$A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$ $2\times2$ $\quad |A| = \begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix} = 1\cdot6 - 2\cdot2 = 2$

$A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} \sim \begin{pmatrix} 2 & 6 \\ 1 & 2 \end{pmatrix} \quad \begin{vmatrix} 2 & 6 \\ 1 & 2 \end{vmatrix} = 2\cdot2 - 6\cdot1 = -2 = -|A|$

$A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}\cdot\frac{1}{2} \sim \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \quad \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1\cdot3 - 1\cdot2 = 1 = \frac{1}{2}|A|$

$A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}^{(-2)} \sim \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix} ✓ \quad \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 1\cdot2 - 2\cdot0 = 2$ ✓

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$B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ $3\times3$

$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{vmatrix} = (1\cdot1\cdot2 + 2\cdot1\cdot3 + 3\cdot2\cdot3) - (3\cdot3\cdot1 + 3\cdot1\cdot1 + 2\cdot2\cdot2) = 26 - 20 = 6$

## Laplaceov rozvoj determinantu

$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ $n \times n$ kde $n \geq 4$

podľa i-teho riadku: $|A| = a_{i1}(-1)^{i+1}D_{i1} + a_{i2}(-1)^{i+2}D_{i2} + a_{i3}(-1)^{i+3}D_{i3} + \dots + a_{in}(-1)^{i+n}D_{in}$

podľa j-teho stĺpca: $|A| = a_{1j}(-1)^{1+j}D_{1j} + a_{2j}(-1)^{2+j}D_{2j} + a_{3j}(-1)^{3+j}D_{3j} + \dots + a_{nj}(-1)^{n+j}D_{nj}$

$|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 1 & 3 & 3 & 2 \end{vmatrix}$

$i = 3$

$|A| = 1\cdot(-1)^{3+1}\cdot\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 2 \end{vmatrix} + 4\cdot(-1)^{3+2}\cdot\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \\ 1 & 3 & 2 \end{vmatrix} + 6\cdot(-1)^{3+3}\cdot\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 2 \end{vmatrix} + 8\cdot(-1)^{3+4}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 3 \end{vmatrix}$

$j = 1$

$|A| = 1\cdot(-1)^{1+1}\cdot\begin{vmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 3 & 3 & 2 \end{vmatrix} + 1\cdot(-1)^{2+1}\begin{vmatrix} 1 & 1 & 1 \\ 4 & 6 & 8 \\ 3 & 3 & 2 \end{vmatrix} + 1\cdot(-1)^{3+1}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 2 \end{vmatrix} + 1\cdot(-1)^{4+1}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix}$

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$|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 1 & 3 & 3 & 2 \end{vmatrix}$ $(-2)$ $\to -2R_2 + R_3$ $= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ -1 & 0 & 0 & 0 \\ 1 & 3 & 3 & 2 \end{vmatrix}$

$i = 3$

$|A| = -1\cdot(-1)^{3+1}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 2 \end{vmatrix} = -1\cdot1\cdot[(6+6+12) - (9+12+4)] = -1(-1) = 1$

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$|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 1 & 3 & 3 & 2 \end{vmatrix}$ $(-1)$ $= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 5 & 7 \\ 0 & 2 & 2 & 1 \end{vmatrix}$

$j = 1$

$|A| = 1\cdot(-1)^{1+1}\cdot\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 2 & 2 & 1 \end{vmatrix} = 1\cdot1\cdot[(5+18+28) - (30+14+6)] = 1(51-50) = 1$

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$x_1 + x_2 - x_3 = 2$
$3x_1 + 2x_2 - 2x_3 = 5$
$4x_1 - 3x_2 + 2x_3 = -1$

$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 3 & 2 & -2 & 5 \\ 4 & -3 & 2 & -1 \end{array}\right)$ $\quad D \neq 0$

$D = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 2 & -2 \\ 4 & -3 & 2 \end{vmatrix} = (4+9-8) - (-8+6+6) = 1 \neq 0$

$D_1 = \begin{vmatrix} 2 & 1 & -1 \\ 5 & 2 & -2 \\ -1 & -3 & 2 \end{vmatrix} = (8+15+2) - (2+12+10) = 1$

$D_2 = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 5 & -2 \\ 4 & -1 & 2 \end{vmatrix} = (10+3-16) - (-20+2+12) = 3$

$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 5 \\ 4 & -3 & -1 \end{vmatrix} = (-2-18+20) - (16-15-3) = 2$

$x_1 = \frac{D_1}{D} = \frac{1}{1} = 1 \quad x_2 = \frac{D_2}{D} = \frac{3}{1} = 3 \quad x_3 = \frac{D_3}{D} = \frac{2}{1} = 2$

$(x_1; x_2; x_3)^T = (1; 3; 2)^T$

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$x_1 + 2x_2 + 3x_3 = 0$
$-3x_1 - 2x_3 = -4$
$x_1 + 8x_2 + 3x_3 = 3$

$D = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 0 & -2 \\ 1 & 8 & 3 \end{vmatrix} = (0-72-4) - (0-16-18) = -42 \neq 0$

$D_1 = \begin{vmatrix} 0 & 2 & 3 \\ -4 & 0 & -2 \\ 3 & 8 & 3 \end{vmatrix} = (0-96-12) - (0+0-24) = -84$

$D_2 = \begin{vmatrix} 1 & 0 & 3 \\ -3 & -4 & -2 \\ 1 & 3 & 3 \end{vmatrix} = (-12-27+0) - (-12-6+0) = -21$

$D_3 = \begin{vmatrix} 1 & 2 & 0 \\ -3 & 0 & -4 \\ 1 & 8 & 3 \end{vmatrix} = (0+0-8) - (0-32-18) = 42$

$x_1 = \frac{D_1}{D} = \frac{-84}{-42} = 2 \quad x_2 = \frac{D_2}{D} = \frac{-21}{-42} = \frac{1}{2} \quad x_3 = \frac{D_3}{D} = \frac{42}{-42} = -1$

$(x_1; x_2; x_3)^T = (2; \frac{1}{2}; -1)^T$