Limita a spojitosť

streda 30. októbra 2024 7:30

$\lim\limits_{x \to 1=a} \sqrt{x-2}$

$D(f): x-2 \geq 0 \iff x \geq 2$

$D(f) = \langle 2, \infty)$

$O_{\frac{1}{10}}(1) = (1-\frac{1}{10}, 1+\frac{1}{10}) = (0{,}9; 1{,}1)$

OKOLIE BODU 1 O POLOMERE $\frac{1}{10}$

V TOMTO OKOLÍ BODU 1 NELEŽÍ ŽIADEN BOD z $D(f)$ $\Rightarrow a=1$ NIE JE HROMADNÝ BOD $D(f)$

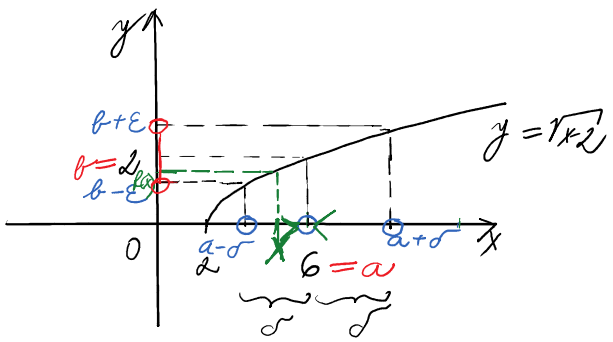
$\Rightarrow \lim\limits_{x \to 1} \sqrt{x-2} \quad \nexists$

$\mathring{O}_{\frac{1}{10}}(1) = (0{,}9; 1{,}1) - \{1\} = (0{,}9; 1) \cup (1; 1{,}1)$

PRSTENCOVÉ OKOLIE BODU 1

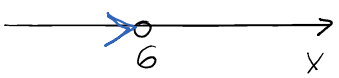
DEFINÍCIA LIMITY

$$\lim_{x \to a} f(x) = b$$



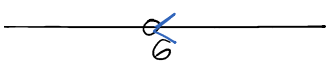
$\lim\limits_{x \to 6=a} \sqrt{x-2} = 2 \neq b$

$\lim\limits_{x \to 6^-} \sqrt{x-2} = 2$



LIMITA ZĽAVA

$\lim\limits_{x \to 6^+} \sqrt{x-2} = 2$



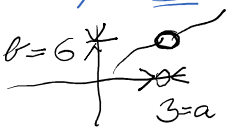
LIMITA SPRAVA

$\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) \Rightarrow \lim\limits_{x \to a} f(x) \quad \nexists$

TYPY NEURČITÝCH VÝRAZOV

$\frac{0}{0}, \pm\frac{\infty}{\infty}, 0.\infty, 1^\infty, 0^0$

PR1 $\lim_{x\to 3} \frac{x^2-9}{x^2-5x+6} \overset{\frac{0}{0}}{=} \lim_{x\to 3} \frac{(x-3)(x+3)}{(x-3)(x-2)} = \frac{6}{1} = 6$



PR2 $\lim_{x\to 0} \frac{\sin x \cdot \operatorname{tg} x}{1-\cos^2 x} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\sin x \cdot \frac{\sin x}{\cos x}}{\sin^2 x} =$

$= \lim_{x\to 0} \frac{\sin^2 x}{\cos x \sin^2 x} = 1$

$\cos^2 x + \sin^2 x = 1$

$1 - \cos^2 x = \sin^2 x$

PR3 $\lim_{x\to 0} \frac{\sin 7x}{\sin 5x} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\sin 7x \cdot \frac{1}{x}}{\sin 5x \cdot \frac{1}{x}} =$

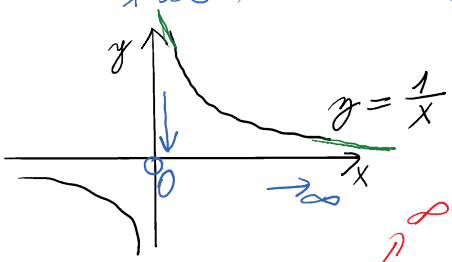
$= \lim_{x\to 0} \frac{\frac{\sin 7x}{x} \cdot \frac{7}{7}}{\frac{\sin 5x}{x} \cdot \frac{5}{5}} = \lim_{x\to 0} \frac{7 \cdot \frac{\sin 7x}{7x}}{5 \cdot \frac{\sin 5x}{5x}} \to 1 = \frac{7}{5}$

$\lim_{t\to 0} \frac{\sin t}{t} = 1$ $\quad \lim_{x\to 0} \frac{\sin x}{x} = 1$

PR4 $\lim_{x\to\infty} \frac{4x^2-3x+2}{3x^2+2x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{x^2\left(4-\frac{3}{x}+\frac{2}{x^2}\right)}{x^2\left(3+\frac{2}{x}\right)} = \frac{4}{3}$

$\lim_{x\to\infty} x = \infty$

$\lim_{x\to\infty} \frac{1}{x} = 0$!

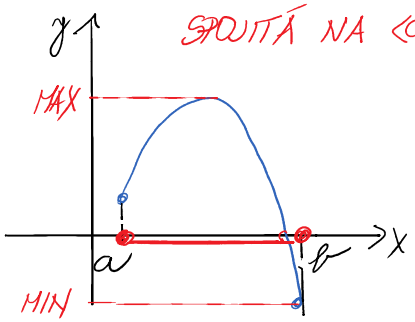


PR 4 $\lim_{x\to\infty} \frac{4x^3-3x+2}{3x^2+2x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{x^3\left(4-\frac{3}{x^2}+\frac{2}{x^3}\right)}{x^2\left(3+\frac{2}{x}\right)} = \frac{\infty}{3} = \infty$

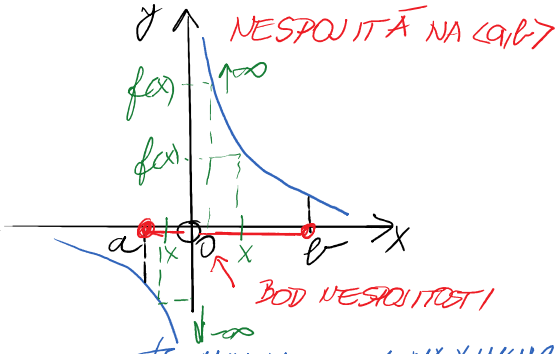
PR5 $\lim_{x\to\infty} \frac{4x^2-3x+2}{3x^3+2x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{x^3\left(\frac{4}{x}-\frac{3}{x^2}+\frac{2}{x^3}\right)}{x^3\left(3+\frac{2}{x^2}\right)} = \frac{0}{3} = 0$

VLASTNOSTI SPOJITÝCH FUNKCIÍ NA UZAVRETOM INTERVALE

VETA 1:

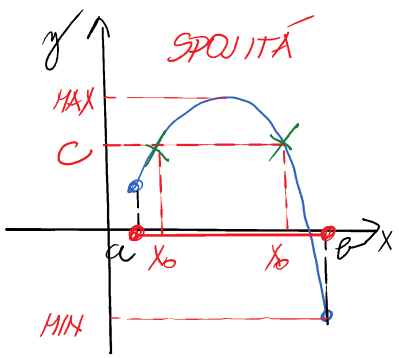


∃ MINIMUM A MAXIMUM

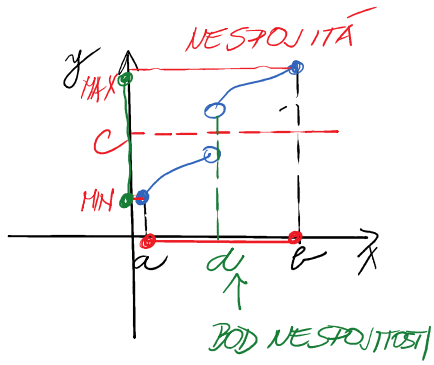


∄ MINIMUM A MAXIMUM

VETA 2: $\forall c \in \langle MIN, MAX \rangle \quad \exists x_0 \in \langle a,b \rangle : f(x_0) = c$

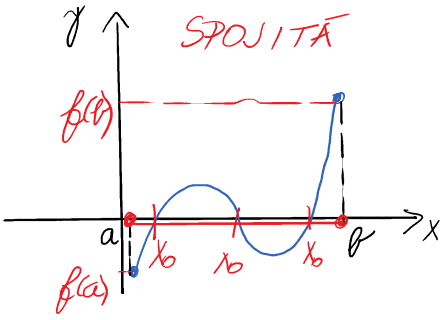


$\exists x_0 \in \langle a,b \rangle : f(x_0) = c$



$\nexists x_0 \in \langle a,b \rangle : f(x_0) = c$

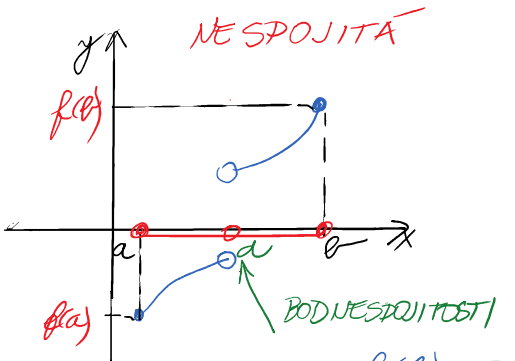
VETA 3 $f(a) \cdot f(b) < 0 \Rightarrow \exists x_0 \in \langle a,b \rangle : f(x_0) = 0$



$f(a) < 0 \quad f(b) > 0$

$f(a) \cdot f(b) < 0$

$\exists x_0 : f(x_0) = 0$



$f(a) < 0 \quad f(b) > 0$

$f(a) \cdot f(b) < 0$

$\nexists x_0 : f(x_0) = 0$