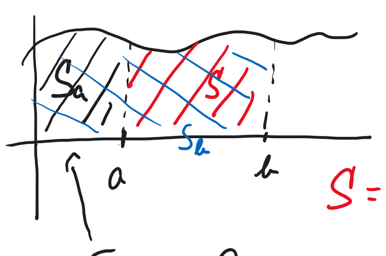
$$\int f(x)\,dx = F(x) + c$$

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$



$S = S_b - S_a$

$F(a) = S_a$

$F(b) = S_b$

Pr: $\int_1^2 (4x^3+3x^2+2)\,dx = \left[\frac{4x^4}{4} + 3\cdot\frac{x^3}{3} + 2x\right]_1^2 =$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$= (2^4+2^3+2\cdot 2) - (1^4+1^3+2\cdot 1) =$

$= (16+8+4) - (4) = \underline{\underline{24}}$

Pr: $\int_3^7 \frac{\frac{2}{2}x}{x^2-4}\,dx = \left[\frac{1}{2}\ln|x^2-4|\right]_3^7 =$

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\boxed{f' = (x^2-4)' = 2x}$

$= \frac{1}{2}\ln|7^2-4| - \frac{1}{2}\ln|3^2-4| =$

$= \frac{1}{2}(\ln 45 - \ln 5) = \frac{1}{2}\ln\frac{45}{5} = \ln\sqrt{9} = \underline{\underline{\ln 3}}$

$\ln a - \ln b = \ln\frac{a}{b}$

$r\ln a = \ln a^r$

Pr: $\int_0^{\frac{\pi}{2}} \sqrt{\sin x - \sin^3 x}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{\sin x(1-\sin^2 x)}\,dx =$

$= \int_0^{\frac{\pi}{2}} \sqrt{\sin x \cdot \cos^2 x}\,dx = \int_0^{\frac{\pi}{2}} \cos x\sqrt{\sin x}\,dx = \left| \begin{array}{ll} \boxed{\sin x = t} & \\ \cos x\,dx = dt & \\ x \to 0 & t \to 0 \\ x \to \frac{\pi}{2} & t \to 1 \end{array} \right|$

$= \int_0^1 \sqrt{t}\,dt = \int_0^1 t^{\frac{1}{2}}\,dt = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \left(\frac{2}{3}\sqrt{1^3}\right) - \left(\frac{2}{3}\sqrt{0^3}\right) = \underline{\underline{\frac{2}{3}}}$

Pr: $\int_0^7 \frac{x-1}{\sqrt[3]{x+1}}\,dx = \left| \begin{array}{ll} \boxed{x+1 = t^3} \to x = t^3 - 1 & \\ dx = 3t^2\,dt & \\ x \to 0 & \boxed{t \to 1} \\ x \to 7 & \boxed{t \to \sqrt[3]{8} = 2} \end{array} \right| =$

$\left\{ \begin{array}{l} x+1 = t^3 \\ 0+1 = t^3 \\ 1 = t^3 \end{array} \right.$

$= \int_1^2 \frac{t^3-1-1}{\cancel{t}} \cdot 3t^{\cancel{2}}\,dt = 3\int_1^2 (t^4 - 2t)\,dt = 3\left[\frac{t^5}{5} - \cancel{2}\cdot\frac{t^2}{\cancel{2}}\right]_1^2 =$

$3\left[\left(\frac{2^5}{5} - 2^2\right) - \left(\frac{1^5}{5} - 1^2\right)\right] = 3\left[\left(\frac{32}{5} - 4\right) - \left(\frac{1}{5} - 1\right)\right] = \underline{\underline{\frac{48}{5}}}$

Pr: $\int_0^1 x e^{1+x^2}\,dx = \left| \begin{array}{ll} \boxed{1+x^2 = t} & \\ 2x\,dx = dt & \\ x\,dx = \frac{dt}{2} & \\ x \to 0 & t \to 1 \\ x \to 1 & t \to 2 \end{array} \right| = \int_1^2 e^t \frac{1}{2}\,dt = \frac{1}{2}\left[e^t\right]_1^2 =$

$= \underline{\frac{1}{2}(e^2 - e)}$

$$\int_a^b u(x)\,v'(x)\,dx = \left[u(x)\,v(x)\right]_a^b - \int_a^b u'(x)\,v(x)\,dx$$

Pr: $\int_2^e x\ln x\,dx = \left| \begin{array}{ll} u = \ln x & v' = x \\ u' = \frac{1}{x} & v = \frac{x^2}{2} \end{array} \right| =$

$\left\{ \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2} = \frac{x}{2} \right.$

$= \left[\frac{x^2}{2}\ln x\right]_2^e - \int_2^e \frac{x}{2}\,dx = \left[\frac{x^2}{2}\ln x\right]_2^e - \left[\frac{1}{2}\frac{x^2}{2}\right]_2^e =$

$= \left(\frac{e^2 \overbrace{\ln e}^{=1}}{2} - \frac{2^2\ln 2}{2}\right) - \left(\frac{e^2}{4} - \frac{2^2}{4}\right) = \frac{e^2}{2} - \underbrace{2\ln 2}_{\ln 2^2} - \frac{e^2}{4} + 1 =$

$= \underline{\underline{\frac{e^2}{4} - \ln 4 + 1}}$

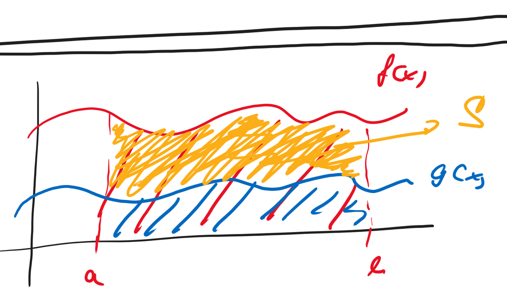
Pr: $\int_0^{\frac{\pi}{2}} x\sin x\,dx = \left| \begin{array}{ll} u = x & v' = \sin x \\ u' = 1 & v = -\cos x \end{array} \right| =$

$= \left[-x\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\,dx = \left[-x\cos x\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}} =$

$= \left(-\frac{\pi}{2}\cdot\underbrace{\cos\frac{\pi}{2}}_{0} - (-0\underbrace{\cos 0}_{1})\right) + \left(\underbrace{\sin\frac{\pi}{2}}_{1} - \underbrace{\sin 0}_{0}\right) = \underline{\underline{1}}$

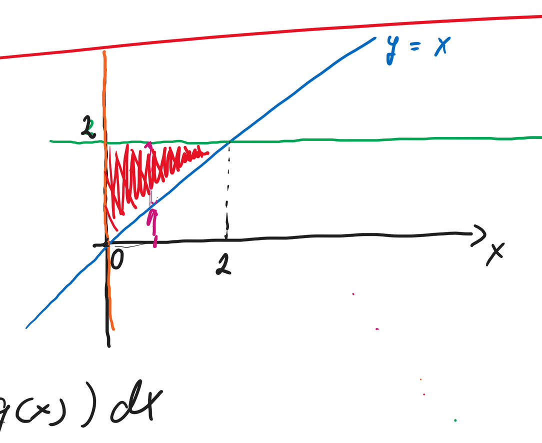
$S = \int_a^b (f(x) - g(x))\,dx$

$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$



$V = \pi\int_a^b (f^2(x) - g^2(x))\,dx$

Pr: $\boxed{y = x}$ $\boxed{y = 2}$ $\boxed{x = 0}$

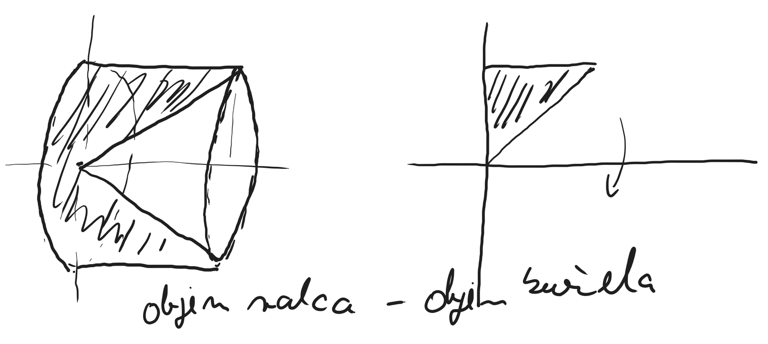


$\overset{a}{0} \le x \le \overset{b}{2}$

$\underset{g(x)}{x} \le y \le \underset{f(x)}{2}$

$S = \int_a^b (f(x) - g(x))\,dx$

$S = \int_0^2 (2 - x)\,dx = \left[2x - \frac{x^2}{2}\right]_0^2 = \left(2\cdot 2 - \frac{2^2}{2}\right) - \left(2\cdot 0 - \frac{0^2}{2}\right) = \underline{\underline{2}}$



objem valca - objem kužela

$V = \pi\int_a^b (f^2(x) - g^2(x))\,dx =$

$V = \pi\int_0^2 (2^2 - x^2)\,dx = \pi\left[4x - \frac{x^3}{3}\right]_0^2 = \pi\left[\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4\cdot 0 - \frac{0^3}{3}\right)\right] = \underline{\underline{\frac{16}{3}\pi}}$