Derivácia

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$c' = 0$

$(x^n)' = n \cdot x^{n-1}$

$(c \cdot f(x))' = c \cdot f'(x)$

$(f(x) + g(x))' = f'(x) + g'(x)$

1) $f(x) = 10$
$f'(x) = 0$

2) $f(x) = 10 \cdot x^3$
$f'(x) = 10 \cdot 3 \cdot x^{3-1} = 30 \cdot x^2$

3) $f(x) = \sqrt[3]{x} + 4x + 8 =$
$= x^{\frac{1}{3}} + 4x^1 + 8$

$f'(x) = \frac{1}{3} x^{\frac{1}{3}-1} + 4 \cdot 1 \cdot x^{1-1} + 0$
$= \frac{1}{3} x^{-\frac{2}{3}} + 4 \; x^{0} = 1 =$
$= \frac{1}{3} \frac{1}{\sqrt[3]{x^2}} + 4$

4) $f(x) = \frac{2}{x^4} + \frac{1}{\sqrt[3]{x}} =$
$= 2 \cdot x^{-4} + x^{-\frac{1}{3}}$
$f'(x) = 2 \cdot (-4) \cdot x^{-4-1} + \left(-\frac{1}{3}\right) \cdot x^{-\frac{1}{3}-1} =$
$= -8 \cdot x^{-5} - \frac{1}{3} x^{-\frac{4}{3}} =$
$= \frac{-8}{x^5} - \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^4}}$

$(x \cdot y)^a = x^a \cdot y^a$
$(x^a)^b = x^{a \cdot b}$
$x^a \cdot x^b = x^{a+b}$

5) $f(x) = \sqrt{x\sqrt{x}} = \left(x \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}$
$= x^{\frac{1}{2}} \cdot \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{2} \cdot \frac{1}{2}} =$ $\frac{1}{4}$
$= x^{\frac{1}{2}+\frac{1}{4}} = x^{\frac{3}{4}}$
$f'(x) = \frac{3}{4} x^{\frac{3}{4}-1} = \frac{3}{4} x^{-\frac{1}{4}} = \frac{3}{4\sqrt[4]{x}}$

$(a^x)' = a^x \ln a$
$(e^x)' = e^x$

6) $f(x) = 2 \cdot 4^x - 3e^x$
$f'(x) = 2 \cdot 4^x \cdot \ln 4 - 3e^x$

$(\log_a x)' = \frac{1}{x \cdot \ln a}$
$(\ln x)' = \frac{1}{x}$

7) $f(x) = \log_2 x - 5 \ln x$
$\frac{1}{} - \frac{5}{}$

7) $f(x) = \ldots$

$$f'(x) = \frac{1}{x \ln 2} - \frac{5}{x}$$

$$(\ln x)' = \frac{1}{x}$$

$$\big(f(x)\cdot g(x)\big)' = f'(x)g(x) + f(x)\cdot g'(x)$$

8) $$f(x) = \underbrace{(x^2-4)}_{f} \cdot \underbrace{\sin x}_{g}$$

$$f'(x) = 2x \cdot \sin x + (x^2-4)\cdot \cos x$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x) g'(x)}{g^2(x)}$$

9) $$f(x) = \operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{\cos x \cdot \cos x - \sin x\,(-\sin x)}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\cos^2 x + \sin^2 x = 1$$

10) $$f(x) = \sqrt{9-x^2} = (\underbrace{9-x^2}_{g(x)})^{\frac{1}{2}} \quad - f(x)$$

$$\big(f(g(x))\big)' = \underbrace{f'(g(x))}\cdot g'(x)$$

$$f'(x) = \frac{1}{2}\left(9-x^2\right)^{\frac{1}{2}-1 = -\frac{1}{2}} \cdot (9-x^2)'$$

$$= \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) = \frac{1}{2\sqrt{9-x^2}} \cdot (-2x)$$

$$= -\frac{x}{\sqrt{9-x^2}}$$

11) $$f(x) = \underset{\uparrow f}{e}^{\,9-x^2 \; g(x)}$$

$$f'(x) = e^{9-x^2} \cdot (9-x^2)'$$

$$= e^{9-x^2}(-2x) = -2x e^{9-x^2}$$

12) $$f(x) = \underset{\uparrow f}{\sin}(\overbrace{9-x^2}^{g(x)})$$

$$f'(x) = \cos(9-x^2) \cdot (9-x^2)' =$$

$$= -2x \cos(9-x^2)$$

$= -2x\cos(9-x^2)$

13) $f(x) = \sqrt{\sin(9-x^2)} = (\underbrace{\sin(9-x^2)}_{g})^{\frac{1}{2}}$ f

$f'(x) = \frac{1}{2}\left(\sin(9-x^2)\right)^{-\frac{1}{2}} \cdot \left(\sin(9-x^2)\right)' =$

$= \frac{1}{2\sqrt{\sin(9-x^2)}} \cdot \left(-2x\cos(9-x^2)\right)$

14) $f(x) = x^3 + 3^x + x^x$ 2.

$(x^3)' = 3 \cdot x^2$ $\quad (x^n)'$

$(3^x)' = 3^x \ln 3$ $\quad (a^x)'$

LOGARITMICKÁ DERIVÁCIA

$f \circ f^{-1} = id$

$\log_a x^n = n \log_a x$

$x^x = e^{\ln x^x} = e^{x \cdot \ln x}$

$(x^x)' = (e^{x\ln x})' = e^{x \cdot \ln x} \cdot [x \cdot \ln x]' =$

$= e^{x\ln x}\left[1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}}\right] =$

$= e^{x\ln x}(\ln x + 1) = x^x(\ln x + 1)$

$f(x) = x^{\sin x} = e^{\ln x^{\sin x}} = e^{\sin x \cdot \ln x}$

$\vdots$