PODMIENKY UDELENIA ZÁPOČTU

aspoň 9 účastí z 13 v dosť dlážke (70 min)

AUTOMATIZOVANÉ

MAIS → 7. 2. 2025

13. 1. 2025 → BN32, 6. posch., 615

KOMPLEXNÉ ČÍSLA

$N \subset Z \subset Q \subset R \subset C$

$C = \{ a + b \cdot i ;\ a, b \in R,\ i^2 = -1 \}$

i … imaginárna jednotka

$z = a + b \cdot i$

$a = Re\, z$ … reálna zložka KČ, $\in R$

$b = Im\, z$ … imaginárna zložka KČ, $\in R$

$i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = i^2 \cdot i = -i$

$i^4 = 1$, $i^5 = i$, $i^6 = -1$, $i^7 = -i$ …

$i^n = i^r$, r – zvyšok po delení 4, $r \in \{0, 1, 2, 3\}$

PR. 1:

a) $i^{82} = (i^4)^{20} \cdot i^2 = 1^{20} \cdot i^2 = -1$

b) $i^{31} = i^3 = -i$

KOMPLEXNE ZDRUŽENÉ ČÍSLO:

$z = a + b \cdot i \longrightarrow \bar{z} = a - b \cdot i$

$z \cdot \bar{z} = (a + b \cdot i) \cdot (a - b \cdot i) = a^2 - (bi)^2 = a^2 + b^2 \in R$

$(A + B) \cdot (A - B) = A^2 - B^2$

PR. 2:

a) $z = 1 - \sqrt{2} \longrightarrow \bar{z} = 1 - \sqrt{2}$

$Re\, z = 1 - \sqrt{2}$

$Im\, z = 0$

b) $z = \pi^2 \cdot i \longrightarrow \bar{z} = -\pi^2 \cdot i$

$Re\, z = 0$

$Im\, z = \pi^2$

c) $z = i^{53} - \sqrt{3} = -\sqrt{3} + i \rightarrow \bar{z} = -\sqrt{3} - i$

$Re\, z = -\sqrt{3}$

$Im\, z = 1$

OPERÁCIE S KČ (ALGEBR. TVARE):

$z_1 \pm z_2$, $k \cdot z$, $k \in R$, $z_1 \cdot z_2$, $\frac{z_1}{z_2}$

PR. 3:

$z_1 = -1 + \sqrt{3} \cdot i$

$z_2 = 2 - i$

a) $\sqrt{3} \cdot z_1 - z_2 = \sqrt{3} \cdot (-1 + \sqrt{3} \cdot i) - (2 - i) =$

$= -\sqrt{3} + 3 \cdot i - 2 + i = (-\sqrt{3} - 2) + 4i$

b) $z_2 \cdot \bar{z}_2 = (2 - i) \cdot (2 + i) = 2^2 + (-1)^2 = 5 =$

$= 4 + 2i - 2i - i^2 = 4 + 1 = 5$ ($-i^2$ = „$-(-1)$“)

c) $z_1 \cdot z_2 = (-1 + \sqrt{3} \cdot i) \cdot (2 - i) = -2 + i +$

$+ 2\sqrt{3}\, i - \sqrt{3} \cdot (-1) = (\sqrt{3} - 2) + (1 + 2\sqrt{3})\, i$

d) $\frac{(\sqrt{3} - 2) + (1 + 2\sqrt{3})\, i}{2 - i} = \frac{(\text{—〃—})}{2 - i} \cdot \frac{2 + i}{2 + i} =$ („1“)

$= \frac{[\sqrt{3} - 2 + (1 + 2\sqrt{3})\, i] \cdot (2 + i)}{5} = \frac{1}{5} \cdot (2\sqrt{3} - 4 + 2i + 4\sqrt{3}\, i +$

$+ \sqrt{3}\, i - 2i - 1 - 2\sqrt{3}) = \frac{1}{5} \cdot (-5 + 5\sqrt{3} \cdot i) = -1 + \sqrt{3}\, i$

Skúška: $\frac{z_1 \cdot z_2}{z_2} = z_1$ ✓ OK

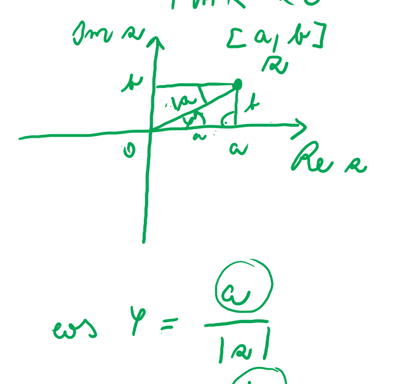
e) $(-1 + \sqrt{3} \cdot i)^5 = ?$ … MOIVROVA VETA

→ k tomu potrebujeme:

GONIOMETRICKÝ TVAR KČ

$z = a + b \cdot i = |z| \cdot (\cos \varphi + i \cdot \sin \varphi) = |z| \cdot e^{i \cdot \varphi}$

($a + b \cdot i$ … ALGEBRAICKÝ TVAR KČ; $|z| \cdot (\cos \varphi + i \cdot \sin \varphi)$ … GON. TVAR; $|z| \cdot e^{i \cdot \varphi}$ … EXP. TVAR KČ)



GAUSSOVA KOMPL. ROVINA

$|z| = \sqrt{a^2 + b^2}$ … MODUL (ABS. HODN.) KČ

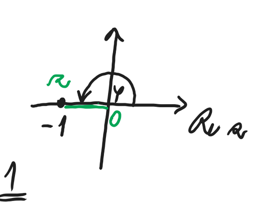
$\varphi \in \langle 0, 2\pi)$ … argument KČ

$\cos \varphi = \frac{a}{|z|}$

$\sin \varphi = \frac{b}{|z|}$

PR. 4: 4/12

b) $z = -1$

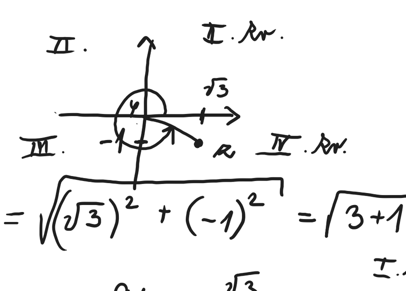


$z = 1 \cdot (\cos \pi + i \cdot \sin \pi)$

$|z| = |-1| = 1$

$\varphi = \pi$

f) $z = \sqrt{3} - i$



$|z| = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3 + 1} = 2$

$\varphi = ?$ $\cos \varphi = \frac{a}{|z|} = \frac{\sqrt{3}}{2}$ … I. kv. $\cos \varphi_0 = \frac{\sqrt{3}}{2}$

cos +

$\sin \varphi = \frac{b}{|z|} = \frac{-1}{2}$ … $\sin \varphi_0 = |-\frac{1}{2}| = \frac{1}{2}$

sin +

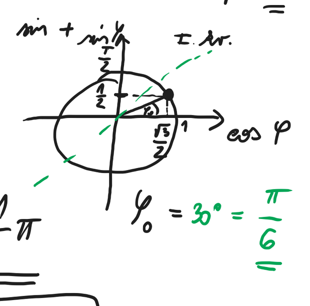
I. $\varphi_0 \in \langle 0; \frac{\pi}{2} \rangle$

II. $\varphi = \pi - \varphi_0$

III. $\varphi = \pi + \varphi_0$

IV. $\varphi = 2\pi - \varphi_0$

I. → IV. kv.: $\varphi = 2\pi - \frac{\pi}{6}$

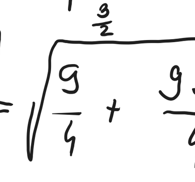


$\varphi = \frac{12\pi - \pi}{6} = \frac{11}{6}\pi$

$\varphi_0 = 30° = \frac{\pi}{6}$

$z = 2 \cdot (\cos \frac{11}{6}\pi + i \cdot \sin \frac{11}{6}\pi)$

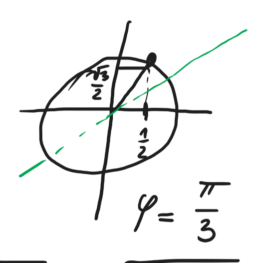
j) $z = \frac{3}{2} + \frac{3\sqrt{3}}{2} \cdot i$



$|z| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{9 \cdot 3}{4}} = \sqrt{\frac{36}{4}} = 3$

$\cos \varphi = \frac{\frac{3}{2}}{\frac{3}{1}} = \frac{3 \cdot 1}{2 \cdot 3} = \frac{1}{2}$

$\sin \varphi = \frac{3\sqrt{3}}{2} \cdot \frac{1}{3} = \frac{\sqrt{3}}{2}$



$\varphi = \frac{\pi}{3}$

$z = 3 \cdot (\cos \frac{\pi}{3} + i \cdot \sin \frac{\pi}{3})$

MOIVROVA VETA (VZOREC)

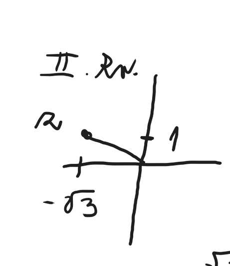
$z = a + b \cdot i = |z| \cdot (\cos \varphi + i \cdot \sin \varphi)$

$z^n = (a + bi)^n = |z|^n \cdot (\cos n \cdot \varphi + i \cdot \sin n \cdot \varphi)$

$n \in N$

PR. 5: 5/13

b) $(i - \sqrt{3})^4 = ?$

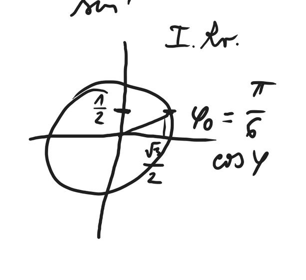


$z = -\sqrt{3} + i$

$|z| = \sqrt{3 + 1} = 2$

$\cos \varphi = \frac{-\sqrt{3}}{2}$

$\sin \varphi = \frac{1}{2}$



II. kv.: $\varphi = \pi - \varphi_0 = \pi - \frac{\pi}{6} = \frac{5}{6}\pi$

$z = 2 \cdot (\cos \frac{5}{6}\pi + i \cdot \sin \frac{5}{6}\pi)$

$z^4 = 2^4 \cdot (\cos 4 \cdot \frac{5}{6}\pi + i \cdot \sin \frac{20}{6}\pi) =$

$= 16 \cdot (\cos \frac{12 + 8}{6}\pi + i \cdot \sin (2\pi + \frac{4}{3}\pi)) =$

$= 16 \cdot (\cos \frac{4}{3}\pi + i \cdot \sin \frac{4}{3}\pi) =$

III. kv. → I. kv.: $\varphi = \pi + \varphi_0$

cos – sin – $\varphi_0 = \varphi - \pi$

$\varphi_0 = \frac{4}{3}\pi - \pi = \frac{\pi}{3}$

$= 16 \cdot (-\cos \frac{\pi}{3} - i \cdot \sin \frac{\pi}{3}) =$

$= 16 \cdot (-\frac{1}{2} - i \cdot \frac{\sqrt{3}}{2}) = -8 - 8\sqrt{3} \cdot i$