Renta odložená, prerušená, nekonečná, so spojitým úrokovaním

streda 20. marca 2024 10:00

ODLOŽENÁ RENTA

PR 1

$${}_5A_{10} = 10\,000$$
$$i = 0,06$$
$$n = 10$$
$$t = 5$$
$$R = ?$$

$${}_tA_n = (1+i)^{-t}\cdot R\cdot \frac{1-(1+i)^{-n}}{i}$$

$$10\,000 = (1+0,06)^{-5}\cdot R\cdot \frac{1-(1+0,06)^{-10}}{0,06}$$

$$R = 10\,000\cdot 1,06^{5}\cdot \frac{0,06}{1-1,06^{-10}}$$

$$\underline{R = 1818,21977}$$

OTÁZKA: AKÁ BY BOLA VÝŠKA SPLÁTKY, AK BY SME SPLÁCANIE NEODLOŽILI?

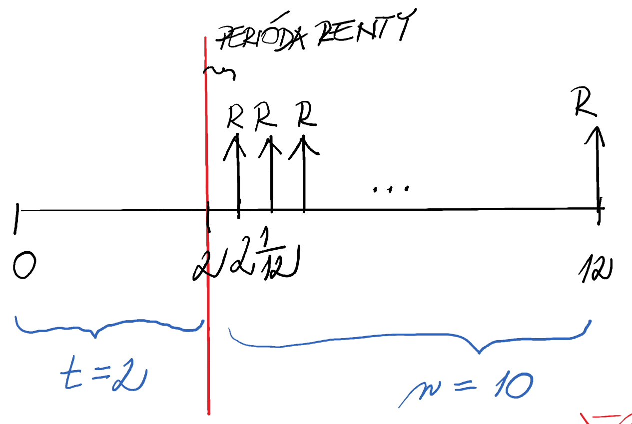
$$\underline{R^{*} = R\cdot 1,06^{-5} = \frac{R}{1,06^{5}} = 1358,67958}$$

VÝŠKA SPLÁTKY

VÝŠKA SPLÁTKY PRI ODLOŽENEJ RENTE

PR 2

$${}_2A_{10} = 3058,25217$$
$$j = 0,1$$
$$n = 10$$
$$t = 2$$
$$p = 12$$
$$m = 4$$
$$R = ?$$



$${}_tA_n = R\left(1+\frac{j}{m}\right)^{-m\cdot t}\cdot\frac{1-\left(1+\frac{j}{m}\right)^{-mn}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1}$$

$$3058,25217 = R\left(1+\frac{0,1}{4}\right)^{-4\cdot 2}\cdot\frac{1-\left(1+\frac{0,1}{4}\right)^{-4\cdot 10}}{\left(1+\frac{0,1}{4}\right)^{\frac{4}{12}}-1}$$

$$R = 3058,25217\cdot 1,025^{8}\cdot\frac{1,025^{\frac{1}{3}}-1}{1-1,025^{-40}}$$

$$\underline{R = 49,07234}$$

VEČNÁ RENTA

$$A_\infty = \lim_{n\to\infty} R\,\frac{1-(1+i)^{-n}}{i} \qquad (1+i)^{-n} \to 0$$

$$\underbrace{(1{,}01)}_{>1}^{-n} = \frac{1}{1{,}01^{n}} \to 0, \quad 1{,}01^n \to \infty$$

$$A_\infty = \frac{R}{i}$$

$p > 1$

$$A_\infty = \lim_{n\to\infty} R\,\frac{1-\left(1+\frac{j}{m}\right)^{-m\cdot n}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} \qquad \left(1+\frac{j}{m}\right)^{-m\cdot n} \to 0$$

$$\frac{1}{\left(1+\frac{j}{m}\right)^{m\cdot \boxed{n}}} \to 0, \quad \left(1+\frac{j}{m}\right)^{m\cdot n} \to \infty$$

$$A_\infty = \frac{R}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1}$$

$$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} R\,\frac{\left(1+\frac{j}{m}\right)^{m\cdot n}-1}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} = \infty, \quad \left(1+\frac{j}{m}\right)^{m\cdot n} \to \infty$$

$$S_\infty = \infty$$

RENTA SO SPOJITÝM ÚROKOVANÍM

$$A_n = \lim_{m\to\infty} R\,\frac{1-\left(1+\frac{j}{m}\right)^{\boxed{m\,n}}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} = \qquad 1^\infty$$

$$\left[\lim_{x\to\infty}\left(1+\frac{\boxed{k}}{x}\right)^{x} = e^{k}\right] \checkmark$$

$$= \lim_{m\to\infty} R\,\frac{1-\Big[\underbrace{\left(1+\frac{j}{m}\right)^{m}}_{\to e^j}\Big]^{-n}}{\Big[\underbrace{\left(1+\frac{j}{m}\right)^{m}}_{\to e^j}\Big]^{\frac{1}{p}}-1} = \qquad !\ (a^b)^c = a^{b\cdot c}$$

$$= R\,\frac{1-(e^j)^{-n}}{(e^j)^{\frac{1}{p}}-1} = R\,\frac{1-e^{-jn}}{e^{\frac{j}{p}}-1}$$

$$S_n = \lim_{m\to\infty} R \, \frac{\left(1+\frac{\delta}{m}\right)^{mn} - 1}{\left(1+\frac{\delta}{m}\right)^{\frac{m}{p}} - 1} =$$

$$= \lim_{m\to\infty} R \, \frac{\Big[\overbrace{\left(1+\frac{\delta}{m}\right)^{m}}^{\to e^{\delta}}\Big]^{n} - 1}{\Big[\underbrace{\left(1+\frac{\delta}{m}\right)^{m}}_{\to e^{\delta}}\Big]^{\frac{1}{p}} - 1} =$$

$$= R \, \frac{\left(e^{\delta}\right)^{n} - 1}{\left(e^{\delta}\right)^{\frac{1}{p}} - 1} = R \, \frac{e^{\delta n} - 1}{e^{\frac{\delta}{p}} - 1}$$

VEČNÁ RENTA SO SPOJITÝM ÚROKOVANÍM

$$A_\infty = \lim_{n\to\infty} R \, \frac{1 - \overbrace{e^{-\delta n}}^{\to 0}}{e^{\frac{\delta}{p}} - 1} \qquad e^{-\delta n} = \frac{1}{\underbrace{e^{\delta n}}_{\to\infty}} \to 0$$

$$e > 1$$

$$A_\infty = \frac{R}{e^{\frac{\delta}{p}} - 1}$$

$$S_\infty = \lim_{n\to\infty} R \, \frac{\overbrace{e^{\delta n}}^{\to\infty} - 1}{e^{\frac{\delta}{p}} - 1} = \infty$$