Marginálna analýza, elasticita

streda 13. novembra 2024 7:30

MARGINÁLNA ANALÝZA

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) \doteq \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

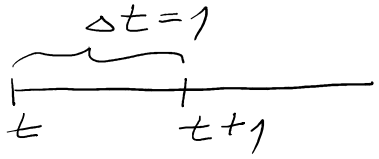
$\Delta x = 1$

$$f'(x) \doteq f(x+1) - f(x)$$

$$Mf(x) = f'(x)$$

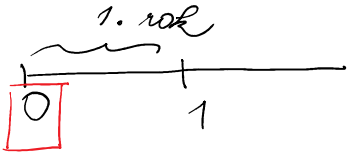
PR. $C(t) = 50t^2 + 100t + 10\,000$

1) tempo rastu po t rokoch
2) prírastok počas 1. roka
3) prírastok počas 6. roka



1) $MC(t) = C'(t)$

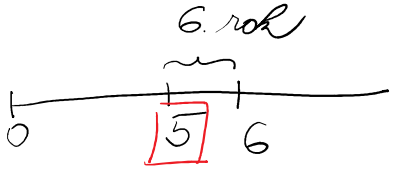
$C'(t) = 100t + 100$



2) $t_0 = 0$

$MC(t_0) = C'(t_0)$

$C'(0) = 100 \cdot 0 + 100 = 100$



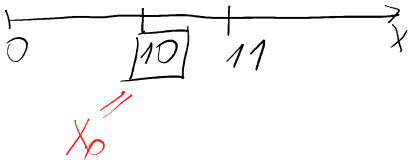
3) $t_0 = 5$

$MC(t_0) = C'(t_0)$

$C'(5) = 100 \cdot 5 + 100 = 600$

PR. $C(x) = x^2 + 15x + 5000$ ✓

1) 11. výrobok odlad
2) presne



$x_0 = 10$

1) $MC(x) = C'(x)$

$MC(10) = ?$

$C'(x) = 2x + 15$

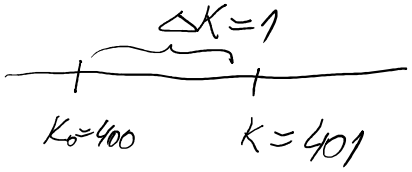
$MC(10) = C'(10) = 2 \cdot 10 + 15 = 35$ približne

2) $C(11) - C(10) = 5286 - 5250 = 36$ presne

PR. $Q(K) = 1200 \cdot \sqrt{K} = 1200 \cdot K^{\frac{1}{2}}$

$K_0 = 400$ kusov

$\Delta K = 1$



$MQ(K_0) = 2$

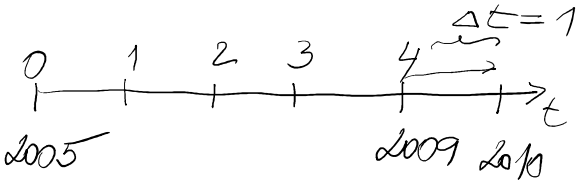
$MQ(K_0) = Q'(K_0)$

$Q'(K) = 1200 \cdot \frac{1}{2} K^{-\frac{1}{2}} = \frac{600}{\sqrt{K}}$

$MQ(400) = Q'(400) = \frac{600}{\sqrt{400}} = \frac{600}{20} = 30$

DENNÁ PRODUKCIA SA ZVÝŠI PRIBLIZNE O 30 KS.

PR. $R(t) = 20t^2 + 1000t + 2000$



1) ročné tempo rastu na začiatku 2009

2) percentuálne ...

1) $MR(4) = 2$

$MR(t) = R'(t)$

$R'(t) = 40t + 1000$

$MR(4) = R'(4) = 40 \cdot 4 + 1000 = 1160$ tisíc eur

2) $\frac{R'(4)}{R(4)} \cdot 100\,\% = \frac{1160}{16\,336} \cdot 100\,\% = 18{,}3\,\%$

ELASTICITA FUNKCIE

$\mathcal{E}(f(x)) = \frac{f'(x)}{f(x)} \cdot x$

PR. $Q(K) = 1200\sqrt{K}$

$\Delta K = \frac{K}{100}$

$\mathcal{E}(Q(K)) = 2$

$Q'(K) = \frac{600}{\sqrt{K}}$

$\mathcal{E}(Q(K)) = \frac{Q'(K)}{Q(K)} \cdot K = \frac{\frac{600}{\sqrt{K}}}{1200\sqrt{K}} \cdot K =$

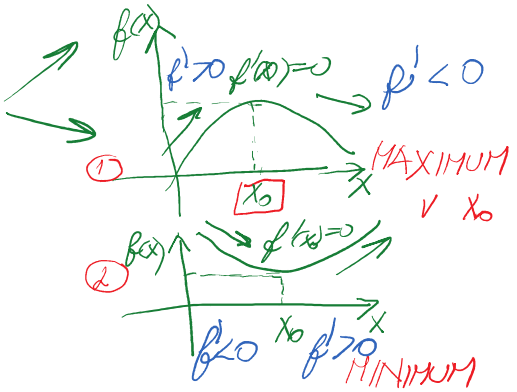
$K^{\frac{1}{2}} \cdot K^{\frac{1}{2}} = K^{\frac{1}{2}+\frac{1}{2}} = 1$

$= \frac{600}{1200 \sqrt{K}\sqrt{K}} \cdot K = \frac{K}{2K} = \frac{1}{2}\,\%$

ELASTICITA DOPYTU

$q = D(p)$ $\quad p > 0, D(p) > 0$

$f'(x) > 0$ FUNKCIA $f(x)$ RASTIE
$f'(x) < 0$ KLESÁ
$f'(x_0) = 0$ NEMENÍ SA



$$E_D(p_0) = -\frac{D'(p_0)}{D(p_0)} \cdot p_0$$

$x > 0$ $f(x) > 0$ $\quad p > 0$ $D(p) = q > 0$

PR $\quad q = 240 - 2p \; (= D(p))$
$0 < p < 120$

$q > 0$
$240 - 2p > 0$
$240 > 2p$
$120 > p$

1) $E_D = ?$
2) $E_D(100) = ?$
3) $E_D(50) = ?$
4) $E_D = 1$

1) $E_D = -\frac{D'(p)}{D(p)} \cdot p = -\frac{(-2)}{240 - 2p} \cdot p = \frac{2p}{240 - 2p}$ ✓ $\quad D'(p) < 0$

$= \frac{2p}{2(120 - p)} = \frac{p}{120 - p}$ ✓

$p = p_0 + \frac{p_0}{100} = 100 + 1 = 101$

2) $E_D(100) = \frac{100}{120 - 100} = \frac{100}{20} = 5\,\% > 1$ ELASTICKÁ

ZVÝŠENIE CENY O 1%, ZNÍŽI POČET PREDANÝCH KS O 5%

3) $E_D(50) = \frac{50}{120 - 50} = \frac{50}{70} \doteq 0{,}71\,\% < 1$ NEELAST.

4) $E_D = 1$

$\frac{p}{120 - p} = 1$

$p = 120 - p$

$2p = 120$

$p = 60$ JEDNOTKOVÁ ELASTICITA

VPLYV ELASTICITY DOPYTU NA CELKOVÉ PRÍJMY

PR $q = 240 - 2p$

$0 < p < 120$

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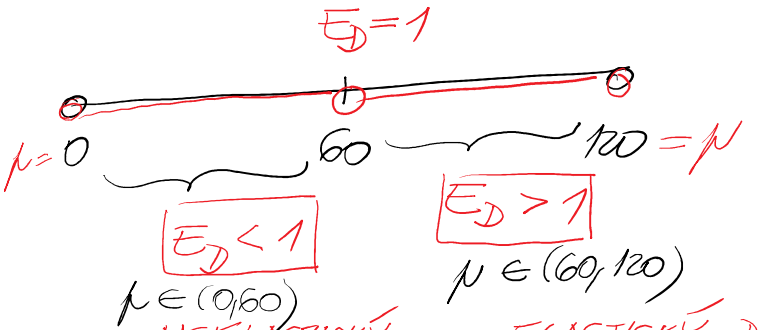
1) $E_D > 1$

$E_D < 1$

$E_D = 1$

$E_D = \frac{p}{120-p} = 1$

$p = 60$



$E_D > 1$ $\quad \frac{p}{120-p} > 1 \quad / \cdot (120-p)$

$p > 120 - p$

$2p > 120$

$p > 60$

$240 - 2p > 0$

$120 - p > 0$

$E_D < 1$ $\quad \frac{p}{120-p} < 1 \quad / \cdot (120-p)$

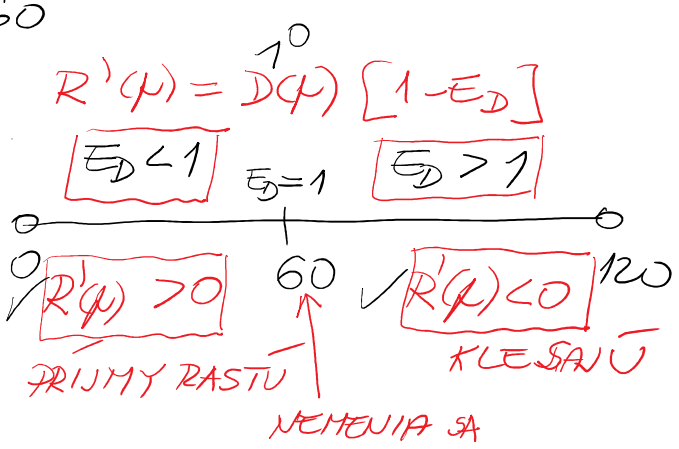
$p < 120 - p$

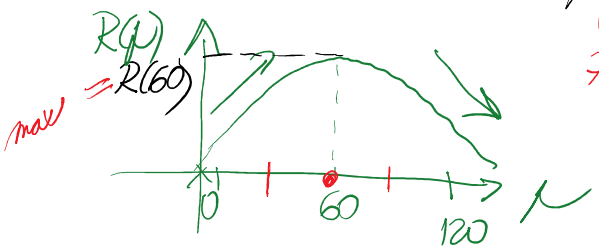
$2p < 120$

$p < 60$

2) intervaly rastu a klesania $R(p)$ pomocou E_D

$R'(p) = D(p)\,[1 - E_D]$





$p \in (0,60)$ PRÍJMY RASTÚ

$p \in (60,120)$ KLESAJÚ

$p = 60$ PRÍJEM JE MAXIMÁLNY

3) $R(p) = p \cdot q = p \cdot D(p) =$

$= p\,(240 - 2p) = 240p - 2p^2$

$R'(p) = 240 - 4p$

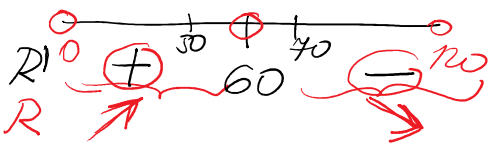
$R'(p) = 0 \qquad 240 - 4p = 0$

$R'(\mu) = 240 - 4\mu$

$R'(\mu) = 0 \qquad 240 - 4\mu = 0$

$4\mu = 240$

$\mu = 60$



METÓDA NULOVÝCH BODOV

$R'(50) = 240 - 4 \cdot 50 = 40 > 0$ $\quad \mu \in (0,60)$ $R' > 0$

$R'(70) = 240 - 4 \cdot 70 = -40 < 0$ $\quad \mu \in (60,120)$ $R' < 0$

KLASICKY

$R'(\mu) > 0 \qquad 240 - 4\mu > 0$

$240 > 4\mu$

$0 < \mu < 60$

$(R(\mu)$ RASTÚCA$)$

$R'(\mu) < 0 \qquad 240 - 4\mu < 0$

$240 < 4\mu$

$60 < \mu (< 120)$

$(R(\mu)$ KLESAJÚCA$)$

PR. $D(\mu) = 4000 - \mu^3$

$0 < \mu < \sqrt[3]{4000}$

$\mu_0 = 11$

$E_D(\mu_0) = ?$

$4000 - \mu^3 > 0$

$\mu^3 < 4000$

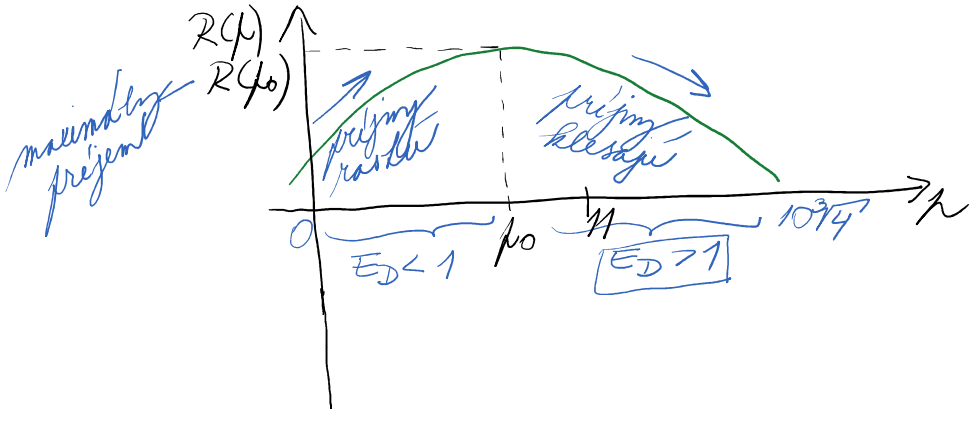
$\mu < \sqrt[3]{4000} = \sqrt[3]{4 \cdot 1000} = 10\sqrt[3]{4}$

$$E_D(\mu_0) = -\frac{D'(\mu_0)}{D(\mu_0)} \cdot \mu_0 = -\frac{-3\mu^2}{4000 - \mu^3} \cdot \mu = \frac{3\mu^3}{4000 - \mu^3}$$

$$E_D(11) = \frac{3 \cdot 11^3}{4000 - 11^3} = 1{,}49 > 1$$

DOPYT JE ELASTICKÝ

ODPORÚČAME ZNÍŽIŤ CENU (NA HODNOTU, KEĎ $E_D = 1$)



$E_D = 1$:

$$\frac{3p^3}{4000 - p^3} = 1 \qquad / \cdot (4000 - p^3)$$

$$3p^3 = 4000 - p^3 \qquad / + p^3$$

$$4p^3 = 4000 \qquad / : 4$$

$$p^3 = 1000$$

$$p_0 = 10$$

$\Longrightarrow$ JEDNOTKOVÁ ELASTICITA PRÍJMY MAXIMÁLNE

$$d f(x_0) = f'(x_0) \cdot \Delta x$$

$$M f(x_0) = f'(x_0) \qquad \boxed{\Delta x = 1}$$