Pr: $f(x) = \frac{x}{x^2-1}$ 1.) $D(f) = \mathbb{R} - \{\pm 1\}$

2.) $\lim_{x\to\infty} \frac{x}{x^2-1} = 0$ $\lim_{x\to -1^-} \frac{x}{x^2-1} = -\infty$ $\lim_{x\to 1^-} \frac{x}{x^2-1} = -\infty$

$\lim_{x\to-\infty} \frac{x}{x^2-1} = 0$ $\lim_{x\to -1^+} \frac{x}{x^2-1} = \infty$ $\lim_{x\to 1^+} \frac{x}{x^2-1} = \infty$

3.) ABS: $x = 1$ ; $x = -1$

ASS: $y = kx + q$ $\boxed{y = 0}$ → ASS

$x\to\infty$ $k: \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{1}{x^2-1} = 0$

$q: \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{x}{x^2-1} - 0\cdot x\right) = \lim_{x\to\infty} \frac{x}{x^2-1} = 0$

$x\to-\infty$ $k: \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{1}{x^2-1} = 0$

$q: \lim_{x\to-\infty} (f(x) - kx) = \lim_{x\to-\infty} \left(\frac{x}{x^2-1} - 0\cdot x\right) = \lim_{x\to-\infty} \frac{x}{x^2-1} = 0$

4.) $f(-x) = \frac{-x}{(-x)^2-1} = -\frac{x}{x^2-1} = -f(x)$ f(x) je nepárna

5.) $O_y$: ($x=0$) $f(0) = \frac{0}{0^2-1} = 0$ $[0;0]$ $O_x$: ($f(x)=0$) $0 = \frac{x}{x^2-1}$ → $x = 0$ $[0;0]$

6.) $f'(x) = \left(\frac{x}{x^2-1}\right)' = \frac{x^2-1-x\cdot 2x}{(x^2-1)^2} = -\frac{x^2+1}{(x^2-1)^2}$ $f' < 0$ je na celom D(f) ⇒ f(x) je na celom D(f) ↘

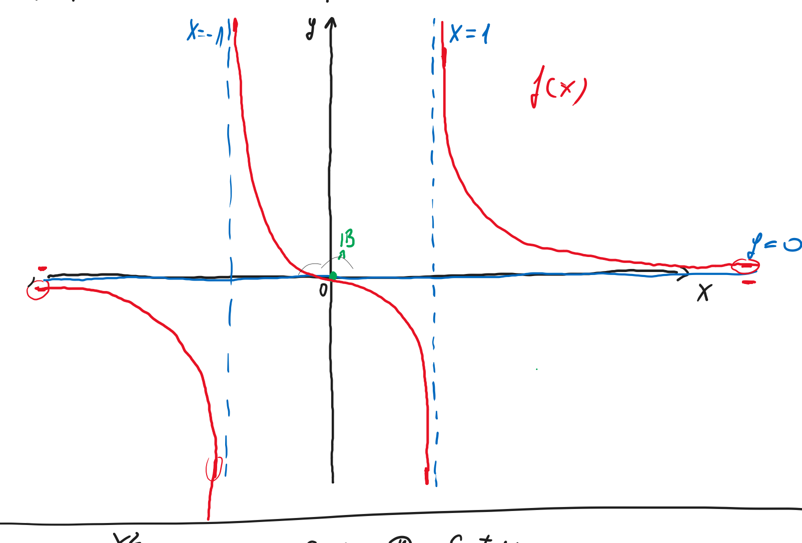
$f'(x) = 0: \emptyset$ $\nexists f': \pm 1$ SB: nemá EXTRÉMY: nemá

7.) $f'' = \left(-\frac{x^2+1}{(x^2-1)^2}\right)' = \frac{-2x(x^2-1)^2 + (x^2+1)2(x^2-1)\cdot 2x}{(x^2-1)^4} = \frac{-2x^3+2x+4x^3+4x}{(x^2-1)^3} =$

$= \frac{2x(x^2+3)}{(x^2-1)^3}$ $f'' = 0: 0$ $\nexists f'': \pm 1$

| | $(-\infty;-1)$ | $(-1;0)$ | 0 | $(0;1)$ | $(1;\infty)$ |
|---|---|---|---|---|---|
| $f''$ | − | + | | − | + |
| f(x) | ∩ | ∪ | IB | ∩ | ∪ |

8.)




Pr: $f(x) = \frac{x^2}{x^2-1}$ $D(f) = \mathbb{R} - \{\pm 1\}$

2.) $\lim_{x\to\infty} \frac{x^2}{x^2-1} = 1$ $\lim_{x\to -1^-} \frac{x^2}{x^2-1} = \infty$ $\lim_{x\to 1^-} \frac{x^2}{x^2-1} = -\infty$

$\lim_{x\to-\infty} \frac{x^2}{x^2-1} = 1$ $\lim_{x\to -1^+} \frac{x^2}{x^2-1} = -\infty$ $\lim_{x\to 1^+} \frac{x^2}{x^2-1} = \infty$

3.) ABS: $x = -1$ ; $x = 1$

ASS: $y = kx + q$ $\boxed{y = 1}$ → ASS

$x\to\infty$ $k: \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x}{x^2-1} = 0$

$q: \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{x^2}{x^2-1} - 0\cdot x\right) = \lim_{x\to\infty} \frac{x^2}{x^2-1} = 1$

$x\to-\infty$ $k: \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x}{x^2-1} = 0$

$q: \lim_{x\to-\infty} (f(x) - kx) = \lim_{x\to-\infty} \left(\frac{x^2}{x^2-1} + 0\cdot x\right) = \lim_{x\to-\infty} \frac{x^2}{x^2-1} = 1$

4.) $f(-x) = \frac{(-x)^2}{(-x)^2-1} = \frac{x^2}{x^2-1} = f(x)$ f(x) je párna

5.) $O_y$: ($x=0$) $f(0) = \frac{0^2}{0^2-1} = 0$ $[0;0]$ $O_x$: ($y=0$) $0 = \frac{x^2}{x^2-1}$ → $x = 0$ $[0;0]$

6.) $f'(x) = \left(\frac{x^2}{x^2-1}\right)' = \frac{2x(x^2-1) - x^2\cdot 2x}{(x^2-1)^2} = -\frac{2x}{(x^2-1)^2}$ $f'(x) = 0: 0$ $\nexists f': \pm 1$ SB: 0

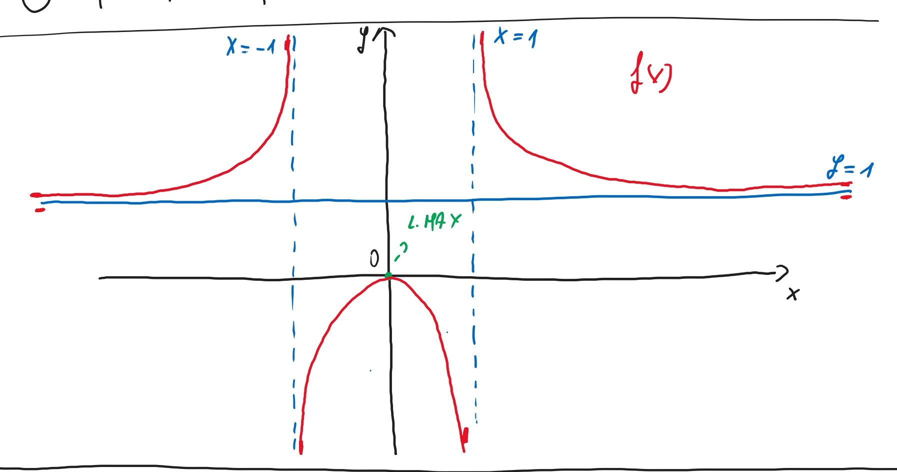
| | $(-\infty;-1)$ | $(-1;0)$ | 0 | $(0;1)$ | $(1;\infty)$ |
|---|---|---|---|---|---|
| $f'$ | + | + | SB | − | − |
| f(x) | ↗ | ↗ | L.MAX [0;0] | ↘ | ↘ |

7.) $f''(x) = \left(-\frac{2x}{(x^2-1)^2}\right)' = \frac{-2(x^2-1)^2 + 2x\cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{2(3x^2+1)}{(x^2-1)^3}$

$f'' = 0: \emptyset$ $\nexists f'': \pm 1$ IB nemá

| | $(-\infty;-1)$ | $(-1;1)$ | $(1;\infty)$ |
|---|---|---|---|
| $f''$ | + | − | + |
| f(x) | ∪ | ∩ | ∪ |

8.)




Pr: $f(x) = \frac{x^3}{2(x+1)^2}$ 1.) $D(f) = \mathbb{R} - \{-1\}$

2.) $\lim_{x\to-\infty} \frac{x^3}{2(x+1)^2} = -\infty$ $\lim_{x\to -1^-} \frac{x^3}{2(x+1)^2} = -\infty$

$\lim_{x\to\infty} \frac{x^3}{2(x+1)^2} = \infty$ $\lim_{x\to -1^+} \frac{x^3}{2(x^2+1)^2} = -\infty$

3.) ABS: $x = -1$ $\boxed{y = \frac{1}{2}x - 1}$ → ASS

ASS: $y = kx + q$

$x\to\infty$ $k: \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2}{2(x+1)^2} = \lim_{x\to\infty} \frac{x^2}{2x^2+4x+2} = \frac{1}{2}$

$q: \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{x^3}{2(x+1)^2} - \frac{1}{2}x\right) = \lim_{x\to\infty} \frac{-2x^2-x}{2(x+1)^2} = -1$

$x\to-\infty$ $k: \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x^2}{2(x+1)^2} = \frac{1}{2}$

$q: \lim_{x\to-\infty} (f(x) - kx) = \lim_{x\to-\infty} \left(\frac{x^3}{2(x+1)^2} - \frac{1}{2}x\right) = \lim_{x\to-\infty} \frac{-2x^2-x}{2(x+1)^2} = -1$

4.) $f(-x) = \frac{(-x)^3}{2(-x+1)^2} = -\frac{x^3}{2(1-x)^2}$ f(x) je ani párna ani nepárna

5.) $O_y$: ($x=0$) $f(0) = \frac{0}{2(x+1)^2} = 0$ $[0;0]$ $O_x$: ($y=0$) $0 = \frac{x^3}{2(x+1)^2}$ ⇒ $x = 0$ $[0;0]$

6.) $f'(x) = \left(\frac{x^3}{2(x+1)^2}\right)' = \frac{3x^2\cdot 2(x+1)^2 - x^3\cdot 4(x+1)}{4(x+1)^4} = \frac{2x^3+6x^2}{4(x+1)^3} = \frac{x^2(x+3)}{2(x+1)^3}$

$f' = 0: 0; -3$ $\nexists f': -1$ SB: 0; −3

| | $(-\infty;-3)$ | −3 | $(-3;-1)$ | $(-1;0)$ | 0 | $(0;\infty)$ |
|---|---|---|---|---|---|---|
| $f'$ | + | SB | − | + | SB | + |
| f(x) | ↗ | L.MAX $-\frac{27}{8}$ | ↘ | ↗ | | ↗ |

$f(-3) = \frac{x^3}{2(x+1)^2} = \frac{(-3)^3}{2(-3+1)^2}$

7.) $f''(x) = \left(\frac{x^3+3x^2}{2(x+1)^3}\right)' = \frac{(3x^2+6x)\cdot 2(x+1)^3 - (x^3+3x^2)\cdot 2\cdot 3(x+1)^2}{4(x+1)^6} =$

$= \frac{3x^3+6x^2+3x^2+6x-3x^3-9x^2}{2(x+1)^4} = \frac{6x}{2(x+1)^4}$

$f'' = 0: 0$ $\nexists f'': -1$

| | $(-\infty;-1)$ | $(-1;0)$ | 0 | $(0;\infty)$ |
|---|---|---|---|---|
| $f''$ | − | − | | + |
| f(x) | ∩ | ∩ | IB [0;0] | ∪ |