

PR:  
 a)  $x$  ... POČET KUSOV MODELU A  
 $y$  ... POČET KUSOV MODELU B  
 $z$  ... POČET KUSOV MODELU C

$40x$  ... MINÚT PRE  $x$  KUSOV MODELU A NA LINKE 1

$30y$  ... MINÚT PRE  $y$  KUSOV MODELU B NA LINKE 1

$25z$  ... MINÚT PRE  $z$  KUSOV MODELU C NA LINKE 1

$$40x + 30y + 25z = 4500$$

$$25x + 20y + 20z = 3050$$

$$10x + 10y + 5z = 1200$$

$$A^* = \left( \begin{array}{ccc|c} 40 & 30 & 25 & 4500 \\ 25 & 20 & 20 & 3050 \\ 10 & 10 & 5 & 1200 \end{array} \right) \cdot \frac{1}{5} \approx \left( \begin{array}{ccc|c} 8 & 6 & 5 & 900 \\ 5 & 4 & 4 & 610 \\ 2 & 2 & 1 & 240 \end{array} \right) \xrightarrow{-2R_3} \approx$$

$$\approx \left( \begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 8 & 6 & 5 & 900 \\ 2 & 2 & 1 & 240 \end{array} \right) \xrightarrow{-8R_1, -2R_1} \approx \left( \begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 6 & -11 & -140 \\ 0 & 2 & -3 & -20 \end{array} \right) \xrightarrow{-2R_2} \approx$$

$$\approx \left( \begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 2 & -3 & -20 \\ 0 & 6 & -11 & -140 \end{array} \right) \xrightarrow{-3R_2} \approx \left( \begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 2 & -3 & -20 \\ 0 & 0 & -2 & -80 \end{array} \right) \cdot \left(-\frac{1}{2}\right)$$

$R(A^*) = 3 = R(A) \Rightarrow \exists$  RIEŠENIE

$R(A^*) = n \Rightarrow \exists$  PRAVE JEDNO RIEŠENIE

$$\approx \left( \begin{array}{ccc|c} 1 & 0 & 2 & 130 \\ 0 & 2 & -3 & -20 \\ 0 & 0 & 1 & 40 \end{array} \right) \xrightarrow{-2R_3, +3R_3} \approx \left( \begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 2 & 0 & 100 \\ 0 & 0 & 1 & 40 \end{array} \right) \cdot \frac{1}{2} \approx$$

JORDANOVA ELIMINÁČIA

$$\approx \left( \begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 40 \end{array} \right) \begin{array}{l} x = 50 \\ y = 50 \\ z = 40 \end{array} \quad \bar{x} = (50, 50, 40)^T$$

JE TREBA VYROBIŤ :  
 50 KS MODELU A  
 50 KS MODELU B  
 40 KS MODELU C

INÝ SPÔSOB :  $x + 2z = 130$   
 $2y - 3z = -20$   
 $z = 40$   
 $2y - 3 \cdot 40 = -20 \Rightarrow 2y = -20 + 120 = 100 \Rightarrow y = 50$   
 $x + 2 \cdot 40 = 130 \Rightarrow x = 130 - 80 = 50$   
 $x = 50$

a)  $40x + 30y + 25z = 4500$   
 $25x + 20y + 20z = 3050$

$$A^* = \left( \begin{array}{ccc|c} 40 & 30 & 25 & 4500 \\ 25 & 20 & 20 & 3050 \end{array} \right) \cdot \frac{1}{5} \approx \left( \begin{array}{ccc|c} 8 & 6 & 5 & 900 \\ 5 & 4 & 4 & 610 \end{array} \right) \cdot 8 - 5R_1 \approx$$

$$\approx \left( \begin{array}{ccc|c} 8 & 6 & 5 & 900 \\ 0 & 2 & 7 & 380 \end{array} \right)$$

$R(A^*) = R(A) = 2 \Rightarrow \exists$  RIEŠENIE

$n = 3 > R(A) \Rightarrow \exists \infty$  VEĽA RIEŠENÍ

$n - R(A) = 3 - 2 = 1$  VOLNÁ PREMENNÁ

$$\left| \begin{array}{l} 8x + 6y + 5z = 900 \\ 2y + 7z = 380 \end{array} \right. \quad \left. \begin{array}{l} z = t \\ 2y + 7t = 380 \end{array} \right. \quad | -7t$$

$$\begin{cases} 8x + 6y + 5z = 900 \\ 2y + 7z = 380 \end{cases}$$

$$\begin{aligned} z &= t \\ 2y + 7t &= 380 & | -7t \\ 2y &= 380 - 7t & | \cdot \frac{1}{2} \\ y &= \frac{380 - 7t}{2} \end{aligned}$$

$$y = 190 - \frac{7}{2}t$$

$$\begin{aligned} 8x + 6 \cdot (190 - \frac{7}{2}t) + 5t &= 900 \\ 8x &= 900 - 6(190 - \frac{7}{2}t) - 5t & | \cdot \frac{1}{8} \\ x &= \frac{1}{8}[-240 + 16t] \end{aligned}$$

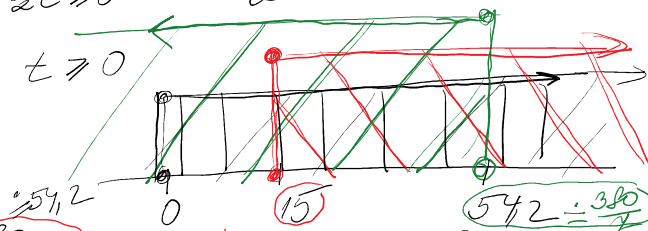
$$x = -30 + 2t$$

$$\vec{x} = (-30 + 2t, 190 - \frac{7}{2}t, t)^T \quad t \in \mathbb{R}$$

$$t=1: \quad x = -30 + 2 \cdot 1 = -28; \quad y = 190 - \frac{7}{2} \cdot 1 = \frac{373}{2} \quad \text{NEVYHODNÉ}$$

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{cases}$$

$$\begin{aligned} -30 + 2t \geq 0 &\Leftrightarrow 2t \geq 30 \Leftrightarrow t \geq 15 & t \in \langle 15, \infty \rangle \\ 190 - \frac{7}{2}t \geq 0 &\Leftrightarrow \frac{7}{2}t \leq 190 \Leftrightarrow t \leq \frac{380}{7} = 54\bar{2} & t \in \langle -\infty, \frac{380}{7} \rangle \\ t \geq 0 & & t \in \langle 0, \infty \rangle \end{aligned}$$



$$t \in \langle 15, \infty \rangle \cap \langle -\infty, \frac{380}{7} \rangle \cap \langle 0, \infty \rangle$$

$$\begin{aligned} t \in \langle 15, \frac{380}{7} \rangle \cap \{2k; k \in \mathbb{Z}\} &= \{16, 18, 20, \dots, 54\} \\ & \rightarrow \{\dots, -4, -2, 0, 2, 4, 6, \dots\} \end{aligned}$$

$$\vec{x} = (-30 + 2t, 190 - \frac{7}{2}t, t)^T \quad t \in \{16, 18, 20, \dots, 54\}$$

20 RIEŠENÍ

PR21

- a)  $x$  ... POČET KUSOV HODINIEK MODELU A  
 $y$  ... POČET KUSOV HODINIEK MODELU B  
 $z$  ... POČET KUSOV HODINIEK MODELU C

$$\begin{cases} 10x + 8y + 12z = 620 \\ 15x + 6y + 16z = 750 \\ 5x + 2y + 8z = 330 \end{cases}$$

$$\begin{aligned} A^* &= \left( \begin{array}{ccc|c} 10 & 8 & 12 & 620 \\ 15 & 6 & 16 & 750 \\ 5 & 2 & 8 & 330 \end{array} \right) \leftarrow \begin{array}{l} \textcircled{5} \cdot 2 \cdot 8 \quad | \quad 330 \\ \textcircled{15} \cdot 6 \cdot 16 \quad | \quad 750 \\ \textcircled{10} \cdot 8 \cdot 12 \quad | \quad 620 \end{array} \begin{array}{l} -3R_1 \\ -2R_1 \end{array} \approx \\ \approx \left( \begin{array}{ccc|c} 5 & 2 & 8 & 330 \\ 0 & 0 & -8 & -240 \\ 0 & 4 & -4 & -40 \end{array} \right) \cdot \begin{array}{l} (\frac{1}{8}) \\ (\frac{1}{4}) \end{array} \leftarrow \approx \left( \begin{array}{ccc|c} \textcircled{5} \cdot 2 & \textcircled{8} & \textcircled{8} & 330 \\ 0 & \textcircled{1} & -1 & -10 \\ 0 & 0 & \textcircled{1} & +30 \end{array} \right) \begin{array}{l} -8R_3 \quad (-2R_2) \\ +2R_3 \end{array} \text{ JORDAN} \end{aligned}$$

$\text{rk}(A) = \text{rk}(A) = 3 = n \Rightarrow \exists$  práve 1 riešenie

$$\begin{aligned} 5x + 2y + 8z &= 330 \\ y - z &= -10 & y - 30 &= -10 \\ & & z &= 30 & y &= 20 \end{aligned}$$

$$\begin{aligned} 5x + 2 \cdot 20 + 8 \cdot 30 &= 330 \\ 5x + 40 + 240 &= 330 \end{aligned}$$

$$5x = 50$$

$$x = 10$$

$$\bar{x} = (10, 20, 30)^T$$

JE TREBA VYROBIT' 10 KUSOV MODELU A  
20 KUSOV MODELU B  
30 KUSOV MODELU C

$$b) \begin{cases} 10x + 8y + 12z = 620 \\ 15x + 6y + 16z = 750 \end{cases}$$

$$A^* = \left( \begin{array}{ccc|c} 10 & 8 & 12 & 620 \\ 15 & 6 & 16 & 750 \end{array} \right) \cdot \frac{1}{2} \approx \left( \begin{array}{ccc|c} 5 & 4 & 6 & 310 \\ 15 & 6 & 16 & 750 \end{array} \right) -3R_1 \approx$$

$$\approx \left( \begin{array}{ccc|c} 5 & 4 & 6 & 310 \\ 0 & -6 & -2 & -180 \end{array} \right) \cdot \left(-\frac{1}{2}\right) \approx \left( \begin{array}{ccc|c} 5 & 4 & 6 & 310 \\ 0 & 3 & 1 & 90 \end{array} \right)$$

$\text{r}(A^*) = \text{r}(A) = 2 \Rightarrow$  J RIESENIE  
 $n = 3 > \text{r}(A) \Rightarrow$  J  $\infty$  VEĽA RIESENÍ  
 $n - \text{r}(A) = 3 - 2 = 1$  VOĽNÁ PREMENNÁ

$$\begin{cases} 5x + 4y + 6z = 310 \\ 3y + z = 90 \end{cases} \quad \begin{cases} y = t \\ z = 90 - 3y = 90 - 3t \end{cases}$$

$$5x + 4t + 6(90 - 3t) = 310$$

$$5x = 310 - 4t - 6(90 - 3t) = 310 - 4t - 540 + 18t =$$

$$= -230 + 14t \quad | \cdot \frac{1}{5}$$

$$x = \frac{-230 + 14t}{5} = -46 + \frac{14}{5}t$$

$$\bar{x} = \left( -46 + \frac{14}{5}t, t, 90 - 3t \right)^T \quad t \in \mathbb{R}$$

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{cases}$$

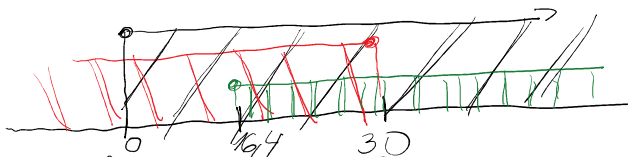
$$\begin{cases} -46 + \frac{14}{5}t \geq 0 \\ t \geq 0 \end{cases}$$

$$90 - 3t \geq 0$$

$$\frac{14}{5}t \geq 46 \quad | \cdot \frac{5}{14} \quad t \geq 16,4 \approx \frac{46 \cdot 5}{14}$$

$$3t \leq 90 \quad t \leq 30$$

$$\begin{cases} t \in \left( \frac{165}{14}, \infty \right) \\ t \in (0, \infty) \\ t \in (-\infty, 30] \end{cases}$$



$$t \in \langle 16,4; 30 \rangle \cap \{5k, k \in \mathbb{Z}\} =$$

CELOČÍSELNÉ NÁSOBKY 5

$$t \in \{20, 25, 30\} \quad \text{3 RIEŠENIA: } \{ \dots, -10, -5, 0, 5, 10, 15, \dots \}$$

$$\begin{aligned} t_1 = 20 \quad \bar{x} &= \left( -46 + \frac{14}{5} \cdot 20; 20, 90 - 3 \cdot 20 \right)^T = (10, 20, 30)^T \\ t_2 = 25 \quad \bar{x} &= \left( -46 + \frac{14}{5} \cdot 25; 25, 90 - 3 \cdot 25 \right)^T = (24, 25, 15)^T \\ t_3 = 30 \quad \bar{x} &= \left( -46 + \frac{14}{5} \cdot 30; 30, 90 - 3 \cdot 30 \right)^T = (38, 30, 0)^T \end{aligned}$$