RzM 3. PR.

RIEŠENIE NEROVNÍC

KVADRATICKÉ NEROVNICE

$L(x) \lesseqgtr P(x)$

$K \subset D \subset O$

EKVIVALENTNÉ ÚPRAVY DU

V MNOŽINE $\mathbb{R}$ RIEŠTE NEROVNICE

Pr. $\frac{3x-1}{5} - \frac{13-x}{2} \geq \frac{7x}{3} - \frac{11(x+3)}{6}$ $/ \cdot 30 > 0$ $D = \mathbb{R}$

$6.(3x-1) - 15(13-x) \geq 10.7.x - 5.11.(x+3)$

$18x - 6 - 195 + 15x \geq 70x - 55x - 165$

$33x - 201 \geq 15x - 165$ $/ -15x + 201$

$18x \geq 36$ $/ : 18 > 0$

$x \geq 2$ $K = \langle 2; \infty)$

Pr. $3x^2 - 2x + 5 > 0$ $D = \mathbb{R}$

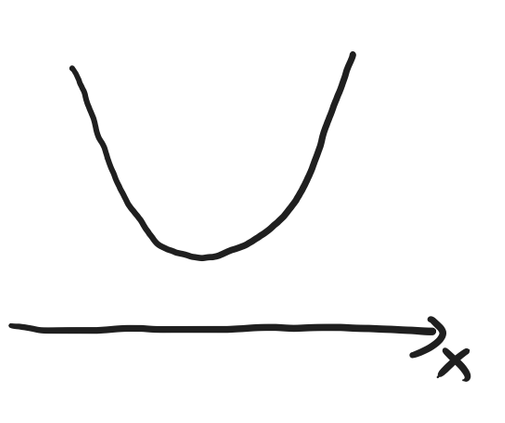
určíme korene kvadrat. rovnice

$3x^2 - 2x + 5 = 0$

$D = (-2)^2 - 4.3.5 = 4 - 60 = -56 < 0$

... nemá v $\mathbb{R}$ korene

$\forall x \in \mathbb{R}$ $3x^2 - 2x + 5 \neq 0$



$V(x) = 3x^2 - 2x + 5$

$V(x) > 0$ $V(x) < 0$

napr. $x = 1$ $V(1) = 3.1^2 - 2.1 + 5 = 6$

$V(x) = 3x^2 - 2x + 5 > 0$ $\forall x \in \mathbb{R}$

$3x^2 - 2x + 5 > 0$ $K = \mathbb{R}$

Pr. $-3x^2 - 7x + 6 \geq 0$ $/ \cdot (-1)$

$3x^2 + 7x - 6 \leq 0$

riešime rovnicu $3x^2 + 7x - 6 = 0$

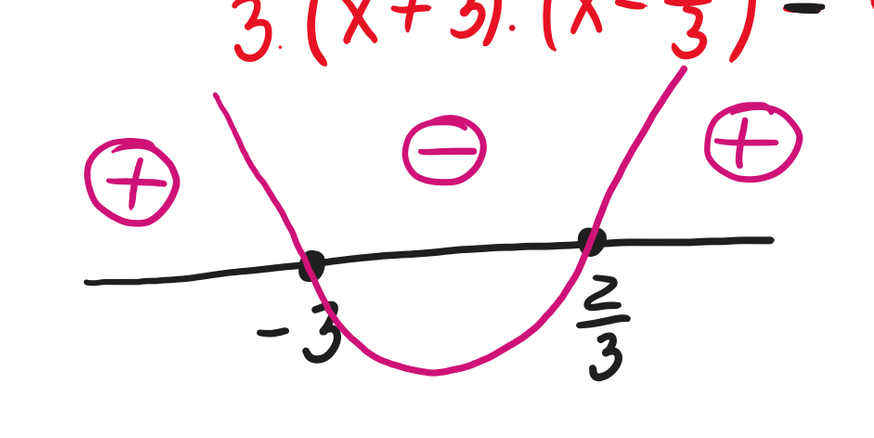
$D = 7^2 - 4.3.(-6) = 49 + 72 = 121$

$x_{1,2} = \frac{-7 \pm \sqrt{121}}{2.3} = \frac{-7 \pm 11}{6} = \begin{cases} \frac{2}{3} \\ -3 \end{cases}$

NULOVÉ BODY (NB): $-3; \frac{2}{3}$

$3x^2 + 7x - 6 = 3.(x+3).(x-\frac{2}{3})$

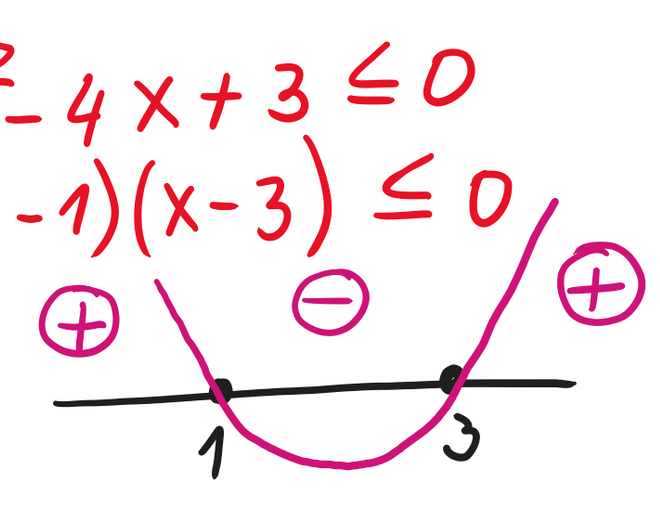
$3.(x+3).(x-\frac{2}{3}) \leq 0$



$K = \langle -3; \frac{2}{3} \rangle$

Pr. $x^2 - 4x + 3 \leq 0$ NB: 1, 3

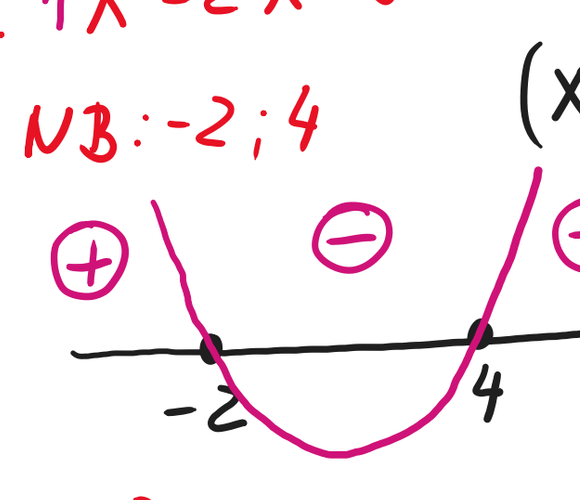
$(x-1)(x-3) \leq 0$



$K = \langle 1; 3 \rangle$

Pr. $1x^2 - 2x - 8 \geq 0$

NB: $-2; 4$ $(x+2)(x-4) \geq 0$

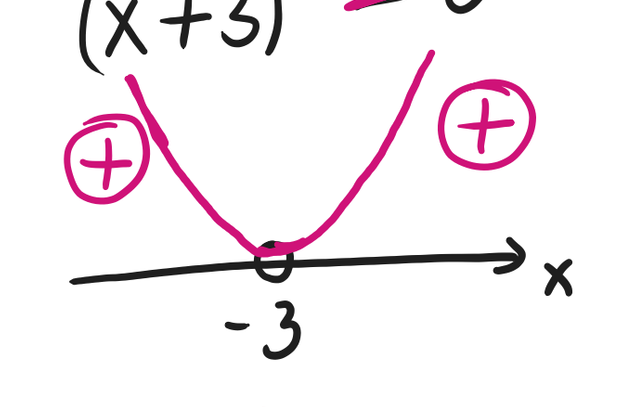


$K = (-\infty; -2\rangle \cup \langle 4; \infty)$

Pr. $-x^2 - 6x - 9 < 0$ $/ \cdot (-1)$

$x^2 + 6x + 9 > 0$

$(x+3)^2 > 0$ NB: -3



$K = \mathbb{R} - \{-3\} = (-\infty; -3) \cup (-3; \infty)$

Pr. $x^4 + x^3 + x + 1 \leq 0$

$x^3(x+1) + x + 1 \leq 0$

$(x+1)(x^3+1) \leq 0$

$a^3 + b^3 = (a+b).(a^2 - ab + b^2)$

$x^3 + 1 = (x+1).(x^2 - x + 1)$

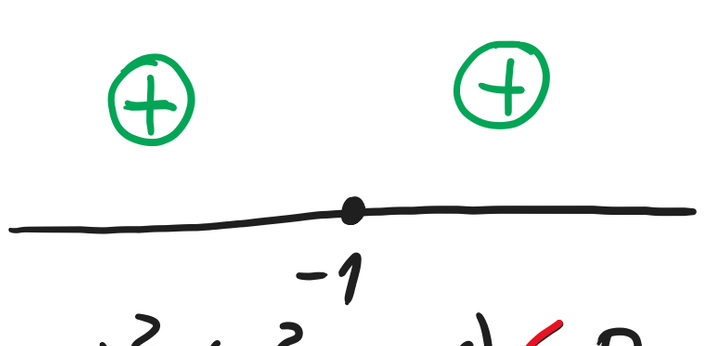
$(x+1).(x+1).(x^2 - x + 1) \leq 0$

NB: $x + 1 = 0$, $x = -1$

$x^2 - x + 1 = 0$

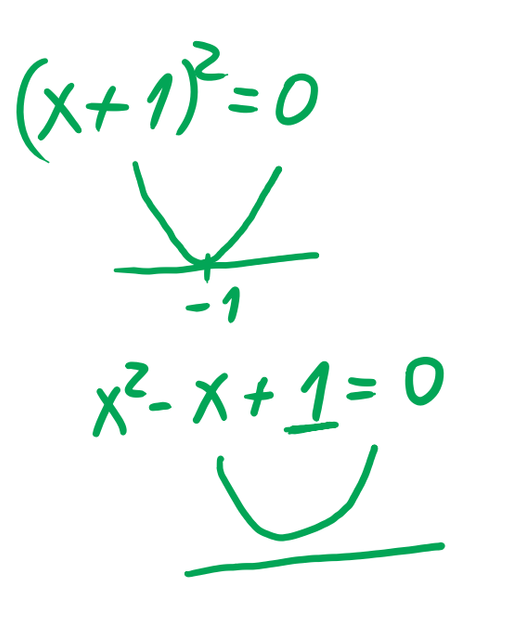
$D = (-1)^2 - 4.1.1 = -3 < 0$

- nemá korene v $\mathbb{R}$



$(x+1)^2 . (x^2 - x + 1) \leq 0$

≥ 0 > 0



$K = \{-1\}$

Pr. $x^4 - 13x^2 + 36 \leq 0$

SUBSTITÚCIA $x^2 = A$

$A^2 - 13A + 36 \leq 0$

$D = (-13)^2 - 4.1.36 = 169 - 144 = 25$

$A_{1,2} = \frac{13 \pm \sqrt{25}}{2.1} = \frac{13 \pm 5}{2} = \begin{cases} 9 \\ 4 \end{cases}$

$x^2 = A$

$x^2 = 9$ $x^2 = 4$

$x = \pm 3$ $x = \pm 2$

$(A - 9).(A - 4) \leq 0$

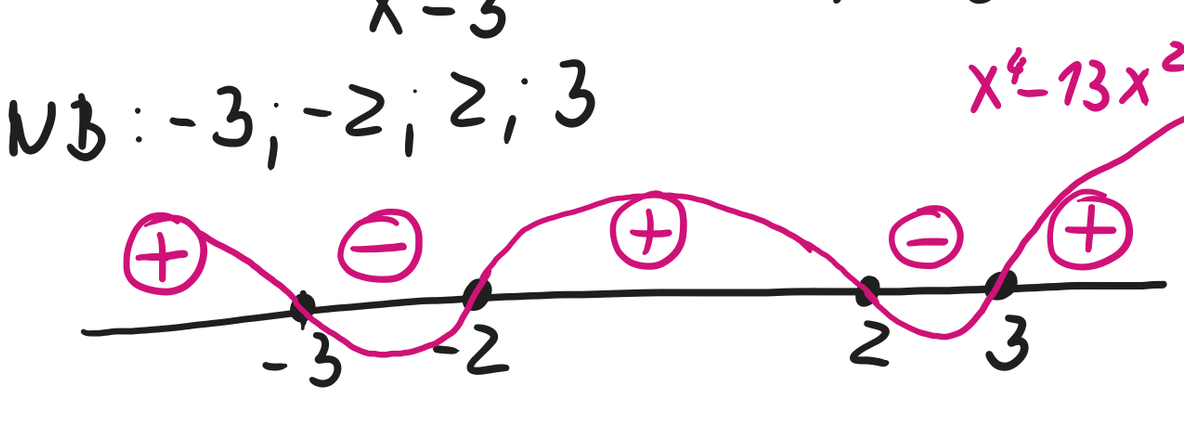
$(x^2 - 9).(x^2 - 4) \leq 0$

$(x-3).(x+3).(x-2).(x+2) \leq 0$

$x - 3 = 0$ alebo $x + 3 = 0$ alebo $x - 2 = 0$ alebo $x + 2 = 0$

$x = 3$ $x = -3$ $x = 2$ $x = -2$

NB: $-3; -2; 2; 3$ $x^4 - 13x^2 + 36 \leq 0$



$K = \langle -3; -2 \rangle \cup \langle 2; 3 \rangle$

	$(-\infty; -3\rangle$	$\langle -3; -2 \rangle$	$\langle -2; 2 \rangle$	$\langle 2; 3 \rangle$	$\langle 3; \infty)$
$x-3$	−	−	−	−	+
$x+3$	−	+	+	+	+
$x-2$	−	−	−	+	+
$x+2$	−	−	+	+	+
$(x-3).(x+3).(x-2)(x+2)$	⊕	⊖	⊕	⊖	⊕

Pr. $\frac{1}{x+2} \leq \frac{3}{x-2}$ $/ - \frac{3}{x-2}$

D: $x + 2 \neq 0$ $x - 2 \neq 0$

$x \neq -2$ $x \neq 2$

$D = \mathbb{R} - \{\pm 2\}$

$\frac{1}{x+2} - \frac{3}{x-2} \leq 0$

$\frac{1.(x-2) - 3.(x+2)}{(x+2).(x-2)} \leq 0$

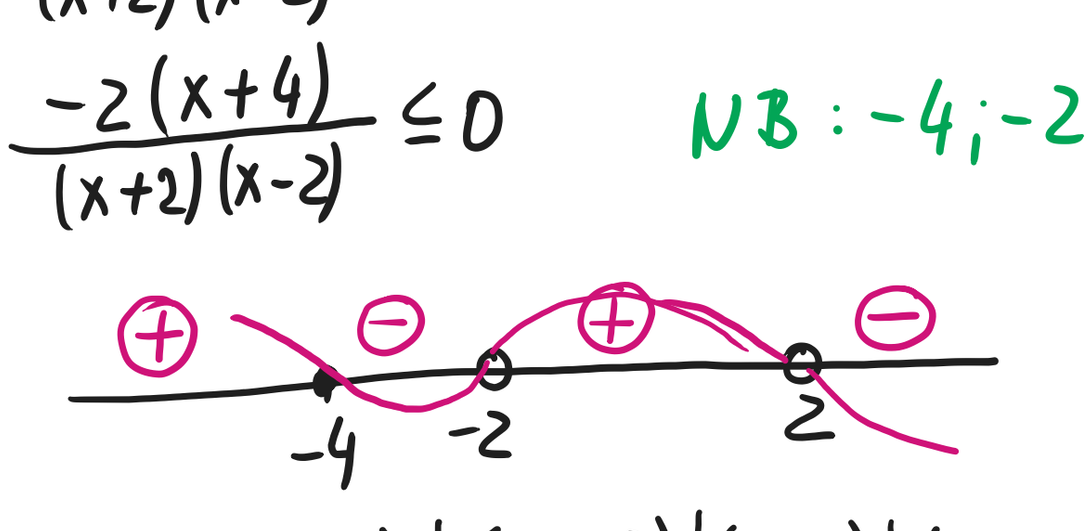
$\frac{x - 2 - 3x - 6}{(x+2)(x-2)} \leq 0$

$\frac{-2x - 8}{(x+2)(x-2)} \leq 0$

NB: $-2(x+4) = 0$, $x = -4$; $x + 2 = 0$, $x = -2$; $x - 2 = 0$, $x = 2$

$\frac{-2(x+4)}{(x+2)(x-2)} \leq 0$

NB: $-4; -2; 2$



$\frac{-2(x+4)}{(x+2).(x-2)} \leq 0$

$K = \langle -4; -2) \cup (2; \infty)$

	$(-\infty; -4\rangle$	$\langle -4; -2)$	$(-2; 2)$	$(2; \infty)$
$x+4$	−	+	+	+
$x+2$	−	−	+	+
$x-2$	−	−	−	+
$\frac{-2.(x+4)}{(x+2).(x-2)}$	⊕	⊖	⊕	⊖

$V(x) = \frac{-2(x+4)}{(x+2)(x-2)}$

$V(-5)$: $\frac{-.-}{-.-} = +$ $V(0)$: $\frac{-.+}{+.-} = +$

$V(-3)$: $\frac{-.+}{-.-} = -$ $V(3)$: $\frac{-.+}{+.+} = -$

Pr. $\frac{x^2 - 3x + 2}{x^2 + 3x + 2} \geq 1$ $/ -1$

D: $x^2 + 3x + 2 \neq 0$

$(x+1).(x+2) \neq 0$

$x \neq -1 \wedge x \neq -2$

$D = \mathbb{R} - \{-1; -2\}$

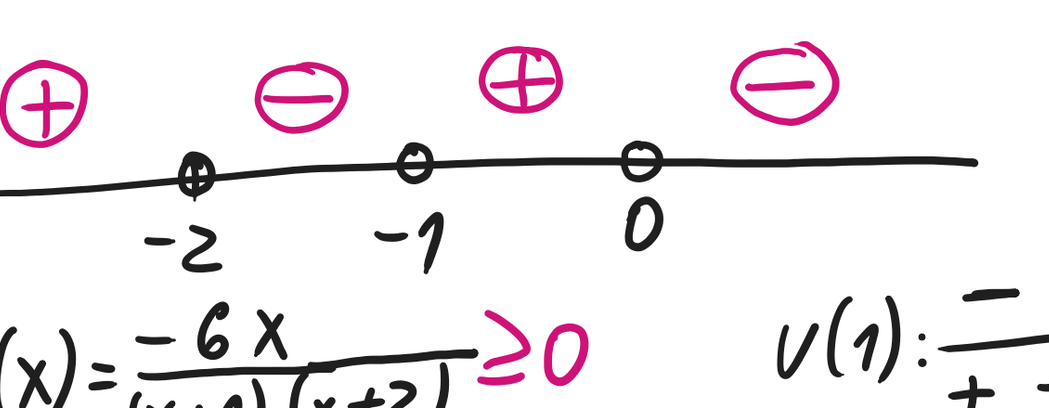
$\frac{x^2 - 3x + 2}{x^2 + 3x + 2} - 1 \geq 0$

$\frac{x^2 - 3x + 2 - x^2 - 3x - 2}{x^2 + 3x + 2} \geq 0$

$\frac{-6x}{x^2 + 3x + 2} \geq 0$

$\frac{-6x}{(x+1)(x+2)} \geq 0$ NB: $-2; -1; 0$

DU - TAB



$V(x) = \frac{-6x}{(x+1).(x+2)} \geq 0$ $V(1)$: $\frac{-}{+.+} = -$

$V(-3)$: $+$

$V(-\frac{3}{2})$: $-$

$V(-\frac{1}{2})$: $+$

$K = (-\infty; -2) \cup (-1; 0\rangle$

V $\mathbb{R}$ riešte sústavu nerovníc

Pr. $2x + 3 \geq 1$

$\frac{1}{x} + \frac{1}{3} < 0$

$D = \mathbb{R} - \{0\}$

$2x + 3 \geq 1$

$2x \geq -2$

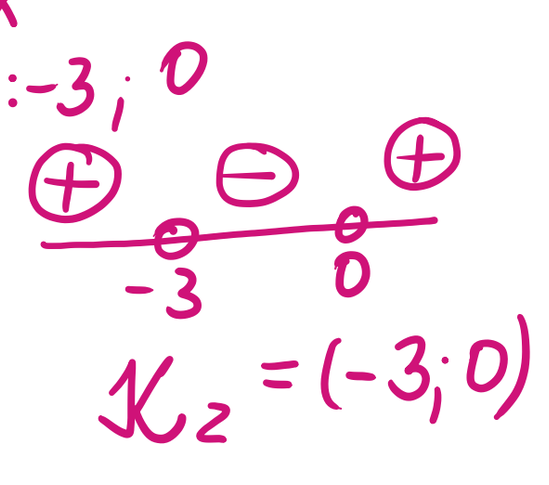
$x \geq -1$

$K_1 = \langle -1; \infty)$

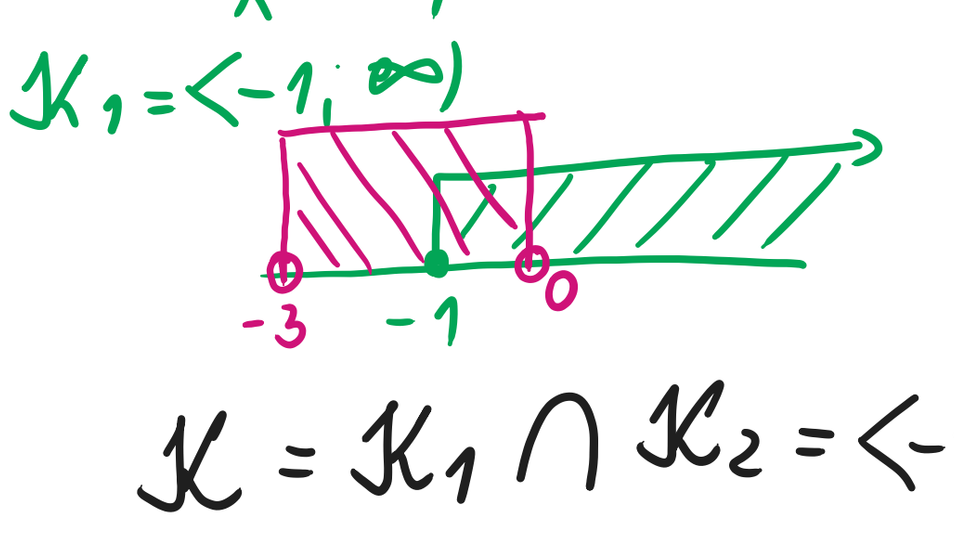
$\frac{1}{x} + \frac{1}{3} < 0$

$\frac{3 + x}{3x} < 0$

NB: $-3; 0$



$K_2 = (-3; 0)$



$K = K_1 \cap K_2 = \langle -1; \infty) \cap (-3; 0) = \langle -1; 0)$