DERIVAČNÉ VZORCE:

$(c)' = 0$

$(x^a)' = a \cdot x^{a-1}$

$a=1$: $(x)' = 1$

$(a^x)' = a^x \cdot \ln a$

$a=e$: $(e^x)' = e^x$

$(\log_a x)' = \frac{1}{x \cdot \ln a}$

$a=e$: $(\ln x)' = \frac{1}{x}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$ $\qquad \operatorname{tg} x = \frac{\sin x}{\cos x}$

$(\operatorname{cotg} x)' = -\frac{1}{\sin^2 x}$ $\qquad \operatorname{cotg} x = \frac{\cos x}{\sin x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ $\qquad (1-x^2) = (1-x) \cdot (1+x)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arccotg} x)' = -\frac{1}{1+x^2}$

PR. 1 str 4/1

$f(x) = 3x^4 - 7x^2 + 5x - 10$

$f'(x) = (3x^4 - 7x^2 + 5x - 10)' \overset{P2}{=}$

$= (3x^4)' - (7x^2)' + (5x)' - (10)' \overset{P1}{=}$

$= 3(x^4)' - 7 \cdot (x^2)' + 5 \cdot (x)' - 10 \cdot (1)' \overset{DV}{=}$

$= 3 \cdot 4 \cdot x^{4-1} - 7 \cdot 2 \cdot x^{2-1} + 5 \cdot 1 - 10 \cdot 0 =$

$= \boxed{12x^3 - 14x + 5}$

PR. 2 str 4/3

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$(x^a)'$

$\frac{1}{x^n} = x^{-n}$

$x^n = \frac{1}{x^{-n}}$

$f(x) = -\frac{3}{x^6} + 3 \cdot \sqrt[3]{x^2} + \frac{2}{\sqrt[2]{x^1}} =$

$= -3 \cdot x^{-6} + 3 \cdot x^{\frac{2}{3}} + 2 \cdot x^{-\frac{1}{2}}$

$f'(x) = -3 \cdot (-6) \cdot x^{-6-1} + 3 \cdot \frac{2}{3} \cdot x^{\frac{2}{3} - \frac{3}{3}} +$

$+ 2 \cdot \left(-\frac{1}{2}\right) \cdot x^{-\frac{1}{2} - \frac{2}{2}} =$

$= 18 \cdot x^{-7} + 2 \cdot x^{-\frac{1}{3}} - x^{-\frac{3}{2}} =$

$= \boxed{\frac{18}{x^7} + \frac{2}{\sqrt[3]{x}} - \frac{1}{\sqrt[2]{x^3}}}$

PR. 3 4/4

$x^m \cdot x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$f(x) = \frac{x}{\sqrt{x}} - \frac{\sqrt{x^3}}{\sqrt{x}} \overset{*}{=} x^1 \cdot x^{-\frac{1}{2}} - x^{\frac{3}{2}} \cdot x^{-\frac{1}{2}} =$

$= x^{1-\frac{1}{2}} - x^{\frac{3}{2} + \left(-\frac{1}{2}\right)} = x^{\frac{1}{2}} - x^1 \overset{*}{=}$

$\overset{*}{=} \frac{x^1}{x^{\frac{1}{2}}} - \frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = x^{1-\frac{1}{2}} - x^{\frac{3}{2} - \frac{1}{2}} = x^{\frac{1}{2}} - x$

$f'(x) = \frac{1}{2} \cdot x^{\frac{1}{2} - 1} - 1 = \frac{1}{2} \cdot x^{-\frac{1}{2}} - 1 =$

$= \boxed{\frac{1}{2 \cdot \sqrt{x}} - 1}$

PR. 4 4/6

$f(x) = 3 \cdot e^x - 5 \cdot 2^x + \frac{1}{3} \cdot \left(\frac{3}{2}\right)^x$

$f'(x) = \boxed{3 \cdot e^x - 5 \cdot 2^x \cdot \ln 2 + \frac{1}{3} \cdot \left(\frac{3}{2}\right)^x \cdot \ln \frac{3}{2}}$

PR. 5 4/8

$f(x) = 2 \cdot \ln x - 3 \cdot \log x + \frac{2}{5} \log_5 x$

$f'(x) = 2 \cdot \frac{1}{x} - 3 \cdot \frac{1}{x \cdot \ln 10} + \frac{2}{5} \cdot \frac{1}{x \cdot \ln 5}$

PR. 6 4/9

$f(x) = 2 \cdot \sin x - 4 \cdot \cos x + 3 \cdot \operatorname{tg} x + \operatorname{cotg} x$

$f'(x) = 2 \cdot \cos x - 4 \cdot (-\sin x) + 3 \cdot \frac{1}{\cos^2 x} + \frac{-1}{\sin^2 x}$

$= \boxed{2 \cos x + 4 \sin x + \frac{3}{\cos^2 x} - \frac{1}{\sin^2 x}}$

PR. 7 4/10

$f(x) = \frac{3}{4} \operatorname{arctg} x - \frac{1}{4} \operatorname{arccotg} x + \arcsin x$

$+ \arccos x$

$f'(x) = \frac{3}{4} \cdot \frac{1}{1+x^2} - \frac{1}{4} \cdot \frac{-1}{1+x^2} + \frac{1}{\sqrt{1-x^2}}$

$+ \frac{-1}{\sqrt{1-x^2}} = \frac{3}{4(1+x^2)} + \frac{1}{4(1+x^2)} + \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$

$= \frac{4}{4(1+x^2)} + \frac{0}{\sqrt{1-x^2}} = \boxed{\frac{1}{1+x^2}}$

PR. 8 4/12

$f(x) = \log x + 10^x - x^{10}$

$f'(x) = \boxed{\frac{1}{x \cdot \ln 10} + 10^x \cdot \ln 10 - 10 \cdot x^9}$

PR. 9 4/15

$f(x) = (e^x - 2^x) \cdot \operatorname{tg} x$

$f'(x) \overset{P3}{=} (e^x - 2^x)' \cdot \operatorname{tg} x + (e^x - 2^x) \cdot (\operatorname{tg} x)' =$

$= \boxed{(e^x - 2^x \cdot \ln 2) \cdot \operatorname{tg} x + (e^x - 2^x) \cdot \left(\frac{1}{\cos^2 x}\right)}$

PR. 10 4/17

$f(x) = (1-x) \cdot \ln x + e^x \ln x =$

$= (1 - x + e^x) \cdot \ln x$

$f'(x) = (0 - 1 + e^x) \cdot \ln x + (1 - x + e^x) \cdot$

$\cdot \frac{1}{x} = \boxed{(e^x - 1) \ln x + \frac{1 - x + e^x}{x}}$

PR. 11 4/18

$f(x) = \frac{\sin x}{x^2 + x + 1}$

$f'(x) \overset{P4}{=} \frac{(\sin x)' \cdot (x^2+x+1) - (\sin x) \cdot (x^2+x+1)'}{(x^2+x+1)^2}$

$= \frac{(x^2+x+1) \cdot \cos x - (2x+1) \cdot \sin x}{(x^2+x+1)^2}$

$= \boxed{\frac{\cos x}{x^2+x+1} - \frac{(2x+1) \cdot \sin x}{(x^2+x+1)^2}}$

PR. 12 4/19

$f(x) = \frac{3x+5}{\operatorname{cotg} x}$

$f'(x) = \frac{3 \cdot \operatorname{cotg} x - (3x+5) \cdot \frac{-1}{\sin^2 x}}{(\operatorname{cotg} x)^2} =$

$= \frac{3}{\operatorname{cotg} x} + \frac{\frac{3x+5}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} =$

$= \boxed{\frac{3}{\operatorname{cotg} x} + \frac{3x+5}{\cos^2 x}}$

PR. 13 4/22

$f(x) = e^{3x} + \frac{2}{5} \cos(5x) - 6 \sin \frac{x}{3}$

$(e^{\heartsuit})' = e^{\heartsuit} \cdot \heartsuit'$

$(\cos \heartsuit)' = -\sin \heartsuit \cdot \heartsuit'$

$f'(x) \overset{P5}{=} e^{3x} \cdot (3x)' + \frac{2}{5} \cdot (-\sin 5x) \cdot (5x)' -$

$- 6 \cdot \left(\cos \frac{x}{3}\right) \cdot \left(\frac{x}{3}\right)' =$

$= 3 \cdot e^{3x} - \frac{2 \cdot 5}{5} \cdot \sin 5x - 6 \cdot \frac{1}{3} \cdot \cos \frac{x}{3}$

$= \boxed{3e^{3x} - 2 \sin 5x - 2 \cos \frac{x}{3}}$