{t^5}}{t^6\cdot \cancel{t^2}(t+1)}\,dt = 6\int \frac{1}{t(t+1)}\,dt = *$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} = \frac{1}{t} - \frac{1}{t+1}$$

$$\frac{1}{t(t+1)} = \frac{A(t+1)+Bt}{t(t+1)}$$

$\frac{7}{4} = \frac{♡}{4} \Rightarrow 7 = ♡$

$1 = A(t+1) + Bt$

$t(t+1) = 0$: $t = -1$, $t = 0$

$t = -1$: $1 = A\cdot 0 + B\cdot(-1)$
$1 = -B$
$-1 = B$

$t = 0$: $1 = A\cdot 1 + B\cdot 0$
$1 = A$

$$* \; 6\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt = 6\left(\ln|t| - \ln|t+1|\right) + c = 6\ln\frac{|t|}{|t+1|} + c =$$

$\int \frac{1}{x}\,dx = \ln|x| + c$ $\quad$ $\int \frac{f'(x)}{f(x)} = \ln|f(x)| + c$

$$= 6\ln\frac{|\sqrt[6]{x}|}{|\sqrt[6]{x}+1|} + c =$$

$$= \ln\frac{|x|}{(\sqrt[6]{x}+1)^6} + c$$

$$\int \frac{\sqrt[3]{x-7}}{(x-7)+\sqrt[6]{(x-7)^5}}\,dx = \left| \begin{array}{l} x-7 = t^6 \\ \sqrt[6]{x-7} = t \\ dx = 6t^5dt \end{array} \right| = \int \frac{t^2}{t^6+t^5}\cdot 6t^5dt =$$

$t^{\frac{6}{3}}$ $\quad$ $t^{\frac{6}{6}\cdot 5}$

$$= 6\int \frac{t^2\cdot \cancel{t^5}}{\cancel{t^5}(t+1)}\,dt = 6\int \frac{t^2-1+1}{t+1}\,dt =$$

$a^2 - b^2 = (a+b)(a-b)$

$t^2 : (t+1) = t - 1 + \frac{1}{t+1}$
$-(t^2+t)$
$\quad -t$
$\quad -(-t-1)$
$\qquad 1$

$$6\int\left(\frac{t^2-1}{t+1} + \frac{1}{t+1}\right)dt =$$

$$6\int\left(\frac{\cancel{(t+1)}(t-1)}{\cancel{t+1}} + \frac{1}{t+1}\right)dt =$$

$$= 6\int\left(t - 1 + \frac{1}{t+1}\right)dt =$$

$\int x^n dx$ $\quad$ $\int \frac{f'(x)}{f(x)}\,dx$

$$= 6\left(\frac{t^2}{2} - t + \ln|t+1|\right) + c = 6\left(\frac{\sqrt[6]{(x-7)^2}}{2} - \sqrt[6]{x-7} + \ln|\sqrt[6]{x-7}+1|\right) + c$$

$$= 3\sqrt[3]{x-7} - 6\sqrt[6]{x-7} + \ln(|\sqrt[6]{x-7}+1|)^6 + c$$

$$\int \frac{\sqrt[6]{6x-5} - 1}{\sqrt[3]{(6x-5)^4} + \sqrt[4]{(6x-5)^5}}\,dx = \left| \begin{array}{l} 6x-5 = t^{12} \\ \sqrt[12]{6x-5} = t \\ \cancel{6}\,dx = \cancel{12}^{\,2}\,t^{11}dt \quad /:6 \\ dx = 2t^{11}dt \end{array} \right| =$$

$t^{\frac{12}{3}\cdot 4}$ $\quad$ $t^{\frac{12}{4}\cdot 5}$

$$= \int \frac{t^2-1}{t^{16}+t^{15}}\,2t^{11}dt = 2\int \frac{(t^2-1)t^{11}}{t^{15}(t+1)}\,dt =$$

$$2\int \frac{\cancel{(t+1)}(t-1)\cancel{t^{11}}}{t^{\cancel{15}\,4}\cancel{(t+1)}}\,dt = 2\int \frac{t-1}{t^4}\,dt = 2\int\left(\frac{1}{t^3} - \frac{1}{t^4}\right)dt$$

t^{-3} $\quad$ t^{-4} $\quad$ $\left\{ \int x^n dx = \frac{x^{n+1}}{n+1} + c \right.$

$$2\left(\frac{t^{-2}}{-2} - \frac{t^{-3}}{-3}\right) + c = -\frac{1}{t^2} + \frac{2}{3t^3} + c =$$

$$= -\frac{1}{\sqrt[12]{(6x-5)^2}} + \frac{2}{3\sqrt[12]{(6x-5)^3}} + c = -\frac{1}{\sqrt[6]{6x-5}} + \frac{2}{3\sqrt[4]{6x-5}} + c$$

$$\int \frac{x^2}{(5x+2)\sqrt{5x+2}}\,dx = \left| \begin{array}{l} 5x+2 = t^2 \to t = \sqrt{5x+2} \\ x = \frac{t^2-2}{5} \\ 5dx = 2t\,dt \\ dx = \frac{2}{5}t\,dt \end{array} \right| = \int \frac{\left(\frac{t^2-2}{5}\right)^2}{t^2\cdot \cancel{t}}\,\frac{2}{5}\cancel{t}\,dt =$$

$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

$$= \frac{2}{5}\cdot\frac{1}{25}\int \frac{t^4-4t^2+4}{t^2}\,dt = \frac{2}{125}\int\left(t^2 - 4 + \underbrace{\frac{4}{t^2}}_{=4\cdot t^{-2}}\right)dt =$$

$\left\{ \int x^n dx \right.$

$$= \frac{2}{125}\left(\frac{t^3}{3} - 4t - \frac{4}{t}\right) + c = \frac{2}{125}\left(\frac{\sqrt{(5x+2)^3}}{3} - 4\sqrt{5x+2} - \frac{4}{\sqrt{5x+2}}\right) + c$$

$$= \frac{2}{125}\sqrt{5x+2}\left(\frac{5x+2}{3} - 4 + \frac{4}{5x+2}\right) + c$$

$$\int \frac{7}{\sqrt{x^2+10x-5}}\,dx = 7\int \frac{1}{\sqrt{x^2+10x-5}}\,dx = *$$

ÚPRAVA NA ŠTVOREC !!!

$(a\pm b)^2 = a^2 \pm 2ab + b^2$

$a = x$ $\quad$ $b = 5$

$x^2+10x-5 =$
$(x+5)^2 - 30$
$(x+5)^2 = x^2+10x+25$

$$* = 7\int \frac{1}{\sqrt{(x+5)^2-30}}\,dx = \left| \begin{array}{l} x+5 = t \\ dx = dt \end{array} \right| = 7\int \frac{1}{\sqrt{t^2-30}}\,dt =$$

$\left\{ \int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln|x + \sqrt{x^2\pm a^2}| + c \right.$

$$= 7\ln|t + \sqrt{t^2-30}| + c$$

$\sqrt{(x+5)^2-30}$

$$= 7\ln|x+5+\sqrt{x^2+10x-5}| + c$$