DR so špeciálnou PS.

Bolo

$$y'' + 2y' + 2y = \underbrace{x^2 + x + 1}_{f(x)}$$

$\varphi(x) = Ax^2 + Bx + C$ ✓

I. prípad

ak $f(x) = \boxed{e^{\alpha x} P_m(x)}$, tak $\varphi(x) = \boxed{e^{\alpha x} Q_m(x)}\, x^s$,

kde α je s-násobný koreň CH. R.

P1

Riešte DR

$$y'' - 5y' + 6y = e^{2x} \cdot 5$$

1 krok

$y'' - 5y' + 6y = 0$ HDR

CHR $\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 3)(\lambda - 2)$

$\lambda_1 = 3 \dots e^{3x}$

$\lambda_2 = 2 \dots e^{2x}$

všeob. $y = \underbrace{c_1 e^{3x} + c_2 e^{2x}}_{\text{všeob. rieš HDR}} + \underbrace{\varphi(x)}_{\text{1 rieš}}$

$\varphi(x) = \underline{A} e^{2x} x^1$ keďže 2 je 1-násob. koreň CHR

je rieš HDR

$\varphi' = 2A e^{2x} \cdot x + A e^{2x}$

$\varphi'' = 4A e^{2x} \cdot x + 2A e^{2x} + 2A e^{2x}$

Keď $y = \varphi$ sú dipol $\varphi(x) = Ae^{2x}$

$0 = e^{2x} 5$ sporná 1.

$y'' - 5y' + 6y = 5e^{2x}$

$$\underset{\sim}{e^{2x}} \left(4Ax + 4A - 5(2Ax + A) + 6Ax \right) = \underset{\sim}{e^{2x}} \cdot 5$$

$4A - 5A = 5$

$-A = 5 \Rightarrow A = -5$

$\varphi(x) = -5e^{2x} \cdot x$

II. prípad

Ak $f(x) = e^{\alpha x} \left[\underline{\underline{A}} \cos \beta x + \underline{\underline{B}} \sin \beta x \right]$, tak

$\varphi(x) = e^{\alpha x} \left[C \cos \beta x + D \sin \beta x \right] x^s$,

kde $\alpha + i\beta$ je s-násobný koreň CHR $\left[\begin{matrix} s=0 \\ s=1 \end{matrix} \right]$

$\alpha = 0,\ A = 78,\ B = 3,\ 3 = 0$

Riešte DR

$$y'' - 5y' + 6y = 78 \cos 3x + 0 \sin 3x$$

HDR $y = \underbrace{c_1 e^{3x} + c_2 e^{2x}}_{\text{všeob. HDR}} + \varphi(x)$

$\alpha + i\beta = 3i$

$\varphi = (C \cos 3x + D \sin 3x)$

$\varphi' = -3C \sin 3x + 3D \cos 3x$

$\varphi'' = -9C \cos 3x - 9D \sin 3x$

$-\underline{\underline{9C}} \cos 3x - 9D \sin 3x - \underline{\underline{5}}(-3C \sin 3x + \underline{\underline{3D}} \cos 3x) + \underline{6(C \cos 3x + D \sin 3x)}$

$= 78 \cos 3x + 0 \sin 3x$

$\cos 3x$: $-9C - 15D + 6C = 78$

$-9D + 15C + 6D = 0$

$-3C - 15D = 78$ /·(−5)

$15C - 3D = 0 \Rightarrow D = 5C$

$-3C - 75C = 78$ $\quad C = -1$