opakovanie:

DOBRÉ RÁNO :)

mali sme: derivácia funkcie $f(x)$ v bode x_0 $(O(x_0))$

$$\lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$$

– pravidlá derivovania.

$$(f \pm g)' = f' \pm g' \quad !$$
$$(k \cdot f)' = k \cdot f' \quad !$$
$$(f\cdot g)' = f'\cdot g + f\cdot g' \quad !$$
$$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2} \quad !$$

! derivácia zloženej funkcie:

$$[f(g(x))]' = f'(g(x))\cdot g'(x)$$

derivácia inverznej funkcie

$$\boxed{[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}}$$

+ derivačné vzorce (derivácie elem. funkcií). !

$$[\mathrm{tg}\,x]' = \frac{1}{\cos^2 x}\ ; \qquad [\mathrm{tg}\,x]' = \left[\frac{\sin x}{\cos x}\right]' = \frac{\cos x\cdot\cos x - \sin x\cdot(-\sin x)}{\cos^2 x} = \frac{\boxed{\cos^2 x + \sin^2 x}^{=1}}{\cos^2 x} = \frac{1}{\cos^2 x}$$

podiel

:)

$$[\mathrm{cotg}\,x]' = -\frac{1}{\sin^2 x}$$

využijem

$$[\sin x]' = \cos x$$
$$[\cos x]' = -\sin x$$

$$[\ln x]' = \frac{1}{x}$$

$$\boxed{f^{-1}(x) = \ln x}$$
$$f(x) = e^x$$
$$f'(x) = e^x$$

$$[f^{-1}(x)]' = [\ln x]' = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

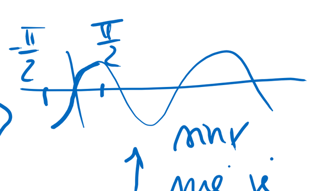
$$f'(f^{-1}(x)) =$$

$$f(x) = \arcsin x$$
$$[\arcsin x]' = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

$$\boxed{[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}}$$

$$\boxed{f(f^{-1}(x)) = x}$$
$$\sin(\arcsin x) = $$
$$= x$$

$$f^{-1}(x) = \boxed{\arcsin x},\ \langle -1, 1\rangle$$
$$f(x) = \sin x\ ;\ x \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle$$



sinx nie je prostá

$$f'(x) = \cos x$$

$$[\arcsin x]' = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-\sin^2(\arcsin x)}} = \frac{1}{\sqrt{1-x^2}}$$

super :)

$$\underset{f^{-1}(x)}{\parallel}$$

$$\cos x = \sqrt{1-\sin^2 x}$$

NIEKOĽKO PRÍKLADOV:

① $f(x) = 5 + 2x - 3x^5 + \frac{7}{x^4} + \sqrt{x^3} - \frac{3}{\sqrt[3]{x^5}} + 2\sin x - \arccos x$

$$[5]' = 0$$
$$[2x]' = 2$$
$$[3 x^5]' = 3\cdot(x^5)' = 3\cdot 5\cdot x^{5-1} =$$
$$= 15\cdot x^4$$

$$[x]' = 1$$
$$[\text{konšt.}\ x]' = \text{konšt.}$$
$$\boxed{(x^{\alpha})' = \alpha\cdot x^{\alpha-1}}$$

vzorec

$$\left[\frac{7}{x^4}\right]' = [7\cdot x^{-4}]' = 7\cdot(-4)\cdot x^{-4-1} = -28\cdot x^{-5}$$

$$(7 + x^{-4})' = 0 + (x^{-4})'$$

$$[\sqrt{x^3}]' = [x^{\frac{3}{2}}]' = \frac{3}{2}\cdot x^{\frac{3}{2}-1} = \frac{3}{2}\cdot x^{\frac{1}{2}}$$

$$\left[\frac{3}{\sqrt[3]{x^5}}\right]' = [3\cdot x^{-\frac{5}{3}}]' = 3\cdot\left(-\frac{5}{3}\right)\cdot x^{-\frac{5}{3}-1} = -5\cdot x^{-\frac{8}{3}}$$

konš. funkcia

$$[2\cdot\sin x]' = 2\cdot\cos x$$
$$[\arccos x]' = -\frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = 2 - 15x^4 - \frac{28}{x^5} + \frac{3}{2}x^{\frac{1}{2}} + 5\cdot x^{-\frac{8}{3}} + 2\cos x + \frac{1}{\sqrt{1-x^2}}$$

② $f(x) = e^x\cdot(x^2 + \cos x)$ ← DERIVÁCIA SÚČINU $[f\cdot g]' = f'\cdot g + f\cdot g'$

$$f'(x) = (e^x)'\cdot(x^2+\cos x) + e^x\cdot(x^2+\cos x)' =$$
$$= e^x\cdot(x^2+\cos x) + e^x\cdot(2x - \sin x)$$

③ $f(x) = \frac{\mathrm{tg}\,x}{3x^3-7}$ ← derivácia podielu $\left[\frac{f}{g}\right]' = \frac{f'\cdot g - f\cdot g'}{g^2}$

$$f'(x) = \frac{(\mathrm{tg}\,x)'\cdot(3x^3-7) - \mathrm{tg}\,x\cdot(3x^3-7)'}{(3x^3-7)^2} =$$

$$(x^3)' = 3x^2$$

$$= \frac{\frac{1}{\cos^2 x}\cdot(3x^3-7) - \mathrm{tg}\,x\cdot(9x^2-0)}{(3x^3-7)^2}$$

④ $f(x) = \cos(e^x + 9x)$ ← derivácia zloženej funkcie $[f(g(x))]' = f'(g(x))\cdot g'(x)$

vonk. zložk.

vnútorná zložka

$$f'(x) = -\sin(e^x+9x)\cdot(e^x+9x)' =$$
$$= -\sin(e^x+9x)\cdot(e^x+9)$$

⑤ $f(x) = \sqrt[3]{\ln\sin(10x)} = (\ln\sin(10x))^{\frac{1}{3}}$

4 zložky: $\sqrt[3]{\ }$, ln, sin, $(\ )^{1/3}$

$$(\square^{\frac{1}{3}})' = \frac{1}{3}\square^{-\frac{2}{3}}\cdot\square'$$

$$f'(x) = \frac{1}{3}(\ln\sin(10x))^{-\frac{2}{3}}\cdot\frac{1}{\sin(10x)}\cdot\cos 10x\cdot 10$$

⑥ $f(x) = x^x$

prípad: $(f(x))^{g(x)}$

$(\text{funkcia})^{\text{funkcia}}$

? $[x^{\alpha}]' = \alpha\cdot x^{\alpha-1}$? NIE

? $[a^x]' = a^x\cdot\ln a$? NIE

musíme upraviť predpis funkcie:

$$(f(x))^{g(x)} = e^{\ln(f(x))^{g(x)}} =$$

$$! \quad \boxed{e^{\ln\heartsuit} = \heartsuit}$$

$$= \boxed{e^{g(x)\cdot\ln f(x)}}$$ ← tento tvar už viem derivovať

$$f(x) = x^x = e^{\ln x^x} = \boxed{e^{x\ln x}}$$

$$[e^{x\ln x}]' = e^{x\cdot\ln x}\cdot(x\cdot\ln x)' =$$

$$[e^{\square}]' = e^{\square}\cdot\square'$$

$$= e^{x\ln x}\cdot\left(1\cdot\ln x + x\cdot\frac{1}{x}\right) = x^x(\ln x + 1)$$