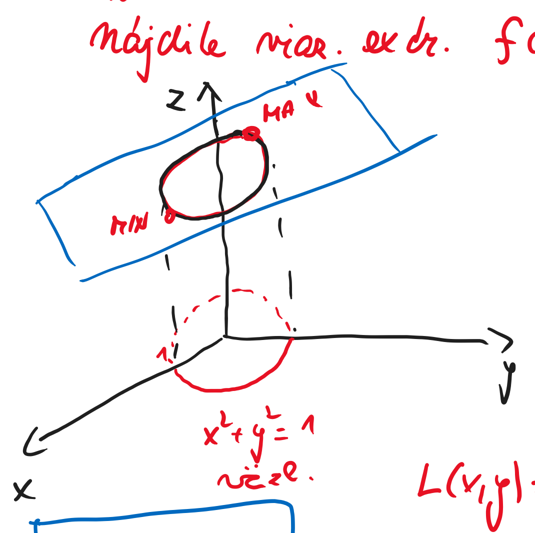
Pr.

Nájdite viaz. extr. $f(x,y) = x + y$, ak $x^2 + y^2 - 1 = 0$



$f(x,y) = x+y$ → rovina

$x^2+y^2-1=0$ → $y = \pm\sqrt{1-x^2}$ ↳ nevieme jednoznačne vyjadriť, t.j. $L(x,y)$

Hľadáme extrémy čiernej krivky

$L(x,y) = x + y + \lambda(x^2+y^2-1)$

1. St. body

$L'_x = 1 + 2\lambda x = 0$
$L'_y = 1 + 2\lambda y = 0$
(L'_λ) $x^2 + y^2 - 1 = 0$

1° vyjadrím λ: $\lambda = -\frac{1}{2x}$, $\lambda = -\frac{1}{2y}$

2° nájdem vzťah medzi x, y: $-\frac{1}{2x} = -\frac{1}{2y}$ ⇒ $\boxed{x = y}$

dosadím do väzby

Problém → sústava nelineár. rovníc

$x^2 + x^2 - 1 = 0$

$2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$ & $x = y$ t.j. má 2 SB

$\lambda = -\frac{1}{2x}$

$$\boxed{A_1 = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \quad \lambda_1 = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}}, \quad \boxed{A_2 = \left[-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right] \quad \lambda_2 = \frac{1}{\sqrt{2}}}$$

dopočítam príslušné λ (objaví sa v L''_{xx}...)

2. test $L''_{xx} = 2\lambda$, $L''_{yy} = 2\lambda$; $L''_{xy} = 0$

pre A_1

$\Delta_1 = -\frac{2}{\sqrt{2}} < 0$ $\quad \Delta_2 = \begin{vmatrix} -\frac{2}{\sqrt{2}} & 0 \\ 0 & -\frac{2}{\sqrt{2}} \end{vmatrix} > 0$ ⇒ v A je lok. max viazané lok. max

pre A_2

$\Delta_1 = \frac{2}{\sqrt{2}} > 0$ $\quad \Delta_2 = \begin{vmatrix} \frac{2}{\sqrt{2}} & 0 \\ 0 & \frac{2}{\sqrt{2}} \end{vmatrix} > 0$ ⇒ v A je lok. min viazané lok. min

POZOR UPRAVÍME ÚLOHU

viaz. ext. $f(x,y) = x + \boxed{y^2}$ ak $x^2 + y^2 - 1 = 0$

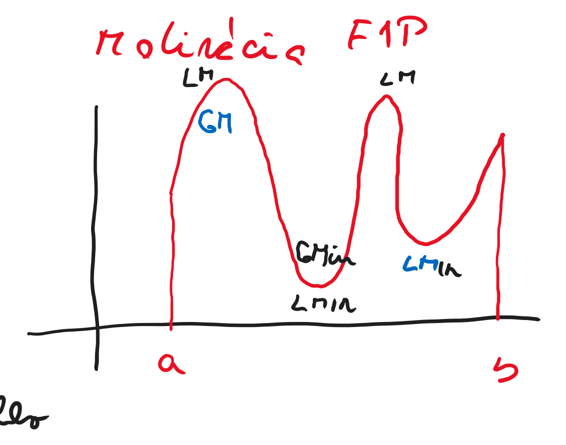
o.k. $y^2 = 1 - x^2$

$f(x,y) = x + (1 - x^2) = F(x)$

netreba $L(x,y)$

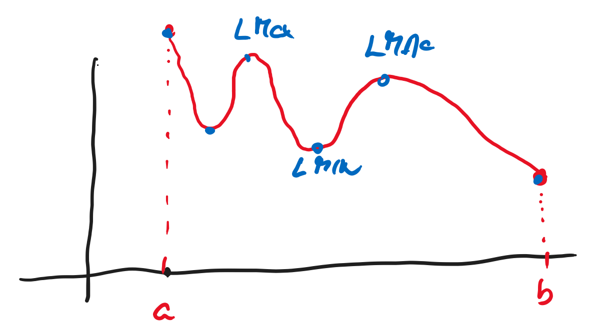
Aj pre $f(x,y)$ viem nájsť lok. extrémy, ako nájsť globálne extrémy

Motivácia F1P



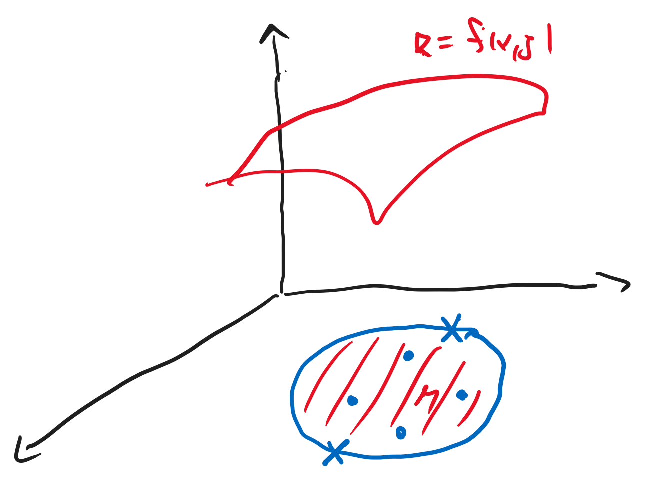
globálny extrém môže byť v lok. extréme

alebo



teraz glob. extrémy sú v a, b

Čo sú a, b ↳ hranica $\langle a, b\rangle$



Nech $f(x,y)$ je spoj ✓ na uz. a ohr. mn M

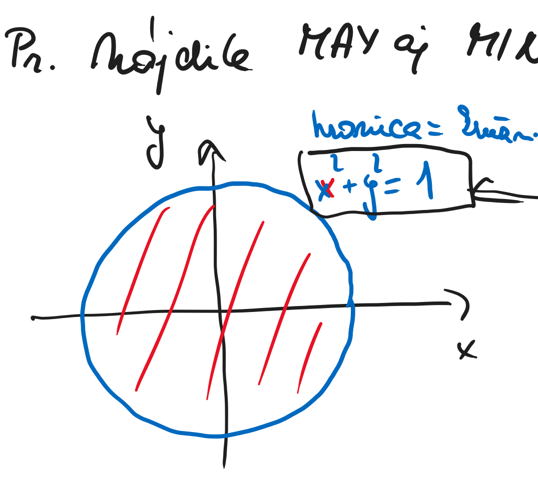
Postup na Gl. ext.

1. nájdem lok. extr. (stačí SB) ktoré patria do (vnútra) M
2. nájdem lok. extr. (stačí SB) na hranici M = viazané extr.
3. vyberiem $\underbrace{\text{najväčšiu}}_{G\ MAX}$ a $\underbrace{\text{najmenšiu hod}}_{G.\ MIN}$

Pr. Nájdite MAX aj MIN $f(x,y) = x+y$ na mn M: $x^2 + y^2 \le 1$

vnútro kruhu

hranica



1. lok. extrémy ∈ M

$f'_x = 1 \neq 0$ t.j. lok. extrémy nie sú

2. extr. na hranici, t.j.

To sme už riešili $f(x,y) = x+y$, ak $x^2+y^2 = 1$

v $A_1 = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ je lok. max = Glob. MAX ✓

v $A_2 = \left[-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$ je lo min = Glob. MIN

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viaz. ex

$f(x,y) = x^2 + y^2 - 12x + 16y$, ak $x^2 + y^2 = 25$

$L(x,y) = x^2 + y^2 - 12x + 16y + \lambda(x^2 + y^2 - 25)$

1. SB

$L'_x = 2x - 12 + 2\lambda x = 0 \iff x(1+\lambda) = 6$ $\quad \boxed{1+\lambda = \frac{6}{x}}$

$L'_y = 2y + 16 + 2\lambda y = 0 \iff y(1+\lambda) = -8$ $\quad 1+\lambda = -\frac{8}{y}$

$x^2 + y^2 - 25 = 0$

$3y = -4x$ $\quad \boxed{y = -\frac{4}{3}x}$

$L''_{xx} = 2 + 2\lambda = 2(1+\lambda)$?

$x^2 + \frac{16}{9}x^2 = 25$

$9x^2 + 16x^2 = 9.25$

$25x^2 = 9.25 \Rightarrow x = \pm 3$ & $y = -\frac{4}{3}x$

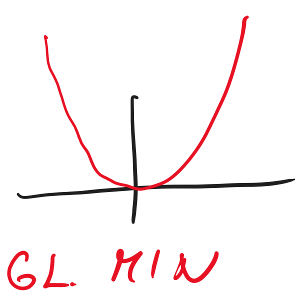
$A_1 = [3, -4]$ $\quad A_2 = [-3, 4]$

$1 + \lambda_1 = ◯$ $\quad 1 + \lambda_2 = □$

2. Test

$L''_{xx} = \ldots$

$f(x) = x^2$



GL. MIN

neex. GL. MAX