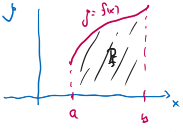
URČITÝ INTEGRÁL

označenie $\int_a^b f(x)\,dx$ → číslo

bob $\int f(x)\,dx = \underbrace{F(x)+c}_{PF}$, kde $F' = f$ ← súvis

Definícia: 1° formulácia problému
2° delenie $\langle a,b\rangle$
3° výberové body (na aproximáciu
4 aproximácia (zjednodušenie)
5 definícia (limitný prechod)

1° Nech $f(x)$ spoj. na $\langle a,b\rangle$; $f(x) \geq 0$



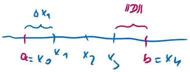
Plocha ?

zhora graf f(x)
zdola os x
zľava x = a
sprava x = b

P_f bude určitý integrál

2° $D = \{x_0, x_1, \dots, x_p\}$ je delenie $\langle a,b\rangle$ ak

$x_0 = a < x_1 < x_2 < \dots < x_p = b$



$\Delta x_i = x_i - x_{i-1}$

$\|D\| = \max_i \Delta x_i$
norma delenia

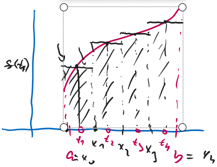
Máme $D_1 = D$ pridáme deliace body, dostaneme $\{D_n\}$

D_1 D_2

$\|D_n\| \to 0$ pre $n \to \infty$

$\langle x_{i-1}, x_i \rangle \longrightarrow$ bod

3° Výberové body $t_i \in \langle x_{i-1}, x_i \rangle$ vyberiem ľubovoľne

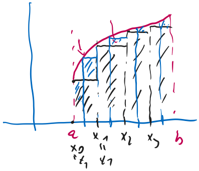


4 Aproximácia $P_f \doteq \sum_{i=1}^{p} f(t_i)\,\Delta x_i$

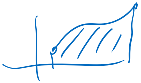
jedna strana

$t_i \doteq x_{i-1}$

$\sum f(t_i)\,\Delta x_i$



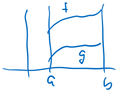
$$P_f = \lim_{n \to \infty} \underbrace{\sum_{i=1}^{n} f(t_i)\,\Delta x_i}_{\text{súčet obsahov obdĺž}} = \int_a^b f(x)\,dx$$



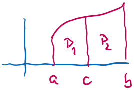
ZÁKL. VLASTNOSTI

V1. Ak $f(x) \geq 0$, tak $\int_a^b f(x)\,dx \geq 0$ (Plocha nemôže byť záporná.

V2 Ak $f(x) \geq g(x) > 0$, tak $\underset{P_f}{\int_a^b f(x)\,dx} \geq \underset{P_g}{\int_a^b g(x)\,dx}$



V3 $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$



V4 $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

V5 $\int_a^b \alpha f(x)\,dx = \alpha \int_a^b f(x)\,dx$

V6 $\int_a^b c_1 f(x) + c_2\, g(x)\,dx = c_1 \int_a^b f(x)\,dx + c_2 \int_a^b g(x)\,dx$

V7 $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\Delta x_i = x_i - x_{i-1}$

$\Delta x_i = x_{i-1} - x_i = \overset{\downarrow}{-}(x_i - x_{i-1})$

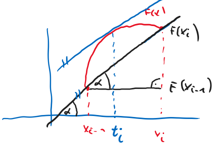
AKO PRAKTICKY VYPOČÍTAŤ

$\int_a^b f(x)\,dx$?

$\int f(x)\,dx$

NEWTON – LEIBNIZ

POUŽIŤ LAGRANGEOVA VETA $F(x)$ na $\langle x_{i-1}, x_i \rangle$ je P.F. k $f(x)$ $(F' = f)$



sečnica

$k_s = \operatorname{tg}\alpha = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}$

dotyčnica

$k_d = F'(t_i)$

Lag. veta

$\exists t_i \in \langle x_{i-1}, x_i \rangle$, taký, že

$F(x_i) - F(x_{i-1}) = F'(t_i)\,\Delta x_i$
$= f(t_i)\,\Delta x_i$

Z definície

$$\int_a^b f(x)\,dx \overset{def.}{=} \lim_{n\to\infty} \sum_{i=1}^{p_n} f(t_i)\Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{p_n} \left(F(x_i) - F(x_{i-1})\right)$$

a
$= x_0$
x_1
x_2
$b = x(p_n)$

$D_1 \; D_2 \; \dots \; D_n$

$x_0, x_1, \dots, x_{p_n}$ $\xrightarrow[n\to\infty]{}$

n = 1

n = 2

$(F(x_1) - \boxed{F(x_0)})$ $\overset{0}{}$

$F(x_2) - F(x_1)$

$F(x_3) - F(x_2)$

$\vdots$

$\boxed{F(x_{p_n})} - F(x_{p_n - 1})$

$= \lim_{n\to\infty}\left(F(b) - F(a)\right) = F(b) - F(a)$

Veta (N-L)

Nech f je spoj. na $\langle a, b \rangle$

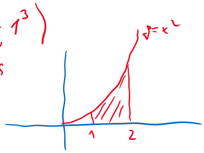
Nech F je PF k f na $\langle a, b \rangle$

Potom $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

na výpočet urč. int. treba nájsť PF
(t.j. neurč. integrál)

Pr. $\int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{1}{3}\left(2^3 - 1^3\right)$ $= P_S$

$\int x^2\,dx = \frac{x^3}{3} + \cancel{c}$



P_2

$\int_0^1 x e^{x^2}\,dx = \frac{1}{2} e^{x^2}\Big|_0^1 = \frac{1}{2}(e - 1)$

1° spôsob

$\frac{1}{2}\int 2x e^{x^2}\,dx = \left| \begin{matrix} x^2 = t & ? \\ 2x\,dx = dt & ! \end{matrix} \right| = \frac{1}{2}\int e^t\,dt = \frac{e^t}{2} + \cancel{c}$

$= \frac{1}{2} e^{x^2}$

SUBSTITÚCIA V URČITOM INTEG.

úvahy VIEM $\boxed{\int_a^b g(x)\,dx = G(x)\Big|_a^b}$ t.j. $G' = g$

TREBA P.F.

Nech F je PF k f t.j. $F' = f$

Potom, ale zlož. $F(\varphi(x))$ je PF k $f(\varphi(x)).\varphi'(x)$

(lebo $\left(F(\varphi(x))\right)' = F'(\varphi(x))\,\varphi'(x) = f(\varphi(x))\,\varphi'(x)$)

TO ale znamená, že môžeme rátať príslušný urč. int

$$\int_a^b \underbrace{f(\varphi(x))\varphi'(x)}\,dx \overset{N-L}{=} F(\varphi(x))\Big|_a^b = F(\varphi(b)) - F(\varphi(a)) = \underline{F}(t)\Big|_{\varphi(a)}^{\varphi(b)}$$

$$= \int_{\varphi(a)}^{\varphi(b)} f(t)\,dt$$

ODVODILI SME:

Nech $\varphi(x)$, $\varphi'(x)$ sú spoj na $\langle a, b\rangle$

Nech $f(t)$ je spoj na $\langle \varphi(a), \varphi(b)\rangle$ Potom

subst. schéma

$$\int_a^b f(\varphi(x))\boxed{\varphi'(x)\,dx} = \left| \begin{array}{ll} \varphi(x) = t & ? \\ \varphi'(x)\,dx = dt & ! \\ \text{ak } x = a \Rightarrow t = \varphi(a) & \\ x = b \Rightarrow t = \varphi(b) & \end{array} \right| = \int_{\varphi(a)}^{\varphi(b)} f(t)\,dt$$

ZMENA HRANÍC

Pr.

$$\frac{1}{2}\int_0^{\sqrt{\pi/2}} 2x \sin(x^2)\,dx = \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ x = 0 \Rightarrow t = 0 \\ x = \sqrt{\frac{\pi}{2}} \Rightarrow t = \frac{\pi}{2} \end{array} \right| = \frac{1}{2}\int_0^{\pi/2} \sin t\,dt = \frac{1}{2}(-\cos t)\Big|_0^{\pi/2} = \frac{1}{2}$$

Per Partes pre určitý integrál

TAHÁK $\int_a^b g(x)\,dx = G(x)\Big|_a^b$, $G' = g$

$$\left(u(x)\,v(x)\right)' = u'(x)\,v(x) + u(x)\,v'(x)$$

t.j. $u(x).v(x)$ je PF k $u'v + uv'$

t.j. $\int_a^b u'(x)v(x) + u(x)v'(x)\,dx = u(x)v(x)\Big|_a^b$

2 int.

t.j. $\boxed{\int_a^b u'(x)\,v(x)\,dx = u(x)\,v(x)\Big|_a^b - \int_a^b u(x)\,v'(x)\,dx}$

Nech $u'(x)$, $v'(x)$ sú spoj. na $\langle a, b\rangle$