RIEŠENIE SÚSTAVY ROVNÍC GAUSSOVOU ELIMINAČNOU METÓDOU:

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 &= 3 \\ 4x_1 + 3x_2 + 5x_3 &= 4 \\ 2x_1 + 3x_3 &= 2 \end{aligned}$$

$$\begin{pmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 2 & 0 & 3 \end{pmatrix} = A \quad h(A)$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ 4 & 3 & 5 & 4 \\ 2 & 0 & 3 & 2 \end{array}\right) = A' \quad h(A')$$

$\to n = 3$

FROBENIOVA VETA:

$h(A) \neq h(A')$ — nemá riešenie

$h(A) = h(A')$ — má riešenie

$h(A) = h(A') = n$ — má 1 riešenie

$h(A) = h(A') < n$ — nekonečne veľa riešení

$$\left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ \boxed{4} & 3 & 5 & 4 \\ \boxed{2} & 0 & 3 & 2 \end{array}\right) (-2)\ (-1) \sim \left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & \boxed{-3} & +1 & -1 \end{array}\right) (-1) \sim \left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 = 1$

$\boxed{0 = 1}$

$n = 3$

$h(A) = 2 \qquad h(A') = 3$

$h(A) \neq h(A')$

nemá riešenie

$$\begin{aligned} 2x_1 + 5x_2 + 4x_3 + x_4 &= 20 \\ x_1 + 3x_2 + 2x_3 + x_4 &= 11 \\ 2x_1 + 10x_2 + 9x_3 + 7x_4 &= 40 \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 &= 37 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 2 & 5 & 4 & 1 & 20 \\ 1 & 3 & 2 & 1 & 11 \\ 2 & 10 & 9 & 7 & 40 \\ 3 & 8 & 9 & 2 & 37 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 11 \\ 2 & 5 & 4 & 1 & 20 \\ 2 & 10 & 9 & 7 & 40 \\ \boxed{3} & 8 & 9 & 2 & 37 \end{array}\right) (-2)\ (-3) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 11 \\ 0 & -1 & 0 & -1 & -2 \\ 0 & \boxed{4} & 5 & 5 & 18 \\ 0 & \boxed{-1} & 3 & -1 & 4 \end{array}\right) /\cdot 4\ (-1) \sim \left(\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 11 \\ 0 & -1 & 0 & -1 & -2 \\ 0 & 0 & 5 & 1 & 10 \\ 0 & 0 & \boxed{3} & 0 & 6 \end{array}\right) (-3)\ /\cdot 5 \quad -3R_3 + 5R_4 \sim$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 11 \\ 0 & -1 & 0 & -1 & -2 \\ 0 & 0 & 5 & 1 & 10 \\ 0 & 0 & 0 & -3 & 0 \end{array}\right)$$

$h(A) = 4$

$h(A') = 4$

$n = 4$

$h(A) = h(A') = n$

MÁM PRÁVE JEDNO RIEŠENIE

$-3x_4 = 0$
$x_4 = 0$

$5x_3 + 1x_4 = 10$
$5x_3 + 1 \cdot 0 = 10$
$5x_3 = 10$
$x_3 = 2$

$-1 \cdot x_2 - 1 \cdot x_4 = -2$
$-1x_2 - 1 \cdot 0 = -2$
$-x_2 = -2$
$x_2 = 2$

$1x_1 + 3x_2 + 2x_3 + 1x_4 = 11$
$x_1 + 3 \cdot 2 + 2 \cdot 2 + 1 \cdot 0 = 11$
$x_1 + 10 = 11$
$x_1 = 1$

$$\boxed{(x_1; x_2; x_3; x_4)^T = (1; 2; 2; 0)^T}$$

$$\begin{aligned} x_1 + x_2 + 2x_3 - x_4 &= 2 \\ 2x_1 + x_2 + x_3 - 2x_4 &= 1 \\ -x_2 - x_3 + x_4 &= 0 \\ 3x_1 + 2x_2 + 3x_3 - 3x_4 &= 3 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 2 \\ \boxed{2} & 1 & 1 & -2 & 1 \\ 0 & -1 & -1 & 1 & 0 \\ \boxed{3} & 2 & 3 & -3 & 3 \end{array}\right) /\cdot(-2)\ (-3) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 2 \\ 0 & -1 & -3 & 0 & -3 \\ 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & -3 & 0 & -3 \end{array}\right) (-1) \sim \left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 2 \\ 0 & -1 & -3 & 0 & -3 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$h(A) = 3$

$h(A') = 3$

$n = 4$

$h(A) = h(A') < n$

MÁ NEKONEČNE VEĽA RIEŠENÍ

$2x_3 + x_4 = 3 \to x_3 = t \quad t \in \mathbb{R}$
$2t + x_4 = 3$
$x_4 = 3 - 2t$

$-x_2 - 3x_3 = -3$
$-x_2 - 3t = -3$
$-x_2 = -3 + 3t$
$x_2 = 3 - 3t$

$x_1 + x_2 + 2x_3 - x_4 = 2$
$x_1 + 3 - 3t + 2t - 3 + 2t = 2$
$x_1 + t = 2$
$x_1 = 2 - t$

$$(x_1; x_2; x_3; x_4)^T = (2 - t; 3 - 3t; t; 3 - 2t)^T \quad t \in \mathbb{R}$$

AK $t = 0$: $(2; 3; 0; 3)^T$

AK $t = 1$: $(1; 0; 1; 1)^T$

$$\begin{aligned} 12x_1 - 6x_2 + 9x_3 + 21x_4 &= 3 \\ 11x_1 - 5x_2 + 10x_3 + 24x_4 &= 1 \\ 7x_1 - 3x_2 + 7x_3 + 17x_4 &= 0 \\ 8x_1 - 6x_2 - x_3 - 5x_4 &= 9 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 12 & -6 & 9 & 21 & 3 \\ 11 & -5 & 10 & 24 & 1 \\ 7 & -3 & 7 & 17 & 0 \\ 8 & -6 & -1 & -5 & 9 \end{array}\right) (-1) \quad -R_2 + R_1$$

$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ \boxed{11} & -5 & 10 & 24 & 1 \\ \boxed{7} & -3 & 7 & 17 & 0 \\ 8 & -6 & -1 & -5 & 9 \end{array}\right) (-11)\ /\cdot(-7)\ /\cdot(-8) \sim \left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 0 & 6 & 21 & 57 & -21 \\ 0 & 4 & 14 & 38 & -14 \\ 0 & 2 & 7 & 19 & -7 \end{array}\right) \begin{array}{l} \\ :3 \\ :2 \\ \\ \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 2 & 7 & 19 & -7 \end{array}\right) (-1) \sim \left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$h(A) = 2$

$h(A') = 2$

$n = 4$

$h(A) = h(A') < n$

nekonečne veľa riešení

$2x_2 + 7x_3 + 19x_4 = -7 \qquad x_4 = t \qquad x_3 = s \qquad t, s \in \mathbb{R}$

$2x_2 + 7s + 19t = -7$

$x_2 = \frac{-7 - 7s - 19t}{2}$

$x_1 - x_2 - x_3 - 3x_4 = 2$

$x_1 - \frac{-7 - 7s - 19t}{2} - s - 3t = 2 \quad /\cdot 2$

$2x_1 + 7 + 7s + 19t - 2s - 6t = 4$

$2x_1 = -3 - 5s - 13t$

$x_1 = \frac{-3 - 5s - 13t}{2}$

$$(x_1; x_2; x_3; x_4)^T = \left(\frac{-3 - 5s - 13t}{2}; \frac{-7 - 7s - 19t}{2}; s; t\right)^T \quad s, t \in \mathbb{R}$$

AK $s = 0$, $t = 0$: $\left(-\frac{3}{2}; -\frac{7}{2}; 0; 0\right)^T$

AK $s = -2$, $t = 1$: $(-3; -6; -2; 1)^T$