

$\frac{P(x)}{Q(x)}$ dt. $P(x) = r$ $r \geq 3 \rightarrow$ racionálna časť
 dt. $Q(x) = s$ $r < 3 \rightarrow$ iracionálna časť

$$\frac{A}{(ax+b)^n} = \frac{Bx+c}{(ax+b)^n} = \frac{A}{(ax+b)} + \frac{Bx+c}{(ax+b)^2}$$

$$\frac{P(x)}{(4x+3)^5} = \frac{A}{4x+3} + \frac{B}{(4x+3)^2} + \frac{C}{(4x+3)^3} + \frac{D}{(4x+3)^4} + \frac{E}{(4x+3)^5}$$

$$P_n: \frac{2x^2+5x^3-x^2+12x+14}{x^4+2x^3-x^2+4x+12} = 2 + \frac{x^2+x^2+4x-10}{x^4+2x^3-x^2+4x+12}$$

$$d) (2x^2+5x^3-x^2+12x+14) : (x^4+2x^3-x^2+4x+12) = 2$$

$$- (2x^4+4x^3-2x^2+8x+24)$$

$$x^3+x^2+4x-10$$

$$x^3+2x^3-x^2+4x+12$$

$$x^3+x^2+4x-10 = (x+2)(x^2-2x+3)$$

$$K = \{ -2 \}$$

$$e) \frac{x^3+x^2+4x-10}{x^4+2x^3-x^2+4x+12} = \frac{x^3+x^2+4x-10}{(x+2)(x^2-2x+3)} = \frac{A}{x+2} + \frac{B}{x^2-2x+3} + \frac{C+D}{x^2-2x+3}$$

$$\frac{x^3+x^2+4x-10}{(x+2)(x^2-2x+3)} = \frac{A(x+2)(x^2-2x+3) + B(x^2-2x+3) + (Cx+D)(x+2)}{(x+2)^2(x^2-2x+3)}$$

$$x^3+x^2+4x-10 = A(x+2)(x^2-2x+3) + B(x^2-2x+3) + (Cx+D)(x+2)$$

$$x^3+x^2+4x-10 = Ax^3-2Ax^2+3Ax+2Ax^2-4Ax+6A+Bx^2-2Bx+3B+Cx^2+4Cx+2Cx+2D$$

$$1x^3 + 0x^2 + 4x - 10 = x^3(A) + x^2(B-2A+3C) + x(-4A+2C+4D) + (6A-2B+2D)$$

$$\begin{cases} A+C=1 \\ B-2A+3C=0 \\ -4A+2C+4D=4 \\ 6A-2B+2D=-10 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & | & 1 \\ 0 & 1 & 4 & 1 & | & 1 \\ 0 & 0 & 13 & 4 & | & 4 \\ 0 & 0 & -18 & 2 & | & -10 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 & | & 1 \\ 0 & 1 & 4 & 1 & | & 1 \\ 0 & 0 & 13 & 4 & | & 4 \\ 0 & 0 & 0 & -12 & | & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 & | & 1 \\ 0 & 1 & 4 & 1 & | & 1 \\ 0 & 0 & 13 & 4 & | & 4 \\ 0 & 0 & 0 & -12 & | & -12 \end{pmatrix}$$

$$\frac{x^3+x^2+4x-10}{x^4+2x^3-x^2+4x+12} = \frac{0}{x+2} - \frac{2}{(x+2)^2} + \frac{x-1}{x^2-2x+3}$$

$$\frac{2x^4+5x^3-x^2+12x+14}{x^4+2x^3-x^2+4x+12} = 2 - \frac{2}{(x+2)^2} + \frac{x-1}{x^2-2x+3}$$

$$P_n: \frac{2x^2+10x-18}{x^2-2x^2-5x+6} = \frac{2x^2+10x-18}{(x-1)(x+2)(x-3)} = \frac{1}{x-1} - \frac{2}{x+2} + \frac{3}{x-3}$$

$$d) \frac{2x^2+10x-18}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$\frac{2x^2+10x-18}{(x-1)(x+2)(x-3)} = \frac{A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)}{(x-1)(x+2)(x-3)}$$

$$2x^2+10x-18 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$$

$$2x^2+10x-18 = Ax^2-2Ax+3Ax+2Ax^2-6Ax+6A+Bx^2-3Bx-3B+Cx^2+2Cx-2Cx+2C$$

$$x=1: 2+10-18 = A \cdot 3 \cdot (-2) + B \cdot 0 \cdot (-2) + C \cdot 0 \cdot (-1) \Rightarrow A = -3$$

$$x=-2: 8-20-18 = A \cdot 0 \cdot (-5) + B \cdot (-3) \cdot (-5) + C \cdot (-3) \cdot 0 \Rightarrow B = -2$$

$$x=3: 18+30-18 = A \cdot 5 \cdot 0 + B \cdot 2 \cdot 0 + C \cdot 2 \cdot 5 \Rightarrow C = 3$$

$$P_n: \frac{x^5-6x^4+5x^3+26x^2-65x+30}{x^5-5x^3-2x^2+12x-8} = x-3 + \frac{-2x^3+8x^2-21x+6}{x^3-5x^2-2x+8}$$

$$d) \frac{-2x^3+8x^2-21x+6}{x^3-5x^2-2x+8} = \frac{-2x^3+8x^2-21x+6}{(x-1)(x-2)(x+2)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$\frac{-2x^3+8x^2-21x+6}{(x-1)(x-2)(x+2)} = \frac{A(x-2)(x+2) + B(x-1)(x+2) + C(x-1)(x-2)}{(x-1)(x-2)(x+2)}$$

$$-2x^3+8x^2-21x+6 = A(x-2)(x+2) + B(x-1)(x+2) + C(x-1)(x-2)$$

$$x=1: -2+8-21+6 = A \cdot (-1) \cdot 3 + B \cdot 0 \cdot 3 + C \cdot 0 \cdot (-1) \Rightarrow A = 3$$

$$x=2: -16+32-42+6 = A \cdot 0 \cdot 4 + B \cdot 1 \cdot 4 + C \cdot 1 \cdot 0 \Rightarrow B = -5$$

$$x=-2: 16+32+42+6 = A \cdot (-4) \cdot 0 + B \cdot (-3) \cdot (-4) + C \cdot (-3) \cdot 0 \Rightarrow C = -2$$

$$x=0: 6 = A \cdot (-2) \cdot 2 + B \cdot (-1) \cdot 2 + C \cdot (-1) \cdot 2 \Rightarrow B = 3$$

$$12 = 4B \Rightarrow B = 3$$

$$VYSLEDOK: \frac{x^5-6x^4+5x^3+26x^2-65x+30}{x^5-5x^3-2x^2+12x-8} = x-3 - \frac{3}{x-1} + \frac{3}{x-2} - \frac{5}{x+2} - \frac{2}{x+2}$$

$$P_n: \frac{4x^5+14x^4+41x^3+42x^2+20x-6}{x^4+2x^3+6x^2+10x+5} = 4x + \frac{-x^2+2x^2-15x-6}{x^4+2x^3+6x^2+10x+5}$$

$$d) \frac{-x^2+2x^2-15x-6}{x^4+2x^3+6x^2+10x+5} = \frac{-x^2+2x^2-15x-6}{(x+1)(x^2+5)} = \frac{A}{x+1} + \frac{B}{x^2+5} + \frac{C+D}{x^2+5}$$

$$\frac{-x^2+2x^2-15x-6}{(x+1)(x^2+5)} = \frac{A(x+1)(x^2+5) + B(x^2+5) + (Cx+D)(x+1)}{(x+1)^2(x^2+5)}$$

$$-x^2+2x^2-15x-6 = A(x+1)(x^2+5) + B(x^2+5) + (Cx+D)(x+1)$$

$$-x^2+2x^2-15x-6 = Ax^3+Ax^2+5Ax+5A+Bx^2+5B+Cx^2+Dx+Dx+D$$

$$\begin{cases} A+C=1 \\ A+B+2C+D=2 \\ 5A+2C+D=-15 \\ 5A+5B+D=-6 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & | & -1 \\ 0 & 1 & 1 & 1 & | & 2 \\ 0 & 0 & 4 & 2 & | & -10 \\ 0 & 0 & 0 & 16 & | & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 & | & -1 \\ 0 & 1 & 1 & 1 & | & 2 \\ 0 & 0 & 4 & 2 & | & -10 \\ 0 & 0 & 0 & 16 & | & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 & | & -1 \\ 0 & 1 & 1 & 1 & | & 2 \\ 0 & 0 & 4 & 2 & | & -10 \\ 0 & 0 & 0 & 16 & | & -6 \end{pmatrix}$$

$$VYSLEDOK: \frac{4x^5+14x^4+41x^3+42x^2+20x-6}{x^4+2x^3+6x^2+10x+5} = 4x - \frac{3}{x+1} + \frac{2}{x+1} + \frac{2x-1}{x^2+5}$$