Example

Write minimal conjunctive form and minimal disjunctive form of a given Boolean function.

a) $f(x, y, z)$ which has value 0 only in arguments (0,0,1), (1,1,0), (1,0,1), (1,0,0) ,
b) $f(p, q, r)$ which has value 1 only in arguments (1,1,1) and (0,0,1),
c) $f(p, q, r, s)$ which has value 0 only in arguments (0,0,1,0), (1,0,0,0), (1,0,1,0), (1,0,0,1), (1,1,0,0), (1,1,0,1) and (1,0,1,1).

Solution:

MINIMAL FORMS

a)

x \ zy	00	10	11	01
0	1	1	1	0
1	0	0	1	0

Minimal conjunctive form is $(\overline{x} \vee z) \wedge (y \vee \overline{z})$.

x \ yz	00	10	11	01
0	1	1	1	0
1	0	0	1	0

Minimal disjunctive form is $(\overline{x} \wedge \overline{z}) \vee (y \wedge z)$.

MINIMAL FORMS

b)

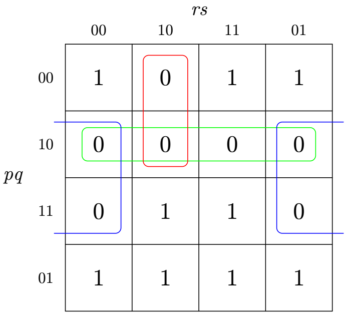
$p \backslash qr$	00	10	11	01
0	0	0	0	1
1	0	0	1	0

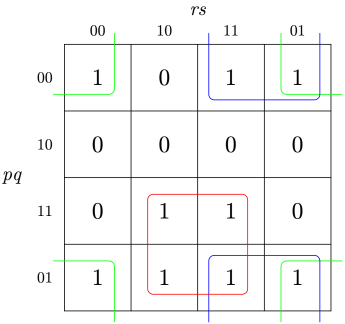
$p \backslash qr$	00	10	11	01
0	0	0	0	1
1	0	0	1	0

Minimal conjunctive form is $(\overline{p} \vee q) \wedge (p \vee \overline{q}) \wedge r$.
Minimal disjunctive form is $(p \wedge q \wedge r) \vee (\overline{p} \wedge \overline{q} \wedge r)$.

MINIMAL FORMS

c)





Minimal conjunctive form is $(\overline{p} \vee q) \wedge (q \vee \overline{r} \vee s) \wedge (\overline{p} \vee r)$.
Minimal disjunctive form is $(\overline{p} \wedge \overline{r}) \vee (q \wedge r) \vee (\overline{p} \wedge s)$.

Example

Without using Karnaugh maps, find minimal disjunctive form of the Boolean function which normal disjunctive form is given.

a) $(x \wedge y \wedge \overline{z}) \vee (\overline{x} \wedge y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z)$,

b) $(x \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge \overline{y} \wedge \overline{z})$,

c) $(x \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z})$.

Solution:

MINIMAL FORMS

a)
$(x \wedge y \wedge \overline{z}) \vee (\overline{x} \wedge y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z) =$
$((x \wedge \overline{z}) \wedge (y \vee \overline{y})) \vee ((\overline{x} \wedge z) \wedge (\overline{y} \vee y)) = (x \wedge \overline{z}) \vee (\overline{x} \wedge z)$

b)
$(x \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge \overline{y} \wedge \overline{z}) =$
$((\overline{y} \wedge z) \wedge (x \vee \overline{x})) \vee ((\overline{x} \wedge z) \wedge (y \vee \overline{y})) \vee ((\overline{y} \wedge \overline{z}) \wedge (x \vee \overline{x})) = (\overline{y} \wedge z) \vee (\overline{x} \wedge z) \vee (\overline{y} \wedge \overline{z})$
$= (\overline{y} \wedge (z \vee \overline{z})) \vee (\overline{x} \wedge z) = \overline{y} \vee (\overline{x} \wedge z)$

c)
$(x \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z}) = ((x \wedge \overline{y}) \wedge (z \vee \overline{z})) \vee (\overline{x} \wedge y \wedge z) = (x \wedge \overline{y}) \vee (\overline{x} \wedge y \wedge z).$

MINIMAL FORMS

Example

Write normal disjunctive form and normal conjunctive form and also both minimal forms for of the Boolean function $f(x, y, z)$ which is given by formula $(x \vee y) \Rightarrow \overline{z}$.

Solution:

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	1

MINIMAL FORMS

Normal disjunctive form is
$(x \wedge y \wedge \overline{z}) \vee (x \wedge \overline{y} \wedge \overline{z}) \vee (\overline{x} \wedge y \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge \overline{y} \wedge \overline{z})$.

Normal conjunctive form is
$(\overline{x} \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee \overline{z})$.

$(x \wedge y \wedge \overline{z}) \vee (x \wedge \overline{y} \wedge \overline{z}) \vee (\overline{x} \wedge y \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z) \vee (\overline{x} \wedge \overline{y} \wedge \overline{z})$
$= (x \wedge \overline{z}) \vee (\overline{x} \wedge \overline{z}) \vee (\overline{x} \wedge \overline{y} \wedge z) = \overline{z} \vee (\overline{x} \wedge \overline{y} \wedge z) = \overline{z} \vee (\overline{x} \wedge \overline{y})$, that is minimal disjunctive form.

$(\overline{x} \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee \overline{z}) = \overline{z} \vee (\overline{x} \wedge (x \vee \overline{y})) = \overline{z} \vee (\overline{x} \wedge \overline{y}) = (\overline{x} \vee \overline{z}) \wedge (\overline{y} \vee \overline{z})$,
that is minimal conjunctive form.

MINIMAL FORMS

x \ yz	00	10	11	01
0	1	1	0	1
1	1	1	0	0

Minimal disjunctive form is $\overline{z} \vee (\overline{x} \wedge \overline{y})$.

Minimal conjunctive form is $(\overline{x} \vee \overline{z}) \wedge (\overline{y} \vee \overline{z})$.

GRAPHS

Definition

A graph is a pair $G = (V, E)$ where V is a nonempty finite set and E is a set of two–element subsets of V.

$G = (V, E)$
$V = \{v_1, v_2, v_3, v_4\}$, $E = \{\{v_1, v_3\}, \{v_2, v_4\}, \{v_1, v_4\}, \{v_2, v_3\}\}$

The elements of V are called the vertices of the graph, and the elements of E are called the edges of the graph.
Let G be a graph. If we neglect to give a name to the vertex set and edge set of G, we can simply write $V(G)$ and $E(G)$ for the vertex and edge sets, respectively.

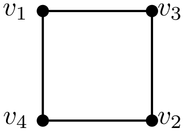
GRAPHS

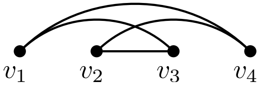
How to draw pictures of graphs? These pictures make graphs much easier to understang.

A drawing of the graph $G = (V, E)$ is a mapping that assing a point in the plane for each vertex and for each edge a continuous curve between its two endpoints.

A drawing of the graph is not the same thing as the graph itself.

The following two drawings both depict the same graph.



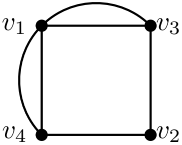


GRAPHS

Definition

A *multigraph* $G = (V, E)$ *consists of a set of vertices* V*, a set of edges* E*, and a function* f *from* E *to* $\{\{u, v\} : u, v \in V, u \neq v\}$*. The edges* e_1 *and* e_2 *are called* *multiple* *or* *parallel edges* *if* $f(e_1) = f(v_2)$*.*

$G = (V, E)$
$V = \{v_1, v_2, v_3, v_4\}$, $E = \{\{v_1, v_3\}, \{v_2, v_4\}, \{v_1, v_4\}, \{v_2, v_3\}, \{v_1, v_4\}\{v_1, v_3\}\}$



GRAPHS

Definition

Two vertices u and v in a graph $G = (V, E)$ are called adjacent in G if $\{u, v\}$ is an edge of G.
If $e = \{u, v\}$, the edge e is called incident with the vertices u and v.

If $\{u, v\}$ is an edge of G, we call u and v the endpoints of the edge.

Definition

Let $G = (V, E)$ be a graph and let $v \in V$. The degree of v is the number of edges with which v is incident. The degree of v is denoted $deg(v)$ or $\delta(v)$.

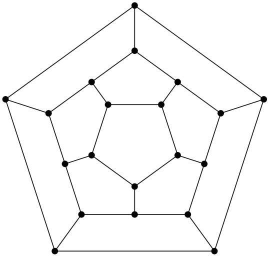
Theorem

Let $G = (V, E)$. The sum of the degrees of the vertices in G is twice the number of edges, that is,

$$\sum_{v \in V} deg(v) = 2 \cdot |E|$$

Definition

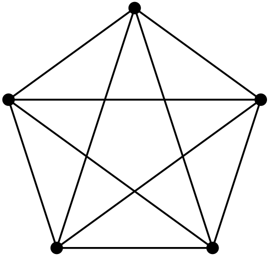
If all vertices in G have the same degree, we call G regular. If a graph is regular and all vertices have degree r, we also call the graph r–regular.



GRAPHS

Definition

Let G be a graph. If all pairs of distinct vertices are adjacent in G, we call G complete. A complete graph on n vertices is denoted K_n.

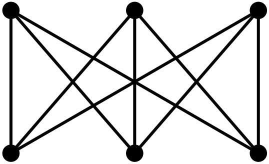


The opposite extreme is a graph with no edges. We call such graphs edgeless.

Definition

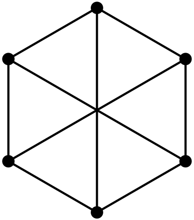
Let $m, n \in \mathbb{N}$. The complete bipartite graph, $K_{m,n}$, is a graph whose vertices can be partitioned $V = V_1 \cup V_2$ such that

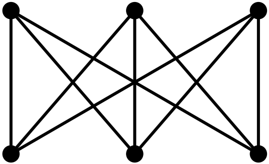
- $|V_1| = m$ *and* $|V_2| = n$
- *for all* $u \in V_1$ *and for all* $v \in V_2$, $\{u, v\}$ *is an edge.*



Definition

Let $G_1 = (V_1, E_1)$ *and* $G_2 = (V_2, E_2)$ *be graphs. We say that* G_1 *is* *isomorphic* *to* G_2 *provided there is a bijection* $f : V_1 \to V_2$ *such that for all* $u, v \in V_1$ *we have* $\{u, v\} \in E_1$ *if and only* $\{f(u), f(v)\} \in E_2$*. The function* f *is called an* *isomorphism* *of* G_1 *to* G_2*.*





GRAPHS

Definition

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be graphs. We call G_1 a subgraph of G provided $V_1 \subseteq V$ and $E_1 \subseteq E$.

Definition

Let $G = (V, E)$ be a graph. We call $G_1 = (V_1, E_1)$ a spanning subgraph of G provided $V_1 = V$ and $E_1 \subseteq E$.

Definition

Let G be a graph. The complement of G is the graph denoted $\overline{G}$ defined by

$$V(\overline{G}) = V(G)$$

$$E(\overline{G}) = \{\{u, v\} : u, v \in V(G), u \neq v, \{u, v\} \notin E(G)\}$$

.

Definition

Let $G = (V, E)$ be a graph. A walk of length n ($n \in \mathbb{N}$) in G is a sequence of vertices $v_0, v_1, \ldots v_n$ of the graph such that $\{v_0, v_1\}, \{v_1, v_2\}, \ldots \{v_{n-1}, v_n\}$ are edges, where $v_0 = u$ and $v_n = v$.
A path of length n in a graph is a walk in which no vertex is repeated.
A cycle is a path of length at least three in which the first and last vertex are the same, but no other vertices are repeated.

(u, v)–path

Definition

A graph $G = (V, E)$ is called connected provided for all $u, v \in V$ there is (u, v)–path.

Definition

Let $G = (V, E)$ be a graph and let $u, v \in V$. The distance from u to v in G is the length of the shortest (u, v)–path. In case there is no such a path, we may either say that the distance is undefined or ∞. The distance from u to v is denoted $d(u, v)$.