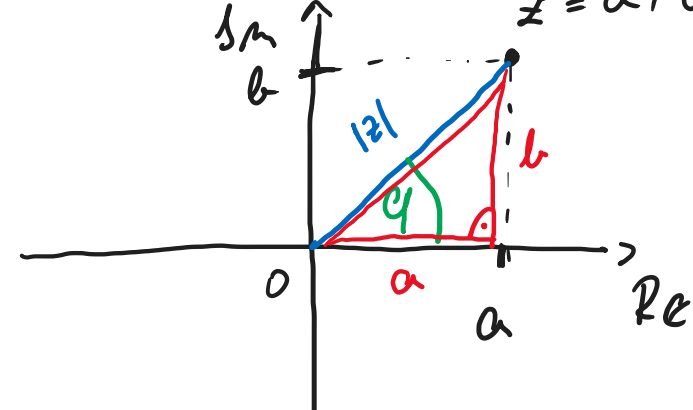


$$z = a + bi \quad a, b \in \mathbb{R}$$

$$\operatorname{Re}(z) = a$$

$$\operatorname{Im}(z) = b$$



$$\sqrt{4} = 2$$

$$\sqrt{-4} = \sqrt{(i) \cdot 4} = \sqrt{(i)} \cdot \sqrt{4} = i \cdot 2$$

$$i = \sqrt{-1} \quad i^2 = -1$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\sin \varphi = \frac{b}{|z|}$$

$$\cos \varphi = \frac{a}{|z|}$$

$$\left. \begin{aligned} i^1 &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \\ i^5 &= i \\ i^6 &= -1 \\ i^7 &= -i \\ i^8 &= 1 \\ i^9 &= i \end{aligned} \right\}$$

$$z_1 = 5 - 5\sqrt{3}i \quad \operatorname{Re}(z_1) = 5 = a_1 \quad \operatorname{Im}(z_1) = -5\sqrt{3} = b_1 \quad z_1 = a_1 + b_1 i$$

$$z_2 = \sqrt{3} + i \quad \operatorname{Re}(z_2) = \sqrt{3} = a_2 \quad \operatorname{Im}(z_2) = 1 = b_2 \quad z_2 = a_2 + b_2 i$$

$$z = z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$

$$z_1 + z_2 = 5 - 5\sqrt{3}i + \sqrt{3} + i = (5 + \sqrt{3}) + (1 - 5\sqrt{3})i$$

$$z_1 - z_2 = (5 - 5\sqrt{3}i) - (\sqrt{3} + i) = (5 - \sqrt{3}) + (-1 - 5\sqrt{3})i$$

$$(5 - 5\sqrt{3}i) - (\sqrt{3} + i)$$

$$z_1 \cdot z_2 = (a_1 + b_1 i)(a_2 + b_2 i) = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i$$

$$z_1 \cdot z_2 = (5 - 5\sqrt{3}i) \cdot (\sqrt{3} + i) = 5\sqrt{3} - 5\sqrt{3}\sqrt{3}i + 5i - 5\sqrt{3}(i \cdot i) = 5\sqrt{3} - 15i + 5i + 5\sqrt{3} = 10\sqrt{3} - 10i$$

$$\frac{z_2}{z_1} = \frac{\sqrt{3} + i}{5 - 5\sqrt{3}i} \cdot \frac{5 + 5\sqrt{3}i}{5 + 5\sqrt{3}i} = \frac{(\sqrt{3} + i)(5 + 5\sqrt{3}i)}{(5 - 5\sqrt{3}i)(5 + 5\sqrt{3}i)} = \frac{5\sqrt{3} + 5i + 5\sqrt{3}\sqrt{3}i + 5\sqrt{3}(i \cdot i)}{25 + 75} = \frac{0 + 20i}{100} = 0.2i$$

$$1 = \frac{z}{z} = \frac{-\sqrt{5}}{-\sqrt{5}} = \frac{x+3}{x+3} = \frac{\heartsuit}{\heartsuit}$$

$$a = 5 \rightarrow a^2 = 25$$

$$b = -5\sqrt{3} \rightarrow b^2 = 25 \cdot 3 = 75$$

$$\begin{aligned} z &= a + bi \\ \bar{z} &= a - bi \end{aligned}$$

$$z \cdot \bar{z} = (a + bi)(a - bi) = a^2 - b^2 i^2 = a^2 + b^2 \quad a, b \in \mathbb{R}$$

$$(r + ik)(r - ik) = r^2 - r^2 i^2 = r^2 + r^2 = 2r^2$$

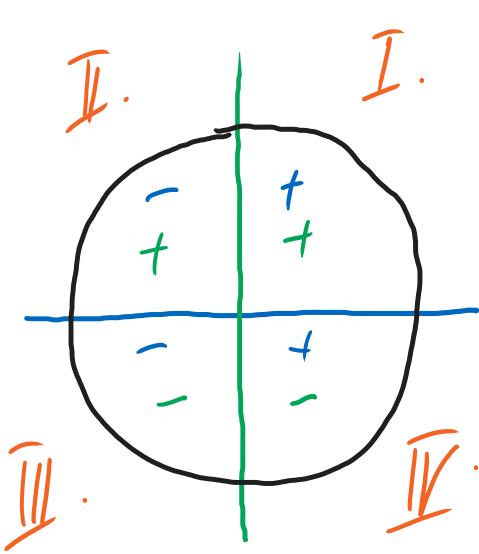
$$i^{-1} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

$$i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$$

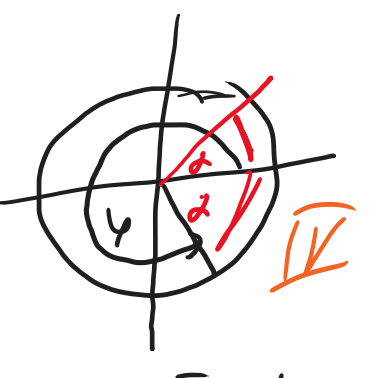
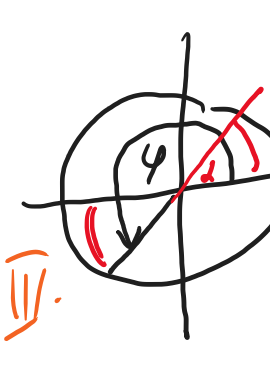
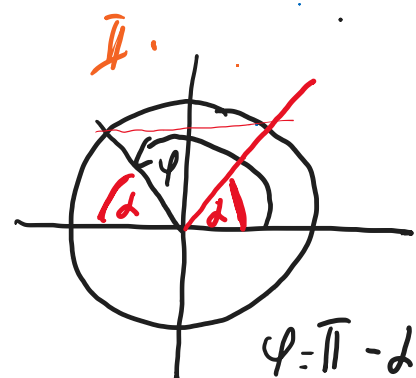
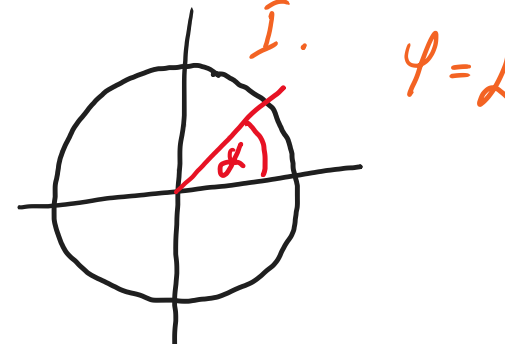
$$i^{-3} = \frac{1}{i^3} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = i$$

$$z = a + bi = |z| (\cos \varphi + i \sin \varphi) = |z| e^{i\varphi}$$

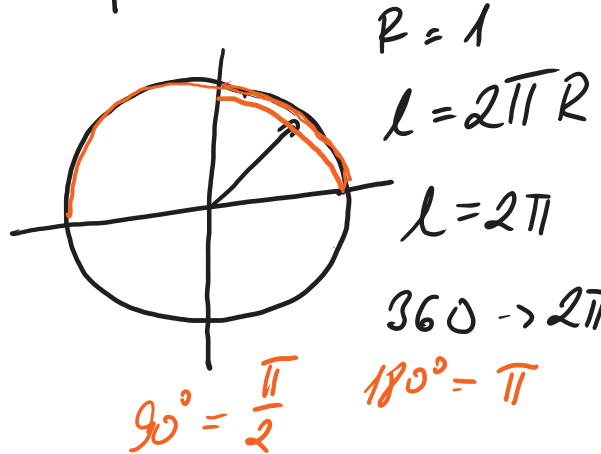
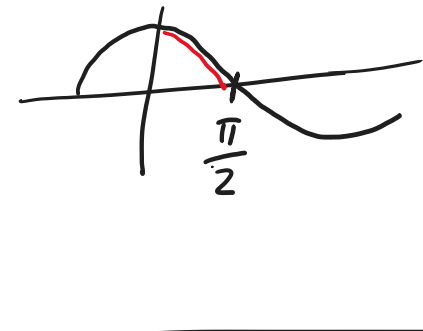
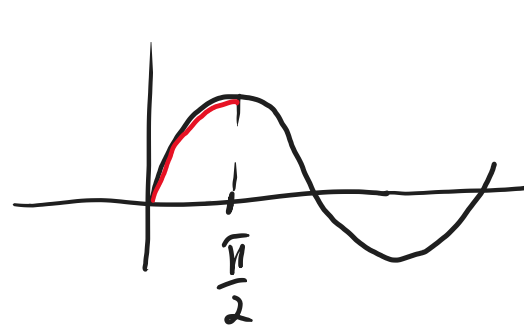
ALGEBRAICKY VVAR GONOMETRICKY VVAR EXPONENCIALNY VVAR



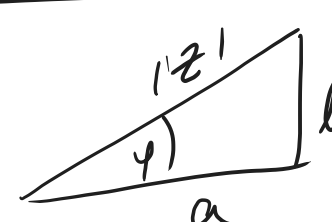
$$\begin{aligned} \cos \varphi \\ \sin \varphi \end{aligned}$$



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\sin \varphi$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$	0	-1	0
$\cos \varphi$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{0}}{2} = 0$	-1	0	1



$$z_1 = 5 - 5\sqrt{3}i$$



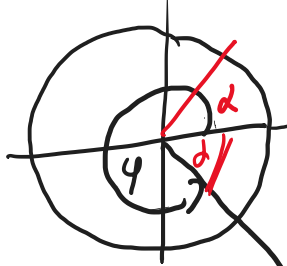
$$z = |z| (\cos \varphi + i \sin \varphi)$$

$$|z| = \sqrt{5^2 + (-5\sqrt{3})^2} = \sqrt{25 + 75} = 10$$

$$|z| = 10$$

$$\cos \varphi = \frac{a}{|z|} = \frac{5}{10} = \frac{1}{2}$$

$$\sin \varphi = \frac{b}{|z|} = \frac{-5\sqrt{3}}{10} = -\frac{\sqrt{3}}{2}$$



$$\varphi = 2\pi - \alpha$$

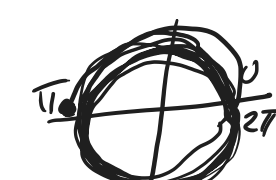
$$\varphi = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$z_1 = 5 - 5\sqrt{3}i = 10 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 10 \cdot e^{i \frac{5\pi}{3}}$$

$$z^n = |z|^n (\cos n\varphi + i \sin n\varphi)$$

$$z_1^3 = (5 - 5\sqrt{3}i)^3 = -10^3$$

$$z_1^3 = 10^3 (\cos 9 \cdot \frac{5\pi}{3} + i \sin 9 \cdot \frac{5\pi}{3}) = 10^3 (\cos 15\pi + i \sin 15\pi) = 10^3 (1 + 0i)$$



$$z_2 = \sqrt{3} + i$$

$$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = 2$$

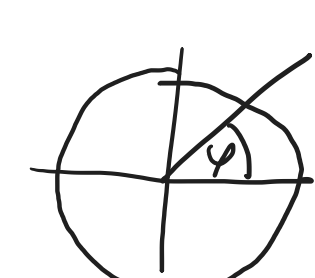
$$z_2^{12} = 2^{12}$$

$$\cos \varphi = \frac{\sqrt{3}}{2}$$

$$\sin \varphi = \frac{1}{2}$$

$$z_2 = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$z_2^{12} = 2^{12} \left(\cos 12 \cdot \frac{\pi}{6} + i \sin 12 \cdot \frac{\pi}{6} \right) = 2^{12} \left(\cos 2\pi + i \sin 2\pi \right) = 2^{12}$$



$$z^4 = 4^4 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

$$z^4 = 4^4 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

$$\frac{4\pi}{3} = \pi + \left(\frac{\pi}{3} \right) = \alpha$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

