Priebeh funkcie

streda 4. decembra 2024 7:30

VYŠETRITE PRIEBEH A NAKRESLITE GRAF FUNKCIE

PRÍKLAD 2)

$f(x) = x^5 - 5x^4 \quad (= x^4(x-5))$

1) $D(f) = \mathbb{R}$

2) $O_y: \; x = 0 \Rightarrow y = f(0) = 0$ $\quad P_1[0,0]$ ✓

$O_x: \; y = 0 \Rightarrow x^5 - 5x^4 = 0$

$x^4 \cdot (x-5) = 0$ $\quad P_2[5,0]$ ✓

$x_1 = 0 \quad x_2 = 5$

$f(-x) = f(x)$ P

$f(-x) = -f(x)$ N

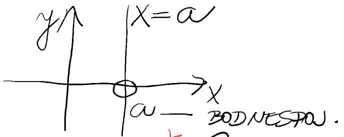
3) $\forall x \in D(f) \; -x \in D(f)$ ✓

$f(-x) = (-x)^5 - 5(-x)^4 = -x^5 - 5x^4 \neq f(x) \Rightarrow$ NIE JE PÁRNA

$-f(x) = -(x^5 - 5x^4) = -x^5 + 5x^4 \neq f(-x) \Rightarrow$ NIE JE NEPÁRNA

4) NEPERIODICKÁ

5) BODY NESPOJITOSTI (BN) NEMÁ $D(f) = \mathbb{R}$



6) AS:

ABS NEMÁ, PRETOŽE NEMÁ BN

ASS: $y = kx + q$ $\quad x \to \pm\infty$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{x^5 - 5x^4}{x} = \lim_{x\to\pm\infty} \frac{\overset{\pm\infty}{\nearrow}\;\overset{\pm\infty}{\nearrow}}{} \frac{x^{\cancel{4}\,3}(x-5)}{\cancel{x}} = \underset{\uparrow}{\infty}$$

NIE JE KONEČNÉ

$\Rightarrow \nexists$ ASS $\quad x \to \pm\infty$

7) $f'(x) = 5x^4 - 20x^3$

S.B.: $f'(x) = 0$

$5x^4 - 20x^3 = 0$

$5x^3(x-4) = 0$

$x_1 = 0 \quad x_2 = 4$

	$(-\infty,0)$	0	$(0,4)$	4	$(4,\infty)$
f'	+	0	−	0	+
f	↗	LOK MAX	↘	LOK. MIN	↗

f' $\quad + \quad \oplus_0 \quad - \quad \oplus_4 \quad +$

$P_3 = [4, -256]$ $\quad = f(4)$

8) FUNKCIA MÁ V BODE $\quad x = 0$ LOK. MAX. $f(0) = 0$

$x = 4$ LOK. MIN. $f(4) = -256$

9) $f''(x) = 20x^3 - 60x^2$

N.B. $\quad f''(x) = 0$

$20x^3 - 60x^2 = 0$

$20\boxed{x^2}(x-3)^1 = 0$

$x_1 = 0 \quad x_2 = 3$

	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f''	−	0	−	0	+
f	∩	—	∩	I.B.	∪

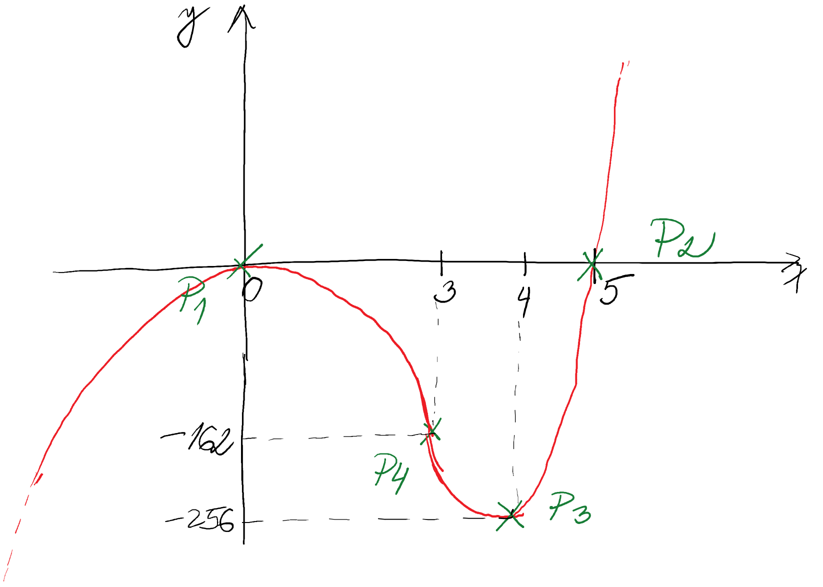
nie je I.B.

f'' $\quad - \quad \oplus_0 \quad - \quad \oplus_3 \quad +$

$P_4 = [3, -162]$ $\quad = f(3)$

10) $x = 3$ JE INFLEXNÝ BOD

10) $X=3$ JE INFLEXNÝ BOD



$H(f)=\mathbb{R}$

PRÍKLAD 3

$$f(x)=\frac{-x^3+x^2+4}{x^2}$$

1) $D(f): x^2\neq 0 \qquad D(f)=\mathbb{R}-\{0\} = (-\infty,0)\cup(0,\infty)$

2) $o_y: x=0 \notin D(f)$

$o_x: y=0 \Rightarrow \frac{-x^3+x^2+4}{x^2}=0$

$-x^3+x^2+4=0 \quad /\cdot(-1)$

$x^3-x^2-4=0$

DELITELE $\boxed{-4}$ → $\alpha\in\{\pm1,\pm2,\pm4\}$

	1	−1	0	−4
2		2	2	4
	1	1	2	$\boxed{0}$

$x^3-x^2-4=(x-2)(x^2+x+2)$

$D=-7$

✓ $x^3-8-x^2+4=0$

$x^3-2^3=(x^3-8)-(x^2-4)=0$

$a^3-b^3=(a-b)(a^2+ab+b^2)$ ✓

$a^2-b^2=(a-b)(a+b)$

$(x-2)(x^2+2x+4)-(x-2)(x+2)=0$

$(x-2)[x^2+2x+4-(x+2)]=0$

$(x-2)\underbrace{(x^2+x+2)}_{\neq 0}=0 \qquad x=2 \qquad P_1[2,0]$

3) $\forall x\in D(f)\ -x\in D(f)$

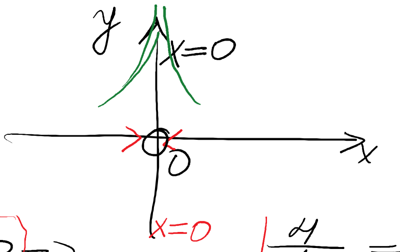
$f(-x)=\frac{-(-x)^3+(-x)^2+4}{(-x)^2}=\frac{x^3+x^2+4}{x^2}\neq f(x)$ ~~P~~

$-f(x)=-\frac{-x^3+x^2+4}{x^2}=\frac{+x^3-x^2-4}{x^2}\neq f(-x)$ ~~N~~

4) NEPERIODICKÁ

5) BN: $x=0$

6) ABS: $x=0$ (PRIAMKA $\parallel o_y$)



$\lim\limits_{x\to 0}\frac{-\overset{\to 0}{x^3}+\overset{\to 0}{x^2}+4 \to 4}{x^2\to 0^+}=\boxed{\infty}\Rightarrow$

$\frac{4}{\frac{1}{10}}=40$

$$\lim_{x\to 0}\frac{-x^3+x^2+4 \to 4}{x^2 \to 0^+} = \infty \Rightarrow$$ $x=0$

$\boxed{x=0 \text{ JE ASS}}$

$$\frac{4}{\frac{1}{10}} = 40 \qquad \frac{4}{\frac{1}{100}} = 400$$

!

ASS: $y = kx+q$

$$k = \lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{\frac{-x^3+x^2+4}{x^2}}{\frac{x}{1}} =$$

$$= \lim_{x\to\infty}\frac{-x^3+x^2+4}{1\cdot x^3} \overset{-\frac{\infty}{\infty}}{\underset{L'H}{=}} l.\lim_{x\to\infty}\frac{-3x^2+2x}{3x^2} \overset{-\frac{\infty}{\infty}}{\underset{L'H}{=}}$$

-1

$$= l.\lim_{x\to\infty}\frac{-6x+2}{6x} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty}\frac{-6}{6} = -1$$ ✓

$$q = \lim_{x\to\infty}(f(x)-k\cdot x) = \lim_{x\to\infty}\left(\frac{-x^3+x^2+4}{x^2} - (-1)\cdot x\right) =$$

$-\infty+\infty$

$+x$

$$= \lim_{x\to\infty}\frac{\cancel{-x^3}+x^2+4\ \cancel{+x\cdot x^2}}{x^2} = \lim_{x\to\infty}\frac{x^2+4}{x^2} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} l.\lim_{x\to\infty}\frac{2x}{2x}$$

$$= \boxed{1}$$ ✓

$y = -1\cdot x + 1$ ASS $x \to \pm\infty$

7) $$f'(x) = \frac{(-3x^2+2x)\cdot x^2 - (-x^3+x^2+4)\cdot 2x}{x^4} =$$

$$= \frac{x\left[-3x^3+2x^2+2x^3-2x^2-8\right]}{x^{\cancel{4}3}} =$$

$$= \frac{-x^3-8}{x^3} = \frac{-(x^3+8)}{x^3} = -\frac{(x+2)\overbrace{(x^2-2x+4)}^{>0}}{x^3}$$

S.B. $f'(x)=0$

$$-\frac{(x^3+8)}{x^3} = 0$$

$$x^3+8 = 0$$ $D=-12$

$$(x+2)\underbrace{(x^2-2x+4)}_{\neq 0} = 0$$

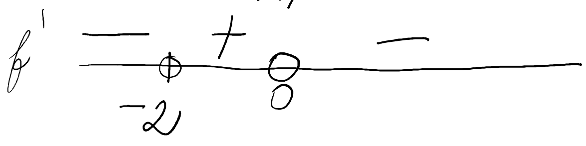
$$x_1 = -2$$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

delitele 8

$\pm1, \pm2, \pm4, \pm8$

	$(-\infty,-2)$	-2	$(-2,0)$	$(0,\infty)$
f'	$-$	0	$+$	$-$
f	↘	LOK. MIN	↗	↘



$P_2[-2,4]$ $=f(-2)$

8) FUNKCIA MÁ V BODE $x=-2$ LOK. MIN. $f(-2)=4$

$8x^{-3}$

9) $$f''(x) = \left[\frac{-(x^3+8)}{x^3}\right]' = -1\left[\frac{x^3+8}{x^3}\right]' = -\left(1+\frac{8}{x^3}\right)' = -(1+8\cdot x^{-3})'$$

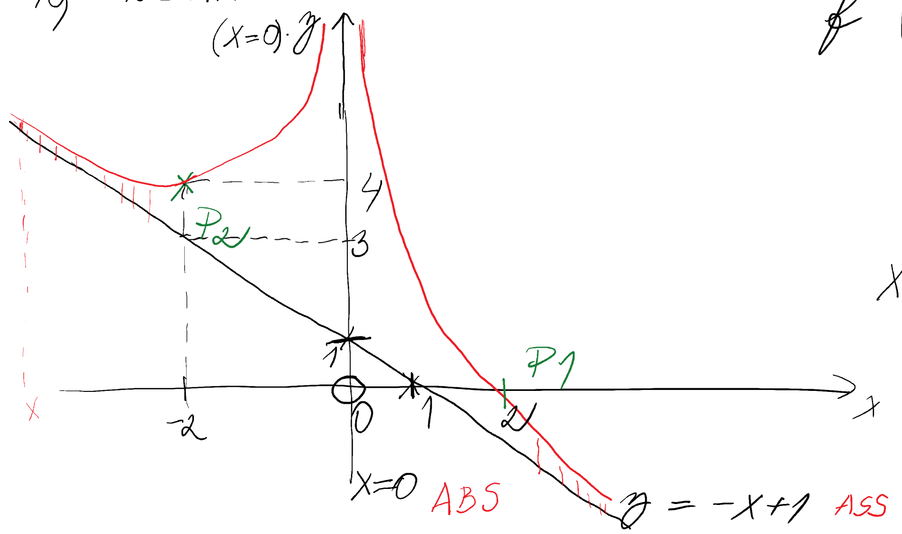
$$= -\left(8\cdot(-3)\cdot x^{-4}\right) = \frac{24}{x^4} > 0$$

	$(-\infty,0)$	$(0,\infty)$
f''	$+$	$+$

10) NEMÁ INFLEXNÉ BODY

f''	$+$	$+$
f	$\cup$	$\cup$

10) NEMÁ INFLEXNÉ BODY



$y = -x+1$

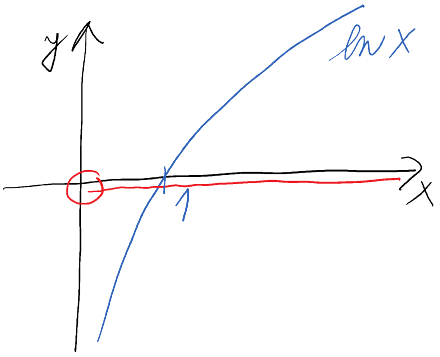
$f(x) = -\dfrac{x^3+x^2+4}{x^2}$

$x=-2: \quad y = -(-2)+1 = 3$

$f(-2) = 4$

PRÍKLAD 4

$f(x) = x \cdot \ln x$



1) $D(f): \ x>0 \qquad D(f) = (0,\infty) = R^+$

2) $\sigma_y: \ x=0 \notin D(f)$

$\sigma_x: \ y=0 \qquad x \cdot \ln x = 0$

$x=0 \ \vee \ \ln x = 0$

$\notin D(f) \qquad x=1$

$P_1 = [1,0]$

3) $\forall x \in D(f) \ -x \in D(f)$ NEPLATÍ, ALE PLATÍ NEGÁCIA:

$\exists x \in D(f) \ -x \notin D(f) \implies$ ANI $\cancel{P}$ ANI $\cancel{N}$

4) NEPERIODICKÁ

5) BODY NESPOJITOSTI NEMÁ

~~$x \to 0^-$~~

6) $\boxed{ABS}$ $\quad x=0$

$D(f) = (0,\infty)$

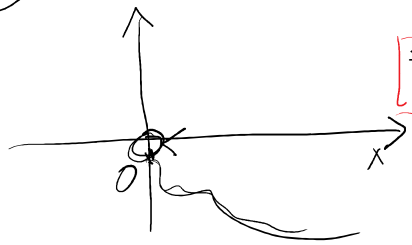
$\lim\limits_{x\to 0^+} f(x) =$

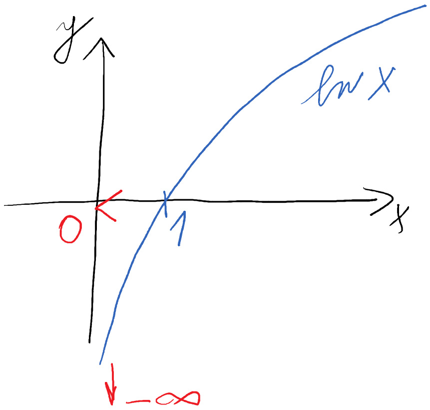
$= \lim\limits_{x\to 0^+} \overset{\to 0^+}{x} \cdot \overset{\to -\infty}{\ln x} = \lim\limits_{x\to 0^+} \dfrac{\overset{-\infty}{\ln x}}{\underset{\to\infty}{\frac{1}{x}}} = \dfrac{-\infty}{\infty} =$

$\underset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+} \overset{-x}{\dfrac{-x^2}{x}} = 0$

KONEČNÉ

$\boxed{\implies x=0 \text{ NIE JE ABS}}$





ASS $\quad y = kx+q \qquad x \to \infty$

$x \to -\infty$ NEMÔŽE

$D(f) = (0,\infty)$

ASS $y = kx + q$ $x \to \infty$ $\to\infty$

$x \to -\infty$ NEMÔŽE
$D(f) = (0, \infty)$

$k = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \frac{\cancel{x} \ln x}{\cancel{x}} = \infty$ NIE JE KONEČNÉ

$\Rightarrow$ ASS $\nexists$

7) $f'(x) = 1 \ln x + x \cdot \frac{1}{x} = \ln x + 1$

S.B.: $f'(x) = 0$
$\ln x + 1 = 0$
$\ln x = -1 \;/e^{()}$
$x = e^{-1} = \frac{1}{e}$

	$(0, e^{-1})$	e^{-1}	(e^{-1}, ∞)
f'	$-$	0	$+$
f	↘	LOK MIN.	↗

$\ln x = -1 \;/e^{()}$
$e^{\ln x} = e^{-1}$
$x = e^{-1}$

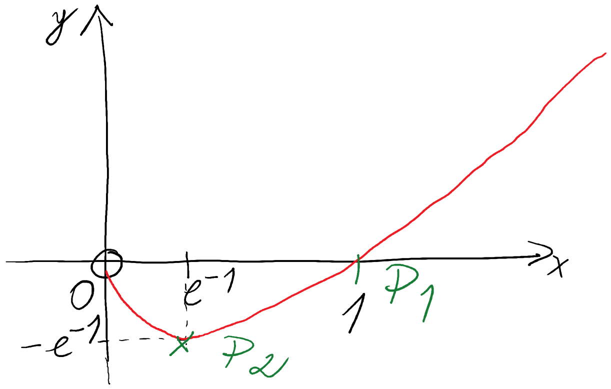
$P_2[e^{-1}; -e^{-1}]$

8) FUNKCIA MÁ V BODE $x = e^{-1}$ LOK. MIN. $f(e^{-1}) = -e^{-1}$

$\ln e^{-2} + 1 = -2 \ln e + 1$
$= -2 + 1 = -1$

9) $f''(x) = (\ln x + 1)' = \frac{1}{x} > 0$ $\forall x \in D(f)$ $\cup$
$= (0, \infty)$

10) NEMÁ INFLEXNÝ BOD



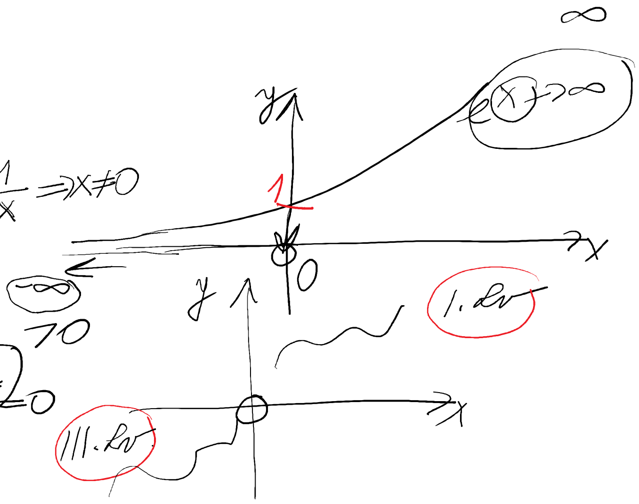
$H(f) = \langle -e^{-1}, \infty)$

PRIKLAD 5

$f(x) = x \cdot e^{\frac{1}{x}}$ > 0

$\frac{1}{x} \Rightarrow x \neq 0$

1) $D(f) = R - \{0\}$

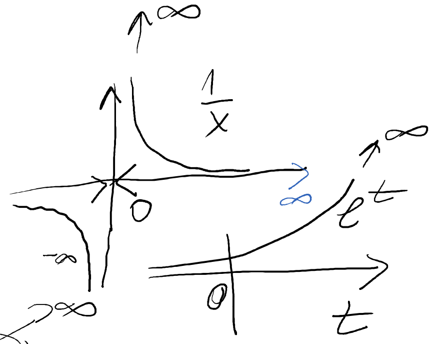


$H(f) = \mathbb{R}^+$

2) σ_y: $x = 0 \notin D(f)$
σ_x: $y = 0$ $x \cdot e^{\frac{1}{x}} = 0$ > 0
$x = 0 \notin D(f)$ $\vee$ $e^{\frac{1}{x}} = 0$ > 0
$\emptyset$ $\emptyset$

NEMÁ PRIESEKY S OSAMI

3) $\forall x \in D(f)$ $\exists -x \in D(f)$
$f(-x) = -x\, e^{-\frac{1}{x}} \neq f(x)$ ANI ~~P~~ ANI ~~N~~
$-f(x) = -x e^{\frac{1}{x}} \neq f(-x)$



4) NEPERIODICKÁ

5) BN $x = 0$

2^{∞} 1^{∞} ∞ $\to\infty$ $\left(\frac{1}{\right)$

4) NEPERIODICKA

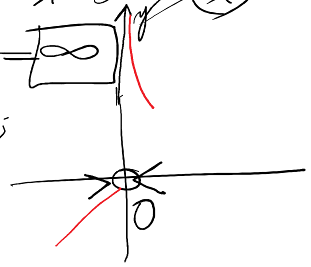
$''$ $\quad 0 \quad t'$

5) BN $\quad x=0$

6) ABS $\quad x=0$

$$\lim_{x\to0^+} \underset{\to 0}{x}\cdot \underbrace{e^{\frac{1}{x}\to\infty}}_{\to\infty} \overset{0\cdot\infty}{=} \lim_{x\to0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to0^+} \frac{e^{\frac{1}{x}}\cdot\cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \lim_{x\to0^+} e^{\frac{1}{x}} = \boxed{\infty}$$

$\Longrightarrow$ $x=0$ JE ABS;



$$\lim_{x\to0^-} \underset{\to0}{x}\, \left(e^{\frac{1}{x}\to -\infty}\right)\to 0 = 0\cdot0 = \underline{\underline{0}}$$

ASS $\quad y = kx+q$

$$k = \lim_{x\to\infty} \frac{\cancel{x}e^{\frac{1}{x}\to0}}{\cancel{x}} = e^0 = \boxed{1}$$

$$q = \lim_{x\to\infty}\left(xe^{\frac{1}{x}} - 1\cdot x\right) = \lim_{x\to\infty} \underset{\to\infty}{x}\left(e^{\frac{1}{x}}-1\right)_{\to 0} \overset{\infty\cdot0}{=}$$

$$= \lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{e^{\frac{1}{x}\to0}\cdot\cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \boxed{1}$$

$$k=\lim_{x\to\infty}\frac{f(x)}{x}$$

$$q=\lim_{x\to\infty}\left(f(x)-k\cdot x\right)$$

$$y=kx+q$$

$$y = 1\cdot x + 1 \qquad x\to\pm\infty$$

7) $f'(x) = e^{\frac{1}{x}} + \cancel{x}\cdot e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^{\cancel{2}}}\right) = e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x} =$ $\quad = \left(\frac{1}{x}\right)' = \left(x^{-1}\right)'$

$$= e^{\frac{1}{x}}\left(1-\frac{1}{x}\right) = \underset{>0}{e^{\frac{1}{x}}}\left(\frac{x-1}{x}\right) \qquad \neq 0$$

S.B. $\quad f'(x)=0$

$\frac{x-1}{x}=0$

$x-1=0$

$x=1$

	$(-\infty,0)$	$(0,1)$	1	$(1,\infty)$
f'	$+$	$-$	0	$+$
f	↗	↘	LOK MIN	↗

8) FUNKCIA MÁ V BODE $x=1$ LOK. MIN. $f(1)=1\cdot e^{\frac{1}{1}}=e$

$$P_1=[1,e]$$

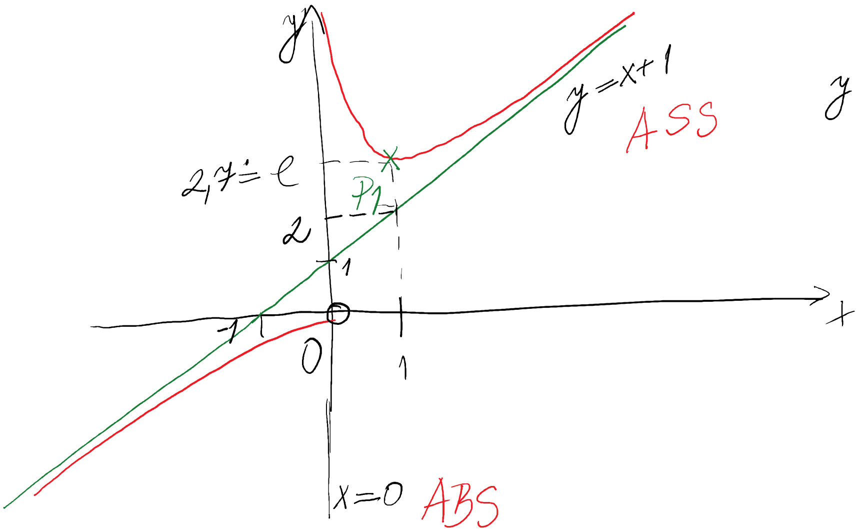
$$\left(1-x^{-1}\right)' = -1\left(-1x^{-2}\right)$$

9) $f''(x) = \left[\left(1-\frac{1}{x}\right)e^{\frac{1}{x}}\right]' = \boxed{\frac{1}{x^2}}\,\boxed{e^{\frac{1}{x}}} + \left(1-\frac{1}{x}\right)\boxed{e^{\frac{1}{x}}}\left(-\boxed{\frac{1}{x^2}}\right) =$ $\quad -\left(1-\frac{1}{x}\right) = \left(-1+\frac{1}{x}\right)$

$$= \frac{1}{x^2}e^{\frac{1}{x}}\left(\underbrace{1-1+\frac{1}{x}}\right) = \frac{e^{\frac{1}{x}}}{x^3} \qquad >0 \quad (\neq 0) \quad \nexists NB$$

10) NEMÁ INFLEXNÉ BODY

	$(-\infty,0)$	$(0,\infty)$
f''	$-$	$+$
f	$\cap$	$\cup$



$y=1+1=2$