

MONOTÓNNOŠŤ FUNKCIE

1) $f(x) = x^4 - 4x^3$ $D(f) = \mathbb{R}$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

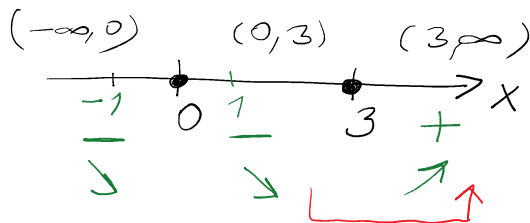
$f'(x) > 0 \Rightarrow$ RASTÚCA \nearrow

$f'(x) < 0 \Rightarrow$ KLESAJÚCA \searrow

METÓDA NULOVÝCH BODOV

$4x^2(x-3) = 0 \checkmark$

$x_1 = 0 \quad x_2 = 3$



x	$(-\infty, 0)$	0	$(0, 3)$	3	$(3, \infty)$
f'	$-$	0	$-$	0	$+$
f	\searrow		\searrow		\nearrow

FUNKCIA JE KLESAJÚCA NA: $(-\infty, 0) \cup (0, 3)$

FUNKCIA JE RASTÚCA NA: $(3, \infty)$

2) $f(x) = x \cdot \ln x$ $x > 0$ $D(f) = (0, \infty) = \mathbb{R}^+$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$ $\checkmark \ln x \stackrel{?}{=} m \ln x$

$f'(x) = 0$
 $\ln x + 1 = 0$

$\ln x = -1 \quad | e^{(\quad)}$

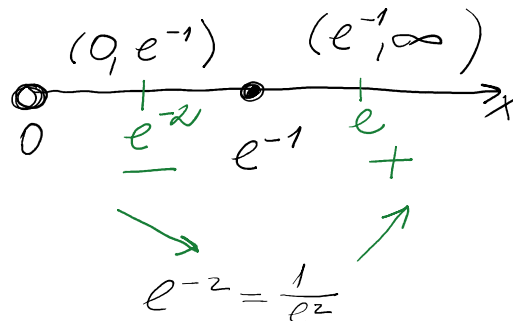
$f \circ f^{-1} = \ln x = e^{-1}$
 $x = e^{-1} = \frac{1}{e}$

inak: $\ln e = 1$
 $\ln x = -1 \cdot \ln e = -1$
 $\ln(x) = \ln e^{-1}$
 $x = e^{-1}$

$\checkmark f'(e^{-2}) = \ln(e^{-2}) + 1 = -2 + 1 = -1$

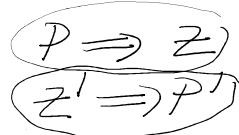
$\checkmark f'(e) = \ln e + 1 = 1 + 1 = 2$

x	$(0, e^{-1})$	e^{-1}	(e^{-1}, ∞)
f'	$-$	0	$+$
f	\searrow		\nearrow



FUNKCIA JE KLESAJÚCA NA: $(0, e^{-1})$

FUNKCIA JE RASTÚCA NA: (e^{-1}, ∞)



STACIONARNE BODY

STACIONARNE BODY

1) $f(x) = x^4 - 4x^3$ $D(f) = \mathbb{R}$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

S.B. $f'(x) = 0$ STACIONARNE BODY

$4x^2(x-3)$

$x_1 = 0$ $x_2 = 3$ ✓

2) $f(x) = x \cdot \ln x$ $D(f) = \mathbb{R}^+$

$f'(x) = \ln x + 1$

S.B. $f'(x) = 0$

$\ln x + 1 = 0$

$x = e^{-1}$

STACIONARNY BOD

3) $f(x) = x^2$ $D(f) = \mathbb{R}$

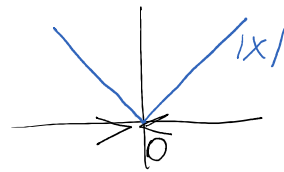
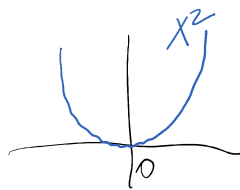
$f'(x) = 2x$

S.B. $f'(x) = 0$

$2x = 0$

$x = 0$

STAC. BOD



4) $f(x) = |x|$ $D(f) = \mathbb{R}$

$|x| = \begin{cases} x & x \geq 0 \checkmark \\ -x & x < 0 \checkmark \end{cases}$

$f'_+(0) = 1$
 $f'_-(0) = -1$ } $\neq f'(0)$

$f'_+(0) = x' = 1$
 $f'_-(0) = -x' = -1$

LOKÁLNE EXTRÉMY POMOCOU MONOTONNOSTI

1) $f(x) = x^4 - 4x^3$ $D(f) = \mathbb{R}$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

S.B. $f'(x) = 0$

$4x^2(x-3) = 0$

$x_1 = 0$ $x_2 = 3$

$f'(x) > 0 \rightarrow$

$f'(x) < 0 \downarrow$

	$(-\infty, 0)$	0	$(0, 3)$	3	$(3, \infty)$	
f'	-	0	-	0	+	
f	\downarrow	lok. extrém	\downarrow	lok. min	\uparrow	

f' MENENI ZNAHRENKO \uparrow MENI ZNAM.

FUNKCIA NEMÁ V BODE $x=0$
 FUNKCIA MÁ V BODE $x=3$

LOKÁLNY EXTRÉM
 LOKÁLNE MINIMUM
 $f(3) = 3^4 - 4 \cdot 3^3 = 3^3(3-4) = -27$

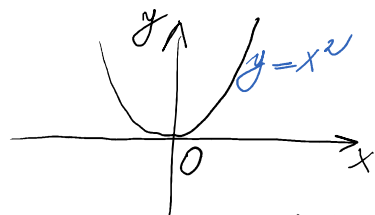
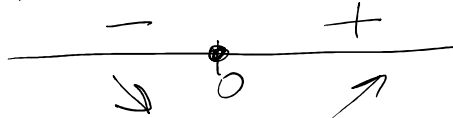
2) $f(x) = x \cdot \ln x$ $D(f) = \mathbb{R}^+$

	$(0, e^{-1})$	e^{-1}	(e^{-1}, ∞)
$f'(x)$	-	0	+
$f(x)$	↘	lok. min.	↗

FUNKCIA NADOBŮDA V BODE $x=e^{-1}$ LOKÁLNE MINIMUM
 $f(e^{-1}) = e^{-1} \cdot \ln e^{-1} = -e^{-1} = -\frac{1}{e}$

3) $f(x) = x^2$
 $f'(x) = 2x$

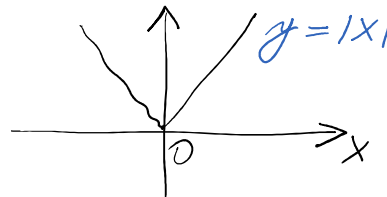
$D(f) = \mathbb{R}$



FUNKCIA NADOBŮDA V BODE $x=0$ LOKÁLNE MINIMUM
 $f(0) = 0$

4) $f(x) = |x|$

$x=0$ \nexists DERIVÁCIA (KRITICKÝ BOD)



$\forall x=0$ MÁ FUNKCIA LOKÁLNE MINIMUM $f(0) = 0$

URČENIE LOK. EXTRÉMU POMOCOU VYŠŠÍCH DERIVÁCIÍ

1) $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

s. B. $f'(x) = 0$

$4x^2(x-3) = 0$

$x_1 = 0$ $x_2 = 3$ ✓

$f''(x) = 12x^2 - 24x$

$f''(3) = 12 \cdot 9 - 24 \cdot 3 = 36 > 0 \Rightarrow$ LOKÁLNE MINIMUM

PÁRNA \Rightarrow V BODE $x=3$ MÁ EXTRÉM

$f''(0) = 0$ ✓

$f'''(x) = 24x - 24$

$f'''(0) = -24 \neq 0$

$$f'''(x) = 24x - 24$$

$$f'''(0) = -24 \neq 0$$

NEPÁRNA \Rightarrow \nexists EXTREM V BODE $x=0$

$$2) f(x) = x^5 - 5x^4 \quad D(f) = \mathbb{R}$$

$$f'(x) = 5x^4 - 20x^3$$

$$\text{s. B. } f'(x) = 0$$

$$5x^3(x-4) = 0$$

$$x_1 = 0 \quad x_2 = 4$$

$$f''(x) = 20x^3 - 60x^2$$

$$f''(0) = 0 \quad \checkmark$$

$$f'''(x) = 60x^2 - 120x$$

$$f'''(0) = 0 \quad \checkmark$$

$$f^{(4)}(x) = 120x - 120$$

$$f^{(4)}(0) = -120 < 0 \Rightarrow \text{LOKÁLNĚ MAXIMUM}$$

PÁRNA \Rightarrow V BODE $x=0$ LOKÁLNÝ EXTREM

PÁRNA \Rightarrow \exists EXTREM

$$f''(4) = 360 > 0$$

V BODE $x=4$ MÁ LOKÁLNĚ MINIMUM

$$3) f(x) = x \cdot \ln x \quad D(f) = \mathbb{R}^+$$

$$f'(x) = \ln x + 1 \quad \checkmark$$

$$\text{s. B. } x = e^{-1} = \frac{1}{e}$$

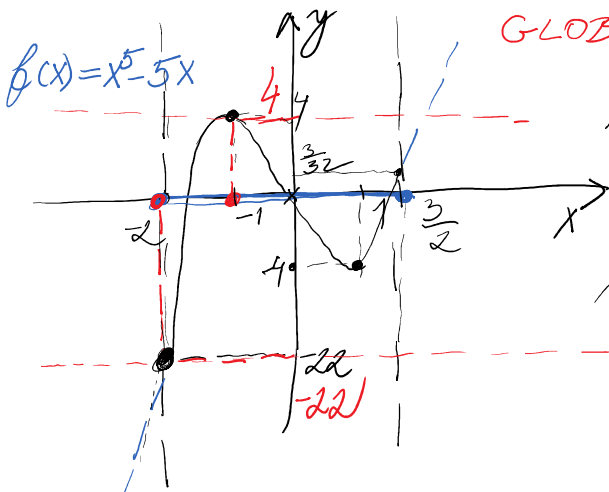
$$f''(x) = \frac{1}{x}$$

$$f''\left(\frac{1}{e}\right) = e > 0 \Rightarrow \text{L. MIN}$$

PÁRNA

MÁ V BODE $x = e^{-1}$ LOKÁLNĚ MINIMUM $f(e^{-1}) = e^{-1} \ln e^{-1} = -e^{-1}$

GLOBALNĚ EXTREMŮ



$$\begin{aligned} \min f(x) &= \min \left\{ f(-1), f(1), f(-2), f\left(\frac{3}{2}\right) \right\} \\ &\left\langle -2, \frac{3}{2} \right\rangle = \min \{ 4, -4, -22, \frac{3}{32} \} = -22 \end{aligned}$$

$$\begin{aligned} \max f(x) &= \max \left\{ f(-1), f(1), f(-2), f\left(\frac{3}{2}\right) \right\} \\ &\left\langle -2, \frac{3}{2} \right\rangle = \max \{ 4, -4, -22, \frac{3}{32} \} = 4 \end{aligned}$$

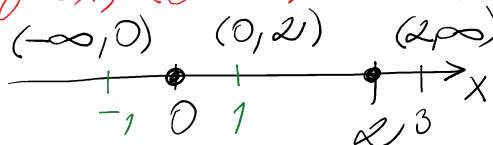
KONVEXNOST / KONKÁVNOST

1) $f(x) = x^4 - 4x^3$ $D(f) = \mathbb{R}$
 $f'(x) = 4x^3 - 12x^2$
 $f''(x) = 12x^2 - 24x = 12x(x-2)$

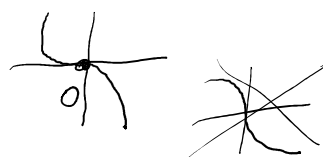
NULOVÉ BODY: $12x^2 - 24x = 0$
 $12x(x-2) = 0$
 $x_1 = 0$ $x_2 = 2$

x	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, \infty)$
f''	+	0	-	0	+
f	U	I.B.	∩	I.B.	U

$f''(x) > 0 \Rightarrow$ KONVEXNÁ U
 $f''(x) < 0 \Rightarrow$ KONKÁVNA ∩



$f''(x)$ + - +
 $f(x)$ U ∩ U
 ZMENAU. $\Rightarrow x=0$ je I.B.
 ZMENAZNAC. $\Rightarrow x=2$ je I.B.

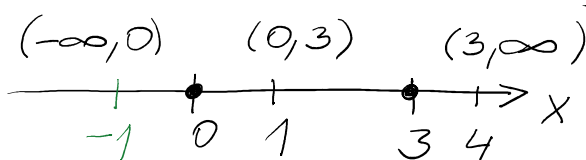


$x=0, x=2$ SÚ INFLEXNÉ BODY
 FUNKCIA JE KONVEXNÁ NA $(-\infty, 0) \cup (2, \infty)$
 FUNKCIA JE KONKÁVNA NA $(0, 2)$

2) $f(x) = x^5 - 5x^4$ $D(f) = \mathbb{R}$

$f'(x) = 5x^4 - 20x^3$
 $f''(x) = 20x^3 - 60x^2$
 N.B. $20x^2(x-3) = 0$
 $x_1 = 0$ $x_2 = 3$

	$(-\infty, 0)$	0	$(0, 3)$	3	$(3, \infty)$
f''	-	0	-	0	+
f	∩	X	∩	I.B.	U



$f''(x)$ - - +
 $f(x)$ ∩ ∩ U
 NIJE ZMENA ZMENA
 $x=0$ NIJE INFLEXNÝ BOD
 $x=3$ JE INFLEXNÝ BOD

FUNKCIA JE KONKÁVNA NA $(-\infty, 0) \cup (0, 3)$
 FUNKCIA JE KONVEXNÁ NA $(3, \infty)$

3) $f(x) = x \cdot \ln x$ $D(f) = \mathbb{R}^+ = (0, \infty)$

$f'(x) = \ln x + 1$
 $f''(x) = \frac{1}{x} > 0 \forall x \in D(f)$ $f''(x) = \frac{1}{x} \neq 0$

FUNKCIA JE KONVEXNÁ NA CELOM \mathbb{R}

FUNKCIA NEMÁ INFLEXNÉ BODY