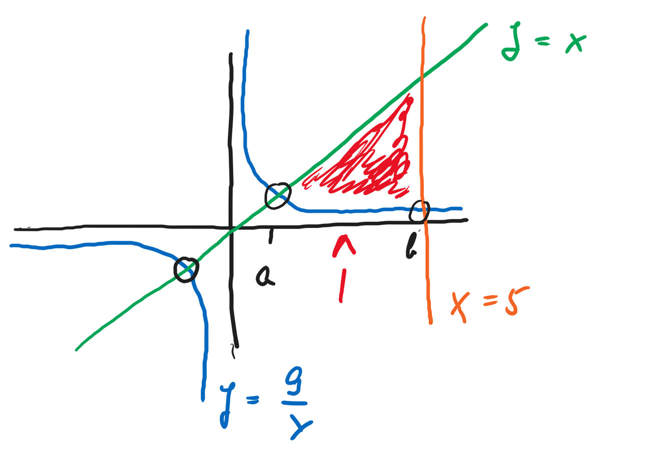
$\underline{Pr}$: $xy - 9 = 0 \quad \to y = f(x) \to y = \frac{9}{x}$

$x - y = 0 \qquad y = x$

$x - 5 = 0 \qquad x = 5$



$\overset{a}{3} \le x \le \overset{b}{5}$

$\underset{g(x)}{\frac{9}{x}} \le y \le \underset{f(x)}{x}$

$y = y$

$\frac{9}{x} = x \quad / \cdot x$

$9 = x^2 \quad / \sqrt{\ }$

$\pm 3 = x$

$S = \int_a^b (f(x) - g(x))\,dx = \int_3^5 \left(x - \frac{9}{x}\right) dx = \left[\frac{x^2}{2} - 9\ln|x|\right]_3^5 =$

$= \left(\frac{5^2}{2} - 9\ln 5\right) - \left(\frac{3^2}{2} - 9\ln 3\right) = \underline{8 + 9\ln\frac{3}{5}}$ $\quad \ln a - \ln b = \ln\frac{a}{b}$

$8 - 9\ln 5 + 9\ln 3 = 8 + 9(\ln 3 - \ln 5) = 8 + 9\ln\frac{3}{5}$

$\to \int -81 \cdot x^{-2} dx = -81\frac{x^{-1}}{-1}$

$V = \pi\int_a^b (f^2(x) - g^2(x))\,dx = \pi\int_3^5 \left(x^2 - \left(\frac{9}{x}\right)^2\right) dx = \pi\int_3^5 \left(x^2 - \frac{81}{x^2}\right) dx =$

$= \pi\left[\frac{x^3}{3} + \frac{81}{x}\right]_3^5 = \pi\left[\left(\underbrace{\frac{5^3}{3} + \frac{81}{5}}_{\frac{625+243}{15} = \frac{868}{15}}\right) - \left(\underbrace{\frac{3^3}{3} + \frac{81}{3}}_{\frac{36 \cdot 15}{15}}\right)\right] = \underline{\underline{\frac{328}{15}\pi}}$

$\pi\int \left(x^2 - \frac{81}{x^2}\right) dx = 81\pi\int\left(\frac{x^2}{81} - \frac{1}{x^2}\right) dx =$

$= \pi\int x^2 dx - \pi\int\frac{81}{x^2}dx = \pi\int x^2 dx - 81\pi\int\frac{1}{x^2}dx$

$\underline{Pr}$: $y + x - 4 = 0 \quad \to y = 4 - x$

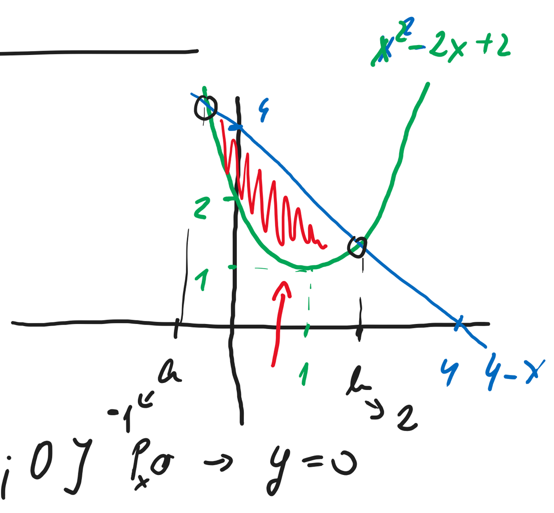
$y = x^2 - 2x + 2 \qquad y = x^2 - 2x + 2 = (x-1)^2 + 1$

$V[1;1]$

$y = 1^2 - 2 \cdot 1 + 2 = 1$

$y' = 2x - 2 = 0$

$\boxed{x = 1}$



$P_{0x}[?;0] \quad P_{0x} \to y = 0$

$P_{0y}[0;?] \quad P_{0y} \to x = 0$

$\overset{a}{-1} \le x \le \overset{b}{2}$

$\underset{g(x)}{x^2 - 2x + 2} \le y \le \underset{f(x)}{4 - x}$

$y = y$

$4 - x = x^2 - 2x + 2$

$0 = x^2 - x - 2$

$0 = (x+1)(x-2)$

$S = \int_a^b (f(x) - g(x))\,dx = \int_{-1}^2 [(4-x) - (x^2 - 2x + 2)]\,dx = \int_{-1}^2 (2 + x - x^2)\,dx =$

$\left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^2 = \left(2 \cdot 2 + \frac{2^2}{2} - \frac{2^3}{3}\right) - \left(2 \cdot (-1) + \frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right) =$

$\left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) = \underline{\underline{4{,}5}}$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(x^2 - 2x + 2)^2 = (x^2 - 2x + 2)(x^2 - 2x + 2)$

$V = \pi\int_a^b (f^2(x) - g^2(x))\,dx = \pi\int_{-1}^2 ((4-x)^2 - (x^2 - 2x + 2)^2)\,dx =$

$= \pi\int_{-1}^2 [(16 - 8x + x^2) - (x^4 - 2x^3 + 2x^2 - 2x^3 + 4x^2 - 4x + 2x^2 - 4x + 4)]\,dx =$

$= \pi\int_{-1}^2 (-x^4 + 4x^3 - 7x^2 + 12)\,dx = \pi\left[-\frac{x^5}{5} + 4\frac{x^4}{4} - 7\frac{x^3}{3} + 12x\right]_{-1}^2 =$

$= \pi\left[\left(-\frac{2^5}{5} + 2^4 - 7\frac{2^3}{3} + 12 \cdot 2\right) - \left(-\frac{(-1)^5}{5} + (-1)^4 - 7\frac{(-1)^3}{3} + 12(-1)\right)\right] = \pi\,\underline{\frac{124}{5}}$

$\frac{d}{dx}f \to f'(x)$

$f(x, y, z) \to \frac{\partial}{\partial x}f \to f'_x$

$\frac{\partial}{\partial y}f \to f'_y$

$\frac{\partial}{\partial z}f \to f'_z$

$\underline{Pr}$: $z = 2x^2y^3(5^y + 3xy)$

$z'_x = 2 \cdot 2xy^3(5^y + 3xy) + 2x^2y^3(0 + 3y) = 4xy^3(5^y + 3xy) + 6x^2y^4$

$z'_y = 2x^2 \cdot 3y^2(5^y + 3xy) + 2x^2y^3(5^y\ln 5 + 3x) = 6x^2y^2(5^y + 3xy) + 2x^2y^3(5^y\ln 5 + 3x)$

$\underline{Pr}$: $z = (2x + 5y)\sin 3x$

$z'_x = (2 + 0)\sin 3x + (2x + 5y)\,3\cos 3x = 2\sin 3x + 3(2x + 5y)\cos 3x$

$z'_y = 5\sin 3x + (2x + 5y) \cdot 0 = 5\sin 3x$

$\underline{Pr}$: $z = \sqrt[4]{(3y - 8x^5 - 2xy)^3} = (3y - 8x^5 - 2xy)^{\frac{3}{4}}$

$(y)'_y = 1 \quad (y)'_x = 0$

$(x)'_x = 1 \quad (x)'_y = 0$

$z'_x = \frac{3}{4}(3y - 8x^5 - 2xy)^{-\frac{1}{4}} \cdot (-40x^4 - 2y) = \frac{-3(40x^4 + 2y)}{4\sqrt[4]{3y - 8x^5 - 2xy}}$

$z'_y = \frac{3}{4}(3y - 8x^5 - 2xy)^{-\frac{1}{4}}(3 - 2x) = \frac{3(3 - 2x)}{4\sqrt[4]{3y - 8x^5 - 2xy}}$

$z = 15x^2y + (2x^3 - 8xy^2)(\arcsin(3x+1) - 3^{x+y}) + \ln \operatorname{tg}(3x + 9y)$

3^{x+y}

$z'_x = 30xy + (6x^2 - 8y^2)(\arcsin(3x+1) - 3^{x+y}) + (2x^3 - 8xy^2)\left(\frac{3}{\sqrt{1 - (3x+1)^2}} - 3^y \cdot 3^x \ln 3\right) +$

$+ \frac{1}{\operatorname{tg}(3x + 9y)} \cdot \frac{1}{\cos^2(3x + 9y)} \cdot 3$

$z'_y = 15x^2 + (-16xy)(\arcsin(3x+1) - 3^{x+y}) + (2x^3 - 8x^2y^2)(0 - 3^x 3^y \ln 3) +$

$+ \frac{1}{\operatorname{tg}(3x + 9y)} \cdot \frac{1}{\cos^2(3x + 9y)} \cdot 9$