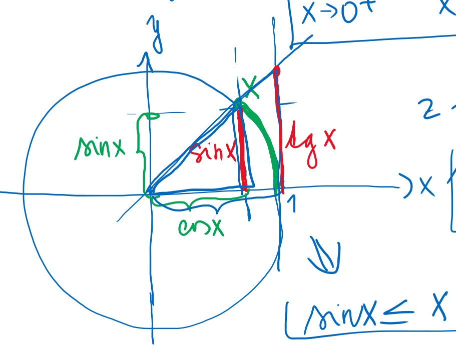
DOBRÉ RÁNO :)

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$\left(\frac{0}{0}\right)$ je to neurčitý výraz

uvediem dôkaz: pre $x \in (0, \frac{\pi}{2})$

dokážeme $\lim_{x\to 0^+} \frac{\sin x}{x} = 1$ (limita zľava podobne).



2 podobnosti △ ⇒ $\frac{\sin x}{\cos x} = \frac{\operatorname{tg} x}{1}$

⇓

$\sin x \le x \le \operatorname{tg} x$,

$\sin x \le x \le \frac{\sin x}{\cos x} \quad \Big/ \cdot \frac{1}{\sin x} \ne 0$

$1 \le \frac{x}{\sin x} \le \frac{1}{\cos x}$

$\cos x \le \frac{\sin x}{x} \le 1$ pre $x \in (0, \frac{\pi}{2})$

$x \to 0^+$: $\cos x \to 1$, $1 \to 1$

na základe vety o policaj.

$$\lim_{x\to 0^+} \frac{\sin x}{x} = 1$$

ĎALŠÍ VZŤAH:

$$\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^{x} = (1^\infty) = e$$

$x \to \infty$, $\frac{1}{\infty} \to 0$, $1+0=1$

(1^∞) neurčitý výraz; e (Eulerova konštanta)

zovšeobecnené

$$\lim_{x\to\infty} \left(1 + \frac{k}{f(x)}\right)^{f(x)} = e^k$$

pre $f(x) \to \infty$ ak $x \to \infty$

príklad

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\cos x}{x^2} \cdot \frac{1+\cos x}{1+\cos x}$$

neurči. výraz; $\frac{1+\cos x}{1+\cos x} = 1$; $(\sin^2 x + \cos^2 x = 1$, $1 - \cos^2 x = \sin^2 x)$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} =$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{1}{1+\cos x}\right) = \frac{1}{2};$$

$\downarrow 1 \quad \downarrow 1 \quad \downarrow \frac{1}{2}$

pr.

$$\lim_{x\to\infty} \left(\frac{3x+1}{3x+4}\right)^{3x+4} = (1^\infty) = \lim_{x\to\infty} \left(\frac{3x+4-3}{3x+4}\right)^{3x+4} =$$

$3x+4 \to \infty$; $\frac{3}{3} = 1$

$$= \lim_{x\to\infty} \left(1 + \frac{-3}{3x+4}\right)^{3x+4} = e^{-3}$$

$$\lim_{x\to\infty} \left(1 + \frac{k}{f(x)}\right)^{f(x)} = e^k$$

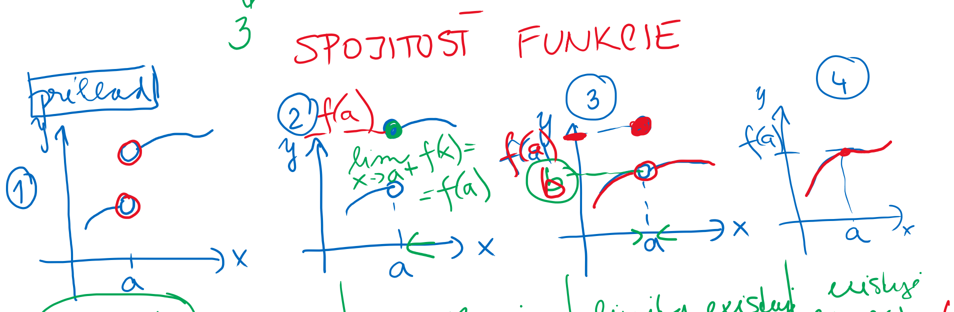
pr.

$$\lim_{x\to\infty} \left(\frac{3x}{x+1}\right)^{4x} = 3^\infty = \infty$$

$4x \to \infty$; $\frac{3x}{x+1} \to 3$

SPOJITOSŤ FUNKCIE

príklad



① $\lim_{x\to a} f(x)$ neexistuje, $f(a)$ neexistuje

② neexistuje limita $x\to a$, $f(a)$ existuje

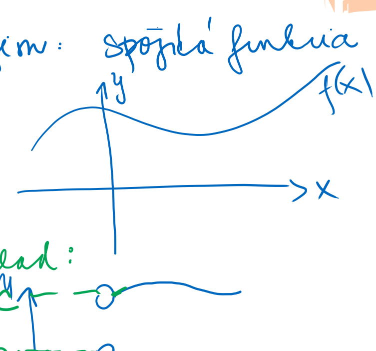
③ limita existuje, $\lim_{x\to a} f(x) = b$, $f(a)$ existuje, $\lim_{x\to a} f(x) \ne f(a)$

④ existuje $\lim_{x\to a} f(x) = f(a)$

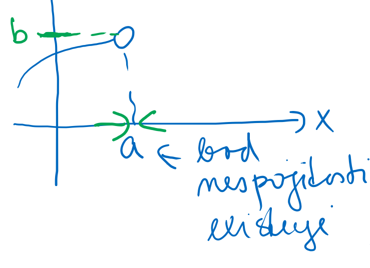
spojitosť funkcie $f(x)$ v bode a : $f(x)$ musí byť def. na $O(a)$ (teda aj v bode)

$$\lim_{x\to a} f(x) = f(a)$$

Pojem: spojitá funkcia (spojitá $\forall x \in D(f)$)



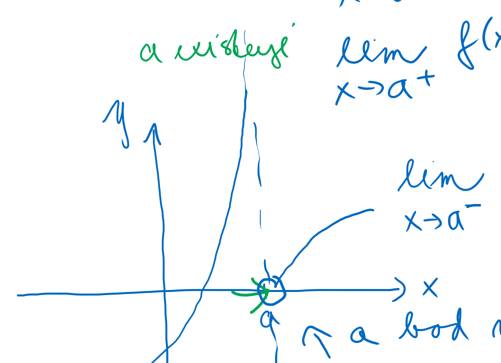
príklad:



a ← bod nespojitosti 1. druhu, lebo existuje $\lim_{x\to a^-} f(x) = b$

a existuje $\lim_{x\to a^+} f(x) = c$

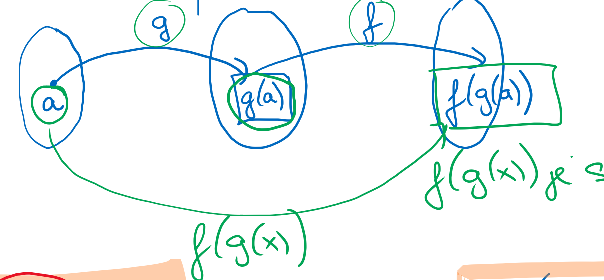
pr.



$\lim_{x\to a^-} f(x) = \infty$

↑ a bod nespojitosti 2. druhu

Spojitosť zloženej funkcie



$f(g(x))$ je spojitá v a

$$\lim_{x\to a} f(g(x)) = f(g(a)) = f\left(\lim_{x\to a} g(x)\right)$$

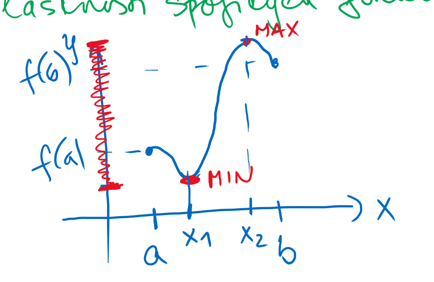
nech je spojitá funkcia!; zo spojitosti; lebo aj $g(x)$ je spojitá funkcia

$$\lim_{x\to a} e^{\square} = e^{\lim_{x\to a} \square}$$

použitie

Vlastnosti spojitých funkcií na uzavretom intervale $\langle a, b\rangle$.

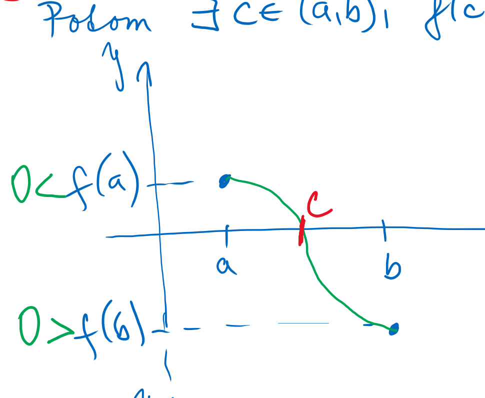
①



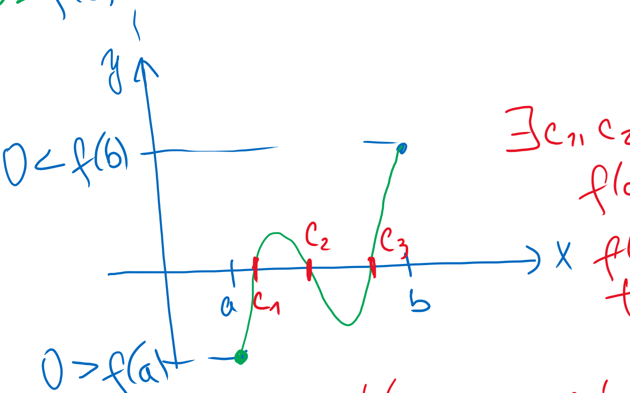
$f(x_1) = \min_{x\in\langle a,b\rangle} f(x)$

$f(x_2) = \max_{x\in\langle a,b\rangle} f(x)$

② nech f je spojitá na $\langle a,b\rangle$. nech $f(a) \cdot f(b) < 0$.
Potom $\exists c \in (a,b)$; $f(c) = 0$.



$\exists c \in (a,b)$
$f(c) = 0$



$\exists c_1, c_2, c_3 \in (a,b)$
$f(c_1) = 0$
$f(c_2) = 0$
$f(c_3) = 0$

význam: približné (numerické riešenie rovníc).