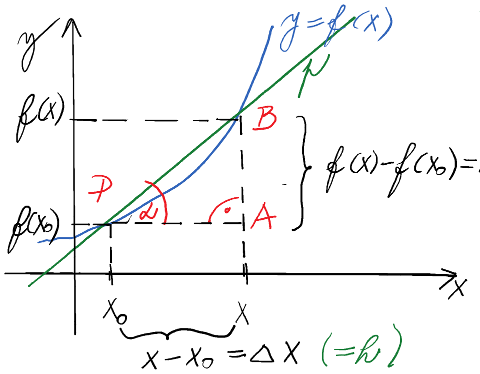
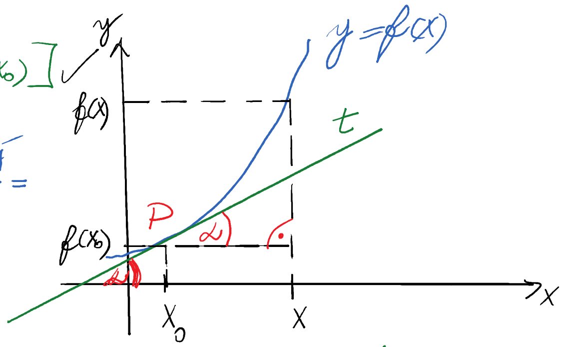
DOTYČNICA KU GRAFU FUNKCIE



t dotyčnica ku grafu funkcie v bode P$[x_0, f(x_0)]$



$\triangle ABP$:

$$\operatorname{tg}\alpha = \frac{\text{PROTIĽAHLÁ}}{\text{PRILĽAHLÁ}} = \frac{\Delta f}{\Delta X}$$

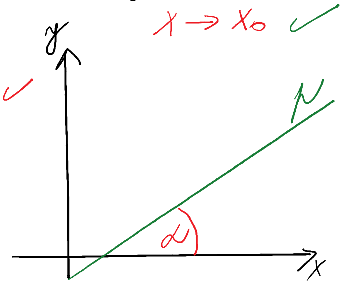
$$\operatorname{tg}\alpha = k$$

$$y = k\cdot x + q$$

$X \to X_0$

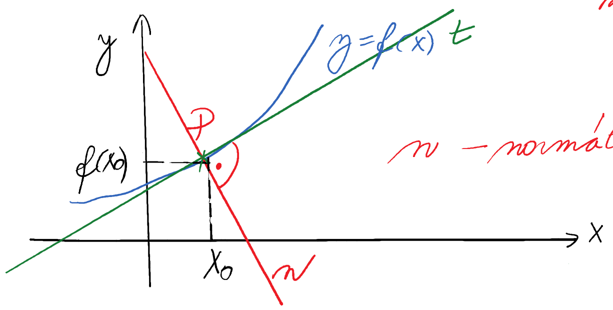
$$f'(x_0) = \lim_{\substack{\Delta X \to 0 \\ h \to 0}} \frac{f(X) - f(x_0)}{X - x_0} = \lim_{\Delta X \to 0} \frac{\Delta f}{\Delta X} = k$$

$$k = f'(x_0)$$



$y = kx + q$

$k = \operatorname{tg}\alpha$



n − normála

$t: y = k_t x + q_t$

$n: y = k_n x + q_n$

! $t \perp n \Rightarrow k_t \cdot k_n = -1$

$$k_n = -\frac{1}{k_t}$$

$$k_n = -\frac{1}{f'(x_0)} \quad !$$

DOTYČNICA t: $y - f(x_0) = f'(x_0)(x - x_0)$ P$[x_0, f(x_0)]$

NORMÁLA n: $y - f(x_0) = -\dfrac{1}{f'(x_0)}\cdot(x - x_0)$

PR $f(x) = x\cdot\ln x$ $D(f) = (0, \infty)$ $\ln x^n = n\ln x$!

$P = [e^2, ?]$ $x_0 = e^2$

t? n?

$f(x_0) = f(e^2) = e^2\cdot\ln e^2 = 2e^2 \boxed{\overset{1}{\ln e}} = 2e^2$

$f(x_0) = 2e^2$

$P = [e^2, 2e^2]$

$f'(x_0) = ?$

súčin

$f'(x) = (x\cdot\ln x)' = 1\ln x + x\cdot\frac{1}{x} = \ln x + 1$

$f'(x_0) = f'(e^2) = \ln e^2 + 1 = 2\,\overset{=1}{\ln e} + 1 = 2+1 = 3$

$f'(x_0) = 3 = k_t$

$t:\ y - 2e^2 = 3(x - e^2)$

$y = kx + q$

$t: \ y - 2e^2 = 3(x - e^2)$

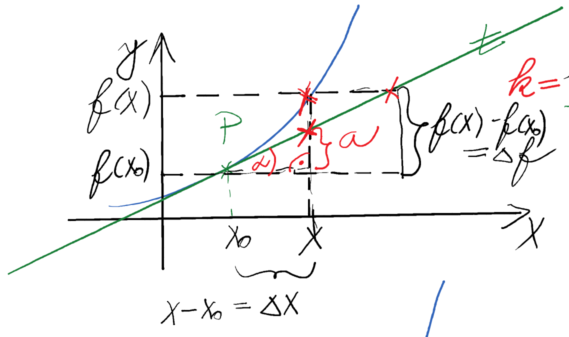
$y = \boxed{3}x - e^2$

$n: \ y - 2e^2 = -\frac{1}{3}(x - e^2)$

$y = kx + q$

$k_n = -\frac{1}{k_t}$

DIFERENCIÁL FUNKCIE

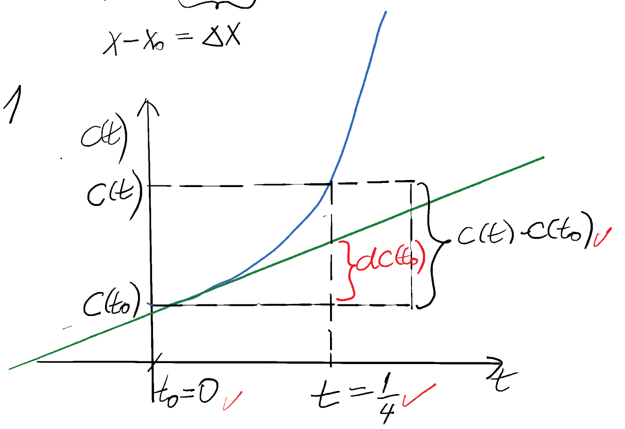


$k = \operatorname{tg}\alpha = \boxed{\frac{a}{\Delta x} = f'(x_0)} \quad / \cdot \Delta x$

$a = f'(x_0)\,\Delta x = df(x_0)$

$a = df(x_0)$

PR 1



$t_0 = 0$

$t = \frac{1}{4}$

$\Delta t = t - t_0 = \frac{1}{4} - 0 = \frac{1}{4}$

$\Delta C = C(t) - C(t_0)$ PRESNE

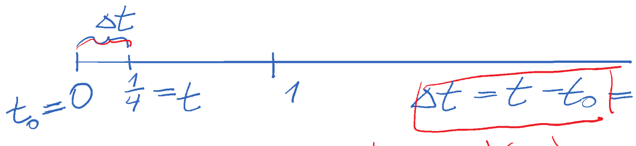
$\Delta C \doteq dC(t_0) = C'(t_0)\Delta t$

ODHAD

1) $t_0 = 0$ $\quad C(t) = -50t^2 + 100t + 10\,000$

$t = \frac{3}{12} = \frac{1}{4}$

$dC(t_0) = ?$



$\boxed{\Delta t = t - t_0} = \frac{1}{4} - 0 = \frac{1}{4}$

$C'(t) = 100t + 100$

$C'(t_0) = C'(0) = \boxed{100}$

$dC(t_0) = \underbrace{C'(t_0)}_{100} \cdot \underbrace{\Delta t}_{\frac{1}{4}} \quad (\doteq \Delta C)$

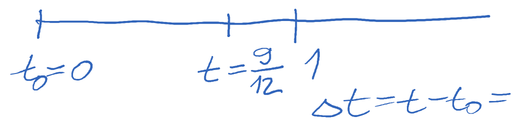
$dC(t_0) = 100 \cdot \frac{1}{4} = 25$ ks

POČAS NASLED. 3 MESIACOV VZRASTIE NÁKLAD O 25 KS. PRIBLIŽNE

2) $t_0 = 0$

$t = \frac{9}{12} = \frac{3}{4}$

$dC(t_0) = ?$



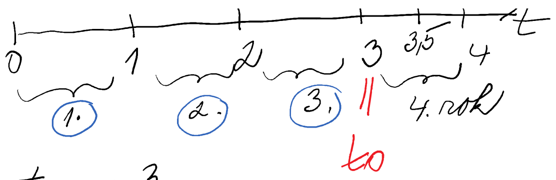
$\Delta t = t - t_0 = \frac{9}{12} - 0 = \frac{3}{4}$

$C'(t_0) = 100$

$dC(t_0) = \underbrace{100}_{C'(t_0)} \cdot \underbrace{\frac{3}{4}}_{\Delta t} = 75$ ks

POČAS NASLEDUJÚCICH 9 MESIACOV VZRASTIE NÁKLAD O 75 KS. PRIBLIŽNE

3)



$\Delta t = t - t_0 = 3{,}5 - 3 = 0{,}5 = \frac{1}{2}$

$t_0 = 3$

$t = 3{,}5$

$dC(t_0) = ?$

$dC(t_0) = C'(t_0)\Delta t$

$\Delta t = t - t_0 = 3{,}5 - 3 = \frac{1}{2}$

$C'(t) = 100t + 100$

$C'(t_0) = C'(3) = 100 \cdot 3 + 100 = 400$

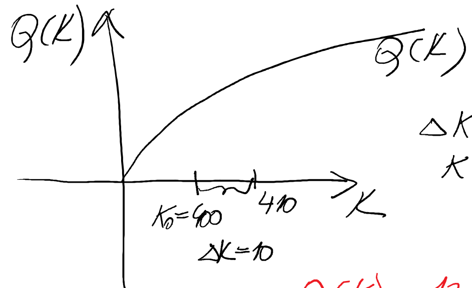
$dC(t_0) = C'(t_0)\cdot\Delta t = 400 \cdot \frac{1}{2} = 200$ ks. $\doteq \Delta C = C(t) - C(t_0)$

ODHAD

PRESNE: $\Delta C = C(t) - C(t_0) = C(3{,}5) - C(3) =$

$= 212{,}5 \doteq 212$ KS PRESNE

PR2: $Q(K) = 1200\sqrt{K} = 1200K^{\frac{1}{2}}$



1) $K_0 = 400$

$\Delta K = 10$

$dQ(K_0) = ?$

$\Delta K = K - K_0$

$K = K_0 + \Delta K$

$dQ(K_0) = Q'(K_0)\Delta K$

$Q(K) = 1200K^{\frac{1}{2}}$

$Q'(K) = 1200 \cdot \frac{1}{2} \cdot K^{\frac{1}{2}-1}$

$Q'(K) = \cancel{1200}^{600} \cdot \frac{1}{2} \cdot K^{-\frac{1}{2}} = \frac{600}{\sqrt{K}}$

$Q'(K_0) = Q'(400) = \frac{600}{\sqrt{400}} = \frac{600}{20} = 30$

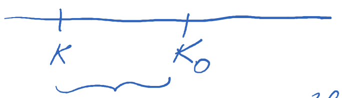
$dQ(K_0) = 30 \cdot 10 = 300$

DENNÁ PRODUKCIA VZRASTIE O 300 KS.

2) $\Delta K = -20$ $\quad K = 380$

$K_0 = 400$

$dQ(K_0) = ?$



$\Delta K = K - K_0 = 380 - 400 = -20$

$Q'(K_0) = Q'(400) = 30$

$dQ(K_0) = 30 \cdot (-20) = -600$

DENNÁ PRODUKCIA SA ZNÍŽI O 600 KS.

3) $K_0 = 400$

$dQ(K_0) = 180$

$K = ?$ $\quad (\Delta K ?)$

$\Delta K = K - K_0$

$\boxed{K = K_0 + \Delta K}$

$400 + 6$

$dQ(K_0) = \underbrace{Q'(K_0)}_{30} \cdot \Delta K = 180$

$30\,\Delta K = 180$

$\Delta K = 6$ (tisíc)

$K = 400 + 6 = 406$

KAP. INV. SA MUSÍ ZVÝŠIŤ NA HODNOTU 406 000

L'HOSPITALOVO PRAVIDLO

$x^3 - 4x^2 = x^2(x-4)$

$$\lim_{x\to a}\frac{f(x)}{g(x)} \qquad \lim_{x\to a}\frac{f'(x)}{g'(x)}\ \exists$$

PR 1 $$\lim_{x\to 0}\frac{\sin x}{x} \overset{\boxed{\frac{0}{0}}}{\underset{L'H}{=}} \lim_{x\to 0}\frac{\cos x}{1} = 1 \qquad \lim_{x\to 0}\frac{(\sin x)'}{x'}$$

PR2 a) $$\lim_{x\to\infty}\frac{x^3-4x^2}{x^2-1} \overset{\boxed{\frac{\infty}{\infty}}}{\underset{L'H}{=}} \lim_{x\to\infty}\frac{3x^2-8x}{2x} \overset{\boxed{\frac{\infty}{\infty}}}{\underset{L'H}{=}} \lim_{x\to\infty}\frac{6x-8}{2} = \boxed{\infty}$$

b) $$\lim_{x\to\infty}\frac{\boxed{5}x^2-2x+1}{\boxed{3}x^2+4x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty}\frac{10x-2}{6x+4} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim \frac{10}{6} = \frac{5}{3}$$

PR 3 $$\lim_{x\to\infty}\frac{x}{e^{x^2}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty}\frac{1}{e^{x^2}\cdot 2x} \quad \frac{1}{\infty} = 0 \qquad \overset{L'H}{\to} \lim_{x\to\infty}\frac{0}{(e^{x^2}\cdot 2x)'} \quad \infty \quad = 0$$

PR4 $$\lim_{x\to 0^+} x^2 e^{\frac{1}{x}} \overset{\boxed{0\cdot\infty}}{=} \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}}{\frac{1}{x^2}} \overset{\boxed{\frac{\infty}{\infty}}}{\underset{L'H}{=}} \qquad \frac{1}{x^2} = x^{-2}$$

$$= \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'}{(x^{-2})'} \overset{L'H}{=} \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}(-1\cdot x^{-2})}{-2x^{-3}} =$$

$$= \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{-\frac{2}{x^3}} = \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}}{2\cdot\frac{1}{x}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$$

$$= \lim_{x\to 0^+}\frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'}{2\left(\frac{1}{x}\right)'} = \lim_{x\to 0^+}\frac{1}{2}\cdot e^{\frac{1}{x}} = \infty$$

PR5 $$\lim_{x\to\infty}\left(\frac{x^3-4x^2}{x^2-1} - x\right) \overset{\infty-\infty}{=} \lim_{x\to\infty}\frac{x^3-4x^2-x(x^2-1)}{x^2-1} =$$

SPOLOČNÝ MENOVATEĽ

$$= \lim_{x\to\infty}\frac{\cancel{x^3}-4x^2-\cancel{x^3}+x}{x^2-1} = \lim_{x\to\infty}\frac{\boxed{-4}x^2+x}{\boxed{1}x^2-1} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}}$$

$$= \lim_{x\to\infty}\frac{-8x+1}{2x} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty}\frac{-8}{2} = -4$$

ASYMPTOTY SO SMERNICOU

ASS: $y = kx + q$

$$k = \lim_{x \to \infty} \frac{f(x)}{x} \qquad f(x) = \frac{x^3 - 4x^2}{x^2 - 1}$$

$$k = \lim_{x \to \pm\infty} \frac{\frac{x^3 - 4x^2}{x^2 - 1}}{x} = \lim_{x \to \pm\infty} \frac{x^{3} - 4x^2}{x^{3} - x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$$

$$= \lim_{x \to \infty} \frac{3x^2 - 8x}{3x^2 - 1} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x \to \infty} \frac{6x - 8}{6x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}}$$

$$= \lim_{x \to \infty} \frac{6}{6} = 1$$

$$q = \lim_{x \to \infty} \left(f(x) - k \cdot x \right)$$

$$q = \lim_{x \to \pm\infty} \left(\frac{x^3 - 4x^2}{x^2 - 1} - 1 \cdot x \right) \overset{\infty - \infty}{=}$$

$$= \lim_{x \to \infty} \frac{x^3 - 4x^2 - x(x^2 - 1)}{x^2 - 1} = \lim_{x \to \infty} \frac{\cancel{x^3} - 4x^2 - \cancel{x^3} + x}{x^2 - 1} =$$

$$= \dots = -4 \text{ (PRÍKLAD 5)}$$

$$y = 1 \cdot x - 4 \qquad x \to \pm\infty$$

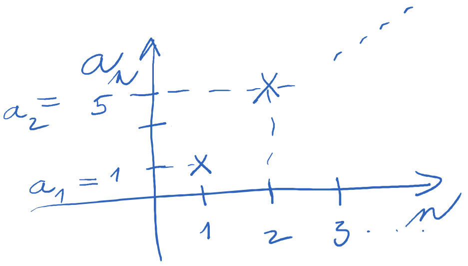
POSTUPNOSTI

1) $a_n = 3n - 1$

$a_1 = 3 \cdot 1 - 1 = 2$

$a_2 = 3 \cdot 2 - 1 = 5$

$\vdots$



2) $a_n = 2^n$

$a_1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

GEOMETRICKÁ POSTUPNOSŤ

$$\frac{a_{n+1}}{a_n} = 2 = q \quad \text{QUOCIENT}$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_n = a_1 \cdot \frac{1 - q^n}{1 - q} \quad \left(= a_1 \frac{q^n - 1}{q - 1} \right)$$

LIMITA POSTUPNOSTI

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty}\left(1+\frac{5}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{5}}\right)^n =$$

$$\frac{n}{5} \to \infty \quad = t \to \infty$$

$$n = 5t$$

$$= \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{5t} = \Big[\underbrace{\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t}}_{e}\Big]^5 =$$

$$= [e]^5 = \underline{\underline{e^5}}$$

$$\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$$