Pr: $f(x,y) = e^{\cos(11x+17y^2)} - \frac{\cotg x + 2\sin y}{4x^2y^3 - 3y^2x^3}$

$f'_x = e^{\cos(11x+17y^2)} \cdot (-\sin(11x+17y^2)) \cdot 11 - \frac{\left(-\frac{1}{\sin^2 x}+0\right)(4x^2y^3-3y^2x^3) - (\cotg x + 2\sin y)(8xy^3-9y^2x^2)}{(4x^2y^3-3y^2x^3)^2}$

$f'_y = e^{\cos(11x+17y^2)} \cdot (-\sin(11x+17y^2)) \, 34y - \frac{2\cos y(4x^2y^3-3y^2x^3) - (\cotg x + 2\sin y)(12x^2y^2-6yx^3)}{(4x^2y^3-3y^2x^3)^2}$

Pr: $f(x,y) = \log_3(x^7y + 14x)$

$f'_x = \frac{7x^6y + 14}{(x^7y+14x)\ln 3}$

$f'_y = \frac{x^7}{(x^7y+14x)\ln 3}$ der. 1. rádu

$f''_{xx} = \frac{42x^5y\ln 3(x^7y+14x) - (7x^6y+14)\ln 3(7x^6y+14)}{\ln^2 3\,(x^7y+14x)^2}$

$f''_{xy} = \frac{7x^6(x^7y+14x)\ln 3 - (7x^6y+14)x^7\ln 3}{\ln^2 3\,(x^7y+14x)^2}$

$f''_{yx} = \frac{7x^6(x^7y+14x)\ln 3 - x^7(7x^6y+14)\ln 3}{\ln^2 3\,(x^7y+14x)^2}$

$f''_{yy} = \frac{0 - x^7 \cdot x^7 \ln 3}{\ln^2 3\,(x^7y+14x)^2}$

der. 2. rádu

Lokálne extrémy

Pr: $z = x^2 + (y+3)^2 - 3$

der. 1. rádu

$z'_x = 2x$ $\qquad 2x = 0 \to x = 0$

$z'_y = 2(y+3)$ $\qquad 2y+6 = 0 \to y = -3$

} Stacionárny bod $A[0; -3]$

der. 2. rádu:

$\boxed{z''_{xx} = 2} > 0$

$z''_{xy} = 0$

$z''_{yx} = 0$

$z''_{yy} = 2$

$\Delta_2(A) = \begin{vmatrix} z''_{xx}(A) & z''_{xy}(A) \\ z''_{yx}(A) & z''_{yy}(A) \end{vmatrix}$ $> 0 \to$ má extrém; $< 0 \to$ nemá extrém, sedlový bod; $= 0 \to$ nevieme

$\Delta_2(A) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 2 \cdot 2 - 0 \cdot 0 = 4 > 0$ má extrém

ak $z''_{xx}(A) > 0 \to$ L. minimum

$< 0 \to$ L. maximum

$\Rightarrow$ v bode A má funkcia L. minimum

Pr: $z = y^2 - x^2 + 3x - 5y + 4$

der. 1. rádu:

$z'_x = -2x + 3$ $\qquad -2x + 3 = 0 \to x = \frac{3}{2}$

$z'_y = 2y - 5$ $\qquad 2y - 5 = 0 \to y = \frac{5}{2}$

$2y = 5$

} stac. bod $A[\frac{3}{2}; \frac{5}{2}]$

der. 2. rádu:

$z''_{xx} = -2$

$z''_{xy} = 0$

$z''_{yx} = 0$

$z''_{yy} = 2$

$\Delta_2(A) = \begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} = -4 < 0$ NEMÁ EXTRÉM

A → sedlový bod

Pr: $z = 24 - x^2 - y^2 + xy + 36y$

der. 1. rádu

$z'_x = -2x + y$ $\to$ $-2x + y = 0$

$z'_y = -2y + x + 36$ $\qquad -2y + x + 36 = 0 \quad /\cdot 2$

$-2x + y = 0$

$2x - 4y = -72$ } +

$-3y = -72$

$y = 24$

$-2x + 24 = 0$

$-2x = -24$

$x = 12$

St. bod $A[12; 24]$

$\boxed{z''_{xx} = -2} < 0$ $\qquad \Delta_2(A) = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 4 - 1 = 3 > 0 \to$ JE EXTRÉM

$z''_{xy} = 1$

$z''_{yx} = 1$ $\qquad$ v A je L. maximum

$z''_{yy} = -2$

Pr: $z = x^3 + y^3 + 3xy + 2$

der. 1. rádu

$z'_x = 3x^2 + 3y$ $\qquad 3x^2 + 3y = 0$

$z'_y = 3y^2 + 3x$ $\qquad 3y^2 + 3x = 0$

$\Rightarrow$ $x^2 + y = 0 \to \boxed{y = -x^2}$

$y^2 + x = 0$

$-(-1)^2 = -1$

$x^4 + x = 0$

$x(x^3 + 1) = 0$

$x = 0 \quad x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

$A = [0; 0]$
$B = [-1; -1]$ } stac. body

	A	B
$z''_{xx} = 6x$	0	$-6 < 0$
$z''_{xy} = 3$	3	3
$z''_{yx} = 3$	3	3
$z''_{yy} = 6y$	0	-6

$\Delta_2(A) = \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} = 0 - 9 = -9 < 0$ NIE JE EXTRÉM

v A sedlový bod

$\Delta_2(B) = \begin{vmatrix} -6 & 3 \\ 3 & -6 \end{vmatrix} = 36 - 9 = 27 > 0$ JE EXTRÉM

v B má L. max

Pr: $z = 2x^3 + y^3 + 3x^2 - 3y - 12x + 1$

der. 1. rádu

$z'_x = 6x^2 + 6x - 12$ $\qquad 6x^2 + 6x - 12 = 0 \to x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$x = 1; -2$

$z'_y = 3y^2 - 3$ $\qquad 3y^2 - 3 = 0$

$y^2 = 1$

$y = \pm 1$

$A[1; 1] \quad B[1; -1] \quad C[-2; 1] \quad D[-2; -1]$

	A	B	C	D
$z''_{xx} = 12x + 6$	$18 > 0$	18	-18	$-18 < 0$
$z''_{xy} = 0$	0	0	0	0
$z''_{yx} = 0$	0	0	0	0
$z''_{yy} = 6y$	6	-6	6	-6

$\Delta_2(A) = \begin{vmatrix} 18 & 0 \\ 0 & 6 \end{vmatrix} = 18 \cdot 6 - 0 > 0$ v A L. min

$\Delta_2(B) = \begin{vmatrix} 18 & 0 \\ 0 & -6 \end{vmatrix} = 18 \cdot (-6) - 0 < 0$ v B sedlový bod

$\Delta_2(C) = \begin{vmatrix} -18 & 0 \\ 0 & 6 \end{vmatrix} = (-18) \cdot 6) - 0 < 0$ v C sedlový bod

$\Delta_2(D) = \begin{vmatrix} -18 & 0 \\ 0 & -6 \end{vmatrix} = 18 \cdot 6 - 0 > 0$ v D lok. max

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Pr: $z = y^3 + 3x^2y - 18x - 30y$

$z'_x = 6xy - 18$ $\qquad 6xy - 18 = 0 \to xy = 3 \to \boxed{x = \frac{3}{y}}$

$z'_y = 3y^2 - 30 + 3x^2$ $\qquad 3y^2 + 3x^2 - 30 = 0 \to x^2 + y^2 = 10$

$\frac{9}{y^2} + y^2 = 10 \quad /y^2$

$y^4 - 10y^2 + 9 = 0 \qquad y^2 = t$

$t^2 - 10t + 9 = 0$

$(t-9)(t-1) = 0$

$t = 9 \quad t = 1$

$y^2 = t$

$y^2 = 9 \qquad y^2 = 1$

$y = \pm 3 \qquad y = \pm 1$

$A[3; 1]$

$B[-3; -1]$

$C[-1; -3]$

$D[1; 3]$

	A	B	C	D
$z''_{xx} = 6y$	6	-6	$-18 < 0$	$18 > 0$
$z''_{xy} = 6x$	18	-18	-6	6
$z''_{yx} = 6x$	18	-18	-6	6
$z''_{yy} = 6y$	6	-6	-18	18

$\Delta_2(A) = \begin{vmatrix} 6 & 18 \\ 18 & 6 \end{vmatrix} = 36 - 18^2 < 0 \to$ sedlový bod

$\Delta_2(B) = \begin{vmatrix} -6 & -18 \\ -18 & -6 \end{vmatrix} = 36 - (-18)^2 < 0 \to$ sedlový bod

$\Delta_2(C) = \begin{vmatrix} -18 & -6 \\ -6 & -18 \end{vmatrix} = (-18)^2 - 36 > 0 \to$ v C lok. max

$\Delta_2(D) = \begin{vmatrix} 18 & 6 \\ 6 & 18 \end{vmatrix} = 18^2 - 36 > 0 \to$ v D lok. min